Optimization of the Transients in Oil Pipelines on a Class of Piecewise Constant Functions

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***Abstract*— Problems of optimal control by the stabilization of transients while transporting oil over trunk pipelines are investigated in the work. The mathematical formulation has the form of a parametric problem of optimal control of a distributed-parameter system where optimized is the transient time, the control actions are represented by the liquid flows at the ends of a linear pipeline section. Control actions are considered on technically easily realizable classes of functions such as piecewise-constant functions.**

***Keywords*— oil pipeline, unsteady motion, optimal time of transient, control actions, range of admissible controls.**

I. INTRODUCTION

The present work considers the problem of optimal control of the transients arising at transportation of the oil along the trunk pipelines at passing from one steady mode to another. These processes arise both at the planned modification of the pumping modes and at the emergency pipeline stoppage. The mathematical formulation has the form of a parametric problem of optimal control of a distributed-parameter system where optimized is the transient time, the control actions are represented by the liquid flows at the ends of a linear pipeline section. The control actions are considered on technically easily realizable classes of functions such as piecewise constant functions. The results of numerical experiments reflecting dependence of the minimal transient-process time from the quantity of the interval of admissible controls are given.

II. PROBLEM STATEMENT

Let us consider the isothermal process of transportation of the single-phase oil with kinematic viscosity v along the linear part of a horizontal pipeline of length l, diameter d, and the coefficient of hydraulic resistance λ for the laminar mode of liquid flow. At both ends of the pipeline segment there are pumping stations supporting the desired mode of pumping.

For the case of subsonic speeds the unsteady motion of liquid in the pipeline is rather adequately described for practical purposes by the following Charniy-linearized system of differential equations [1]:

$$\begin{aligned} -\frac{\partial p}{\partial x} &= \rho\left(\frac{\partial \omega}{\partial t} + 2a\omega\right), \\ -\frac{\partial p}{\partial t} &= c^2 \rho \frac{\partial \omega}{\partial x}, \end{aligned} \qquad x \in (0,l), \quad t > 0 \tag{1}$$

where $p = p(x,t)$, $\omega = \omega(x,t)$ are the pressure and fluid flow velocity at the pipeline point $x \in (0,l)$ at the time $t > 0$, c is the speed of sound in the medium, ρ is the liquid density which may be regarded as constant for the dropping liquid, and $2a = \lambda\omega/2d$ is the linearized friction coefficient.

We assume that in the pipeline until the time instant $t = 0$ there was a steady mode defined by the initial conditions

$$\omega(x,t) = \omega_0 = const, \quad x \in [0,l], \quad t \le 0 \tag{2}$$

$$p(x,t) = p_0(x), \quad x \in [0,l], \quad t \le 0 \tag{3}$$

Conditions (2) and (3) are satisfied by the pump stations maintaining the modes

$$\omega(0,t) = \omega(l,t) = \omega_0, \quad t \le 0, \tag{4}$$

that are for $t \le 0$ the boundary conditions for system (1).

It deserves noting that in practice it is impossible to observe precisely the stabilization conditions (2) and (3) because in the pipeline there are always minor perturbations caused by the irregularities of equipment operation and giving rise to relative small deviations of the stabilization conditions (2) and (3):

$$\begin{aligned} &\left|\omega(x,t) - \omega_0\right| \le \delta_\omega, \\ &\left|p(x,t) - p_0(x)\right| \le \delta_p, \quad x \in (0,l), \quad t \le 0, \end{aligned} \tag{5}$$

where δ_ω, δ_p are the given positive small values defined by fractions or per cent, respectively, of ω_0 and $p_0(x)$ of some steady mode.

In this connection, by the δ-steady mode is meant the mode of raw stock transportation through the pipeline such that conditions (5) are satisfied for it.

The problem of optimal control of transients is as follows: it is desired to control the modes of the pumping stations at the pipeline ends so as to drive in the minimal possible time T the mode (2), (3) to a new preassigned mode

$$\omega(x,t) = \omega_T = const, \quad t \ge T, x \in [0,l], \tag{6}$$

$$p(x,t) = p_T(x), \quad t \ge T, \quad x \in [0,l], \tag{7}$$

where T is the time after which the new steady mode (6), (7) begins.

