

# About One Task of Optimal Location of Service Centers of Municipal Transport by Fuzzy Graphs

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**Abstract**— This electronic document is a "live" template. The various components of your paper [title, text, heads, etc.] are already defined on the style sheet, as illustrated by the portions given in this document. In the paper the problem of location, implying putting of planning of co-operating centers in different parts of the city into intelligent management of urban transport is considered. The adequate model of the choice of location area for service centers using fuzzy graphs is constructed. Optimality criteria are taken into account.

**Keywords**— technical service centers; vertices; degree of vitality; optimal location

From Japan International Cooperation Agency (JICA), Cabinet of Ministers and Executive Power of Baku city of Azerbaijan Republic it was held the joint project, which main purpose was the problem of urban transport in the city of Baku. From October 2000 to March 2002 the project team conducted researches to develop the plan for improving city transport for the period until 2020, as well as to conduct the technical and economic feasibility on priority projects, technology development and implementation of the study.

According to statistics, in 1999 the number of passengers carried on only by city transport (excluding railways and private buses) accounted for about half million passengers a day. The average number of trips per person per day in 2020 will amount to 2.71-th of visits compared to 2.04-th in 2000. [1] The results also showed high road traffic because of high traffic intensity compared to road capacity. It was also found that in Baku city arterial roads (avenues) form a system of radial and ring roads, and roads in the center of city are arranged in a lattice. With the exception of the central roads their state is poor, and continuous operation of vehicles leads to rapid damage.

This problem requires entering the planning of co-acting centers in different parts of the city to intelligent management of urban transport, which is a problem of location.

There are many tasks to deploy the service centers [2,3,7]. However, for these classes of problems, information can be either approximate, or not credible [4]. Another problem is that the assessment of quality of service centers of any part of the territory may be held by several, including conflicting criteria [5]. Therefore, in our opinion an adequate model of the task of area location's choice of service centers is the use of fuzzy graphs [6], since precise graphs are powerless in the case of approximation and lack of initial information. When placing

technical service centers in the fuzzy graph, it should solve the problem of finding the optimal location in one sense or another. We assume that the service centers are located only on the vertices of oriented fuzzy graph. Optimality criteria will be refined further. To address this problem, we introduce the following notion.

**Definition 1.** Let  $\tilde{G} = (X, \tilde{U})$  if the fuzzy directed graph [4], with the number of vertices  $|X| = n$ . Suppose we have  $k$  service center ( $k < n$ ), placed at the vertices of the subset  $Y, |Y| = K, Y \subset X$ . Let  $\tau(x_i, x_j)$  - the degree of reachable of vertex  $x_i$  by vertex  $x_j$ .

Value

$$V_{\tilde{G}}(Y) = \bigwedge_{\forall y \in X/Y} \left( \bigvee_{\forall x_i \in Y} \tau(x_i, x_j) \bigwedge \tau(x_j, x_i) \right) \quad (1)$$

is called the degree of persistence of fuzzy graph  $\tilde{G}$  [4] during its maintenance by  $k$  centers from set  $Y$  of vertices.

The value  $V_{\tilde{G}}(Y)$  determines the minimum value of the strong connection between each vertex of set  $X / Y$  and one of the centers located in  $Y$ .

In terms of optimal location of service centers of maintenance, we can "get out" of a vertex of subset  $Y$ , reach any vertex of the graph, "serve it", "return back" to the starting vertex and thus the conjunctive strength of the path will not be less than the quantity  $V_{\tilde{G}}(Y)$ .

We can easily prove the following property quantity  $V_{\tilde{G}}(Y)$ .

Theorem 1.  $V_{\tilde{G}}(Y)$  depends on number of  $k$  centers and location of centers on the vertices of graph  $\tilde{G}$  (i.e. on choice of set  $Y$ ) and  $0 \leq V_{\tilde{G}}(Y) \leq 1$ .

