# Control Problem of Elastic Rod Oscillations

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***Abstract*— It is showed in this work for control problem of elastic rod oscillations with the aid of boundary governing functions that initial disturbances can be defaced in a final time.**

***Keywords*— *oscillation equation; elastic rod; boundary governing functions; transient time***

## I. INTRODUCTION

It is known that the control problems of oscillating processes have a quantity of applications. Among these control problems the control problem of of elastic rod oscillations have a special place. It is to be noted, that we come to the equation of lateral oscillations in a number of tasks on elastic rod oscillations – in the task on eigen oscillation of tuning fork, in calculation of stability of rotating shafts and in studies on ships vibrations [6].

## II. PROBLEM STATEMENT

Let the controlled process in the rectangle

$$Q = \{0 < x < l, 0 < t < T\}$$

is described by the following boundary value problem:

$$a^2 \frac{\partial^2 u}{\partial t^2} + \frac{\partial^4 u}{\partial x^4} = 0, \quad (x,t) \in Q, \tag{1}$$

$$u(x,0) = \varphi(x), \quad \frac{\partial u(x,0)}{\partial t} = \psi(x), \quad 0 < x < l, \tag{2}$$

$$u(0,t) = \mu_0(t), \quad \frac{\partial^2 u(0,t)}{\partial x^2} = \mu_2(t), \quad 0 < t < T, \tag{3}$$

$$u(l,t) = \nu_0(t), \quad \frac{\partial^2 u(l,t)}{\partial x^2} = \nu_2(t), \quad 0 < t < T. \tag{4}$$

It is assumed that these tasks (1)-(4) satisfy the conditions:

$$\varphi \in W_2^2[0,1], \psi \in W_2^1[0,1], \mu_0 \in W_2^2[0,T],$$

$$\nu_0 \in W_2^2[0,T], \mu_2 \in L_2[0,T], \nu_2 \in L_2[0,T].$$

But control $(\mu_0(t), \nu_0(t), \mu_2(t), \nu_2(t))$ is belonged to set

$$U = \{(\mu_0(t), \nu_0(t), \mu_2(t), \nu_2(t)) \in W_2^2[0,T] \times W_2^2[0,T] \times L_2[0,T] \times L_2[0,T] | \mu_0(0) = \varphi(0), \nu_0(0) = \varphi(l), \mu_0'(0) = \psi(0) = 0, \nu_0'(0) = \psi(l) = 0\}.$$

The following control problem is considered: it is required to define the instant of time $T > 0$ and such appropriate governing functions $\mu_k(t)$ and $\nu_k(t)$, $k = 0,2$, the definable decision $u(x,t)$ of which will satisfy the boundary value problem (1)-(4)

$$u(x,t) \equiv 0, \quad \frac{\partial u(x,T)}{\partial t} \equiv 0, \quad 0 \prec x \prec l.$$

## III. CONCLUSION

This wok shows, that in the presented control problem of elastic rod oscillations with the aid of boundary governing functions, initial disturbances can be defaced in a final time

$$T = al^2 / \pi,$$

assuming

$$\nu_0(t) \equiv 0, \mu_2(t) \equiv 0, \mu_0(t) = \mu_0^0(t), \nu_2(t) = \nu_2^0(t),$$

where $\mu_0^0(t)$ and $\nu_2^0(t)$ are defined by the certain formulas.

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