The pumping station modes are controlled by varying the volume flows at the ends of the linear pipeline segment which amounts to varying the liquid velocity, that is, the boundary conditions

$$\omega(0,t) = u_1(t), \quad \omega(l,t) = u_2(t), \quad t \in [0,T], \tag{8}$$

provided that some technological and technical constraints (with allowance for the pump wear characteristics) are satisfied:

$$\underline{u}_1 \le u_1(t) \le \overline{u}_1, \quad \underline{u}_2 \le u_2(t) \le \overline{u}_2, \quad t \in [0,T], \tag{9}$$

where $u_1(t), u_2(t)$ are the control actions belonging to a certain class of functions.

From the conditions for pipeline strength, the following technological constraints on the value of maximal pressure at transportation through the pipeline must be observed over the entire time interval of control of transients:

$$\underline{p} \le p(x,t) \le \overline{p}, \qquad x \in (0,l), \qquad t \in [0,T], \tag{10}$$

where $\overline{p}$ is the assigned maximal permissible pressure depending on the characteristics of the pipeline material and $\underline{p}$ is the pressure below which the undesirable oil cavitation (boiling) occurs.

Constraints (10) may be transformed into those on the maximum permissible values of the linear velocity ω:

$$\underline{\omega} \le \omega(x,t) \le \overline{\omega}, \qquad x \in (0,1), \qquad t \in [0,T]. \tag{11}$$

Taking into consideration the impossibility of precise determination of mode (6), (7), in addition to the transient time T the minimized objective functional includes a term following the mean-square deviation of the values taken on by the functions of velocity and pressure from the desired values during a certain time interval after the instant of δ-stabilization of the process:

$$J(u,T)=T+\int_T^{T+DT}\int_0^l\left\{r_1\left[p(x,t)-p_T(x)\right]^2+r_2\left[\omega(x,t)-\omega_T\right]^2\right\}dxdt \to \min \tag{12}$$

where DT is the preassigned duration of the time interval over which the process and occurrence of the δ-stabilized mode are observed.

It is obvious, that at the decision of a considered problem in the class of sectionally continuous functions $u(t)=(u_1(t),u_2(t))$ there are difficulties on practical realization of the received control actions, connected with continuous change of operating modes of pumps in time. On the other hand, frequent switching of pumps causes their damage and premature deterioration. In this connection the present work considers the problem of boundary control of process (1) in the readily realizable classes of functions such as piecewise-constant time functions like

$$\begin{gathered} u_i(t) = \mathrm{v}_{ij} = const,\ t \in [\tau_{j-1}, \tau_j),\ i = 1,2,\ j = \overline{1,L}, \\ \tau_0 = 0, \quad \tau_L = T + DT, \quad \tau_j = \tau_{j-1} + \Delta\tau_j,\ j = \overline{1, L-1}. \end{gathered} \tag{13}$$

In this case, the optimal control problem consists in determining finite-dimensional vector $\mathrm{v} \in R^{2L}$

$\mathrm{v} = (\mathrm{v}_{11},...,\mathrm{v}_{1L},\mathrm{v}_{21},...,\mathrm{v}_{2L})$,

under the constraints:

$\underline{u}_i \le \mathrm{v}_{ij} \le \overline{u}_i, \quad i=1,2,\ j=\overline{1,L}.$

Such case, when each pump station has its own number and/or duration of intervals of a constancy, is possible also, i.e.:

$$\begin{gathered} u_i(t) = \mathrm{v}_{ij} = const,\ t \in [\tau_{ij-1}, \tau_{ij}), \\ \tau_{i0} = 0,\ \tau_{iL} = T + DT,\ i = 1,2,\ j = 1,...,L_i \end{gathered}. \tag{14}$$

As for the time instants τ_{ij} of control switching and, correspondingly, the intervals $\Delta\tau_{ij}$ of constant controls, they may be determined in different ways. For a given number L of switchings of the piecewise-constant control actions, cases are possible of defining the switching times τ_{ij}, for example, by the condition of uniform intervals: $\tau_{ij} = j*(T/L)$, that is, $\Delta\tau_{ij} = \Delta\tau = const,\ i=1,2,\ j=\overline{1,L-1}$.

When the instants of control switchings τ_{ij} themselves are also optimized, two cases are possible.