Abovementioned task of location of maintenance centers ( $k < n$ ) for a given fuzzy graph  $\tilde{G}$  is reduced to finding such value of the degree of vitality  $V_{\tilde{G}}(Y)$  that attains its maximum value, i.e. values

$$V_{\tilde{G}}(K) = \max_{\substack{Y \subseteq X \\ |Y|=k}} \{V_{\tilde{G}}(Y)\} \quad (2)$$

Definition 2. The fuzzy set

$$V_{\tilde{G}} = \left\{ \langle V_{\tilde{G}}(1)/1 \rangle, \langle V_{\tilde{G}}(2)/2 \rangle, \dots, \langle V_{\tilde{G}}(n)/n \rangle \right\},$$

defined on the set of n vertices, is called the fuzzy set of vitality of fuzzy graph.

Fuzzy set of vitality  $\tilde{V}_{\tilde{G}}$  is defined by most degree of vitality of graph  $\tilde{G}$  for its maintenance by 1,2, ..., n centers.

To find the optimal location of service centers at which the degree of vitality of fuzzy graph reaches its maximum value, consider the following well-known theorem, which is based on the method of solving this problem.

Theorem 2. In order to value  $V_o \in [0,1]$  was the highest degree of vitality of fuzzy graph  $\tilde{G}$  in the presence of k centers and at the same time  $V_{\tilde{G}}(k-1) < V_o$ , necessary and sufficient existing of a unique partition K of set of vertices X in  $\ell$  classes so that fuzzy subgraphs  $\tilde{G}_1 = (X_1, \tilde{U}_1), \tilde{G}_2 = (X_2, \tilde{U}_2), \dots, \tilde{G}_\ell = (X_\ell, \tilde{U}_\ell)$ , defined by these classes are maximal subgraphs with degrees of internal and external vitality respectively

$$V_{\text{вн}}(\tilde{G}_1), V_{\text{вн}}(\tilde{G}_2), \dots, V_{\text{вн}}(\tilde{G}_\ell), V_{\text{вн}}(\tilde{G}_1), V_{\text{вн}}(\tilde{G}_2), \dots, V_{\text{вн}}(\tilde{G}_\ell),$$

which would satisfy the state:

$$\max_{j \in 1, \ell} (V_{\text{вн}}(\tilde{G}_j)) < \min_{i \in 1, \ell} (V_{\text{вн}}(\tilde{G}_i)) \quad (3)$$

and

$$\max_{j \in 1, \ell} (V_{\text{вн}}(\tilde{G}_j)) = V_o \quad (4)$$

Based on this theorem, we consider an approach to the location of service centers of buses and taxis in Baku. In order to “optimally” locate k centers in the fuzzy graph with n vertices (k<n) with highest degree of vitality, we have to make partitioning of the graph into  $\ell$  maximal, disjoint subgraphs which satisfy the state (3).

Next, place each center of service to any of vertices of the selected subgraphs. The degree of vitality of all subgraph will not be possible to determine by minimum degree of the subgraphs, and this means that we can “serve” the graph, without reducing the degree of vitality and smaller (compared to  $\ell$ ) number of centers.

Consider the method of finding all the minimal sets of maintenance centers, with the greatest degree of vitality. This method is a generalization of Mago’s method for precise graphs [5,6,7] and is similar to the method of finding of minimal and externally stable sets for fuzzy graphs [6].

Let Y be a subset of vertices of fuzzy graph  $\tilde{G} = (X, \tilde{U})$ , in which there are centers and the degree of vitality is  $V$  [4,5,6].

Theorem 3. For every vertex  $x_i \in X$  is done either one of the following two states, or both simultaneously:

a) the vertex  $x_i$  belongs to the considerate set

b) there is a vertex  $x_j$ , which belongs to Y, and at the same there is inequality

$$\tau(x_i, x_j) \geq V \quad \text{è} \quad \tau(x_j, x_i) \geq V$$

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