In the first, when at both ends of the pipeline segment the pumping station modes are changed simultaneously in an optimal (for both stations) time. In this case, we optimize the $3L-1$-dimensional vector:

$$q = (\mathrm{v},\tau) = (\mathrm{v}_{11},...,\mathrm{v}_{1L},\mathrm{v}_{21},...,\mathrm{v}_{2L},\tau_1,...,\tau_{L-1}) \tag{15}$$

satisfying the constraints which follows from the restrictions on the control actions (9)

$0 \le \tau_j \le T + DT, \quad \tau_j \le \tau_{j+1},\ j=1,...,L-1,$

$\underline{u}_i \le \mathrm{v}_{ij} \le \overline{u}_i, \qquad i=1,2, \quad j=1,...,L.$

In the second case the values v_{ij} of piecewise-constant controls on the intervals of constancy $[\tau_{ij-1},\tau_{ij})$, as well as the instants of control switchings $\tau_{ij},\ i=1,2,\ j=\overline{1,L}$ are optimized.

In this case, we optimize the finite-dimensional vector $q=(\mathrm{v},\tau) \in R^{4L-2}$ of the following form

$$q = (\mathrm{v}_{11},...,\mathrm{v}_{1L},\mathrm{v}_{21},...,\mathrm{v}_{2L},\tau_{11},...,\tau_{1L-1},\tau_{21},...,\tau_{2L-1}) \tag{16}$$

under the constraints:

$0 \le \tau_{ij} \le T + DT, \quad \tau_{ij} \le \tau_{ij+1},\ j=1,...,L-1,$

$\underline{u}_i \le \mathrm{v}_{ij} \le \overline{u}_i, \qquad i=1,2, \quad j=1,...,L.$

The problem of optimizing the number L of switchings, which in this paper assumes a preset, is also of practical interest.

III. Formulas for Numerical Solution to the Problem

We present the formulas obtained for the components of the gradient of the functional on the optimizing pumping station modes ($u(t)$, (v,τ)), as well as on the parameter T time of the process completion.

Using the method of variation of the optimized functions [3] and the parameter T, we can obtain the following formulas:

$$grad_{u_1} J(u,T) = c^2\psi_2(0,t),\ 0 \le t \le T+DT, \tag{17}$$

$$grad_{u_2} J(u,T) = c^2\psi_2(l,t),\ 0 \le t \le T+DT, \tag{18}$$

$$\begin{aligned} grad_T J(u,T) = 1 &+ \int_0^l r_1(\omega(x,T+DT)+\omega(x,T)-2\omega_T)(\omega(x,T+DT)-\omega(x,T))dx + \\ &+ \int_0^l r_2(p(x,T+DT)+p(x,T)-2p_T(x))(p(x,T+DT)-p(x,T))dx + \\ &+ \int_0^l \{R_1[\max(0,\omega(x,T)-\overline{\omega})]^2 + R_2[\max(0,-\omega(x,T)+\underline{\omega})]^2\}dx \end{aligned} \tag{19}$$

Here $\psi_1(x,t), \psi_2(x,t)$ are the solutions to the following conjugate boundary problem:

$$\frac{\partial\psi_1}{\partial t}=\begin{cases}-c^2\frac{\partial\psi_2}{\partial x}+2a\psi_1-2r_1(\omega(x,t)-\omega_T), & T\le t\le T+DT,\\ -c^2\frac{\partial\psi_2}{\partial x}+2a\psi_1-2R_1\left[\max(0,\omega(x,t)-\overline{\omega})\right]+ & \\ +2R_2\left[\max(0,-\omega(x,t)+\underline{\omega})\right], & 0\le t<T,\end{cases}$$

$$\frac{\partial\psi_2}{\partial t}=\begin{cases}-\frac{\partial\psi_1}{\partial x}-2r_2(p(x,t)-p_T(x)), & T\le t\le T+DT,\\ -\frac{\partial\psi_1}{\partial x}, & 0\le t<T,\end{cases} \tag{20}$$

$$\psi_1(0,t)=\psi_1(l,t)=0,\quad 0\le t<T+DT, \tag{21}$$

$$\psi_1(x,T+DT)=0,\quad \psi_2(x,T+DT)=0,\ 0\le x\le l \tag{22}$$

On the class of piecewise-constant control actions like (14), the gradient of the functional with respect to the values of the control parameters v_{ij} obeys the formulas:

$$\frac{dJ}{dv_{1j}}=c^2\int_{\tau_{1j-1}}^{\tau_{1j}}\psi_2(0,t)dt,\quad \frac{dJ}{dv_{2j}}=c^2\int_{\tau_{2j-1}}^{\tau_{2j}}\psi_2(l,t)dt,\ j=\overline{1,L}.$$

In the case of optimization of the constancy intervals of control actions themselves, that is, the control switching instants τ_{ij}, $i=1,2$, $j=\overline{1,L-1}$, the gradient of the functional with respect to the switching instants obeys the following formulas:

$$\frac{dJ}{d\tau_{1j}}=c^2\ \psi_2(0,\tau_{1j})(v_{1j}-v_{1j+1}),$$

$$\frac{dJ}{d\tau_{2j}}=c^2\ \psi_2(l,\tau_{2j})(v_{2j}-v_{2j+1}),\ j=\overline{1,L-1}.$$

To solve the problem of optimal control (1)–(3), (8)–(12), the above formulas enable one to use in the space of the optimized parameters the efficient numerical methods of first-order optimization such as gradient projection, conjugate gradient, penalty functions, and their combinations [3]. The method of external penalty was used to allow for the constraints on the phase state (11) [3], and that of projection of the conjugate gradient, to allow for the control constraints (9). The direct (1)–(3) and conjugate (20)-(22) boundary problems were solved numerically with application of a method of finite differences by means of approximation on a three-layer nine-point symmetrical pattern [5].

IV. THE RESULTS OF NUMERICAL EXPERIMENTS

Further we give some results from numerous computational experiments that were carried out to detect the regularities of the dependence of the minimal transient time on the interval of permissible controls for various values of the initial and final steady modes. The problem of control of the transients was studied under the constraints on the control actions and the state functions (9), (11).

In all numerical experiments on control of the transients, control was applied to the left end (beginning) of the segment, the boundary condition corresponding to the desired steady mode being set up at the opposite end.

TABLE 1
PARAMETERS OF THE CONSIDERED PROBLEMS OF TRANSIENT CONTROL

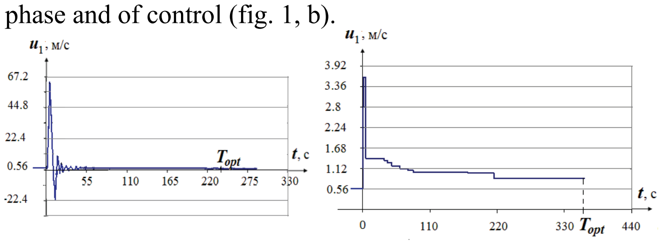
№	In dimensional units							
	$2a$ $1/s$	q_0 m^3/h	ω_0 m/s	q_T m^3/h	ω_T m/s	$p_0(0)$ **atm**	$p_T(0)$ **atm**	l **km**
I	0.0192	400	0.56	600	0.84	30	43	132
II	0.0192	400	0.56	600	0.84	30	43	264
III	0.0192	400	0.56	600	0.84	23	33	132
IV	0.0288	400	0.56	600	0.84	23	33	132
V	0.0192	400	0.56	600	0.84	18	24	132
VI	0.0192	300	0.42	600	0.84	14	24	132
VII	0.0192	200	0.28	800	1.12	10	29	132
VIII	0.0192	600	0.84	400	0.56	24	18	132
IX	0.0192	600	0.84	300	0.42	24	14	132

Table 1 compiles the values of the parameters of the initial and final steady modes of oil motion for the following problems of transient control. Here, q_0 and q_T are, respectively, the values of flow for the initial and final steady modes; $p_0(0)$ and $p_T(0)$ are the values at the left end of the segment, respectively, at the initial and final steady modes. Taking into consideration that the liquid flow and velocity are obviously related by $q(x,t)=\omega(x,t)\cdot S$, $S=\pi d^2/4$, we go from the flow values q_0 and q_T to the velocities ω_0 and ω_T, which will be used as the parameters of the initial and final modes (Table 1).

Numerical study of the transients under constrained states and control actions demonstrated that the time of stabilization of the optimal transient process depends on the interval of permissible values of controls $[\underline{u},\overline{u}]$ for the preassigned values of the initial ω_0 and final ω_T steady modes, where $\underline{u}=\omega_0-\Delta\underline{u}$, $\overline{u}=\omega_T+\Delta\overline{u}$, and $\Delta\underline{u}$, $\Delta\overline{u}$ are called, respectively, the lower and upper tolerances on the control actions.

By the example of the problem V, move on to the investigation of transient processes under the assumption that the control is given on a class of piecewise constant functions; at that we optimize not only the values of the controls v_j, $j=\overline{1,L}$, but the intervals of constancy $\Delta\tau_j$, $j=\overline{1,L-1}$, or the switchings moments of the control τ_j as well (see table 4). Suppose that L (the number of intervals of constancy of the control) is given (particularly, $L=10$ in all the computations presented). In case if the intervals of constancy $\Delta\tau_j=\tau_{j+1}-\tau_j$, or the difference between the adjacent values of the controls are small quantities, i.e. $\Delta\tau_j<\Delta\tau_{\min}$, or $\left|v_j-v_{j+1}\right|<\varepsilon$, where $\Delta\tau_{\min}$, ε are sufficiently small (particularly, in this problem $\Delta\tau_{\min}$=0.5 s and ε=0.005 m/s), then we can "stick together" some intervals of constancy of the control function and, therefore, reduce the total number of these intervals (i.e. reduce L).

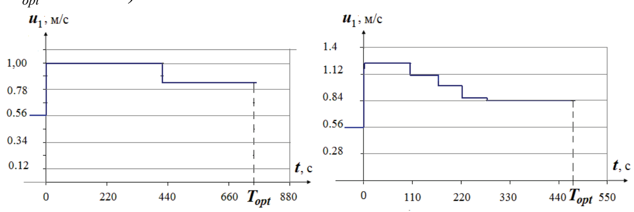
First, we give the optimal control obtained when investigating the problem V on a class of piecewise constant control functions without any constraints on the functions of phase and of control (fig. 1, b).



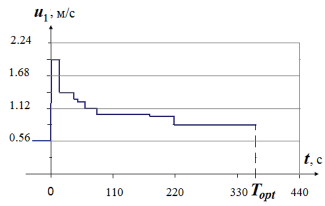
a) b)

Fig. 1. Graphs of transient process control for unconstrained problem V for (a) sectionally continuous and (b) piecewise-constant controls

As it is evident from comparison with the respective graph obtained for piecewise continuous control under the same conditions (figure 1, a), here we do not observe such strong oscillation of the function, although the transient period slightly increases (on fig. 1, a, $T_{opt} \approx 220$ s; on figure 1, b, $T_{opt} \approx 374$ s).



a) b)



c)

Fig. 2. Graphs of the piecewise-constant optimal control for problem V for (a) $\overline{u_1} = 1$ m/s, (b) $\overline{u_1} = 1.2$ m/s, (c) $\overline{u_1} = 2.5$ m/s.

As it is evident from the graphs (fig. 2, a, b) and table 2, under the values $\overline{u} < 1.5$ m/s there is significant reduction of the number of intervals of constancy mainly due to the switchings moments of the controls being optimized (when $\overline{u} = 1$ m/s the control takes place at two intervals, when $\overline{u} = 1.2$ m/s – at five intervals of constancy).

When $\overline{u} \geq 1.5$ m/s (fig. 2, c) the picture significantly changes: the number of the intervals increases, the transient period becomes equal to the minimal ($T_{opt} \approx 374$ s) and does not change anymore with the increase of the range of upper permissible level of u and in this case the transient period coincides with the transient period for the problem without constraints on the functions of phase and of control (fig. 1, b).

TABLE 2

DEPENDENCE OF THE TRANSIENT TIME FROM $\overline{u}_1$ IN THE PROBLEM V WITH $\underline{u}_1$ =0.28 m/s FOR PIECEWISE-CONSTANT CONTROLS

$\overline{u_1}$ (m/s)	τ_{L-1} (s)	T_{opt} (s)	L
0.95	612.7	902	2
1.	417	726	3
1.12	319	616	3
1.18	297	484	4
1.23	278.3	473	5
1.34	252	462	7
1.4	243	396	8
1.5	220	374	9(10)

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