Problem of Corporate Service in Communication Systems in Non-Stationary Conditions

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***Abstract*— The report analyzes the traditional problems of mass servicing (MS), identified a number of shortcomings that limit the practical aspects of such problems. The generalized problem of corporate MS consist of identification of gaps in the first place, the nature of non-stationary and an algorithm for its solution is suggested.**

***Keywords*— mass servicing; communications; equipment of service; application rate; buffer; corporate network; capacity; optimization**

I. INTRODUCTION

With the development of communications and computer technology (CCT), the interest of researchers in mass servicing theory (MS) is significantly increased. This is due, on the one hand, the need for the formulation and solution of many problems MS in the design and the operation of communication networks increase (CN), and the other - with the identification of new properties and characteristics of the tasks of the MS and the development of science and technology, creating the preconditions for improvement and development of MS theory. Quite a lot of work devoted to both theoretical and applied aspects of MS in different directions were done and obtained are important results in these directions.

Problems similar to the MS occur mainly at the nodes of the CN when performing encoding and decoding, switching, transmission of information via communication channels, etc. MS theory is widely used in other areas, particularly in the industrial, agricultural, military, economic, environmental and natural areas, in the field of public service. The main elements of any MS system are:

1) application (surveys) to service;
2) buffers for queuing and waiting for orders;
3) instruments (channels) to service requests;
4) The mathematical apparatus for studying, analyzing and solving problems of the MS and discipline of service requests. Basic framework of such an apparatus for the traditional tasks of Defense is the theory of probability and statistical methods. For new tasks, it is also possible to use the technical and economic criteria, optimization methods, choices and decisions.

Traditional methods, models and algorithms for MS theory classes are limited to the tasks defined by the system introduced by J. Kendall, in the fifties of the twentieth century and is in the form of four characters: $A(t)|B|n|m$ [1], where A is characterizes the distribution of the interval time of receipt of applications;B is service time application; n isnumber of identical parallel units of s to wait for applications in the buffer and $m\geq0$. At $m=\infty$ possible service requests and $n\geq1$; m is number of number of seat omit the symbol/

Of these systems MS (SMS) introduced by J. Kendall most studied are $M|M|1$; $M|M|\infty$; $M|M||1|m, m>1$; $M|M||n|m$, $n>1$, $m>0$.

Each element of the SMS has a number of indicators, properties, and rules governing their operation in certain situations.

The main assumptions in the SMS are considered stationary, orderly and without the effects of flow applications; the subordination of the distribution of time between applications and the service time of individual applications to the exponential law, the homogeneity of channels, uniformity of service and specificity of the average service time for all applications.

The task of the SMS is to identify balanced average characteristics of the main indicators of its elements.

The author conducted an analysis of tasks of the MS for a number of specific sites. Result of analysis show that many of the assumptions adopted in the SMS in real situations are not satisfied. In particular, the flow of applications are often non-stationary, there are situations when the channels are heterogeneous, the average service time of individual applications is not typical for many, necessary to determine the probability characteristics of indicators with a certain reliability and possibly with unit probability, there is a desire to consideration of a number of technical and economic criteria, limitations, there is the possibility to use resources from other SMS and imposed by J. Kendall classes of systems are not considered varieties of applications and features simultaneous service channels (devices) as a single phase and multiphase applications. Under the multiphase application refers to an application that requires the implementation of at least two types of service.

II. STATEMENT OF THE PROBLEM

The type and nature of the service model of multiphase applications is largely dependent on the structure of channels and the order of service. Here, a more specific you can specify the following typical situation.

1. Multiphase homogeneous application, i.e. the number and nature of their phases aren't different and each

channel is able to handle such applications, i.e. channels are uniform.

2. Multiphase heterogeneous applications, i.e. number and (or) the nature of the phases are different and the channels are grouped to serve certain types of multiphase applications. It is assumed that both the first and second phases of the case of applications in each channel is maintained consistently in a certain order. In this case it is understood that the application, which began servicing the channel does not leave until all of its phases.
3. Multiphase heterogeneous applications and channels for the classified service of the individual phases. That is, each application is received in the service of a certain phase of the channel leaves him and becomes part of another channel to serve the next phase of the known and established order. Consequently, each multiphase application is served successively in different channels. This implies that a consistent service between the channels are placed buffers and probabilistic characteristics of applications and channels, in general, differ.

Consideration of such properties in the analysis and design of the SMS requires the formulation of new problems of the MS non-traditional. Characters such tasks defined the possible combinations of violations of assumptions made in the theory of traditional SMS. In [2] in the light of these observations are defined typical situation more adequately reflect the reality and the appropriate problem of the MS.

The report focuses on the optimal MS for some non-traditional situations, namely in situations where:

1) The flow of orders is not Poisson, i.e. non-stationary and possibly with delay. Moreover, unsteady flow applications is reflected in changes in their intensity and uniformity. The latter property gives rise to different flow time and cost of service;
2) Services are heterogeneous devices - have a different time, cost and quality of service, both single phase and multiphase applications.
3) Given that the SMS are located in different nodes are elements of a unified system, it is interesting to corporate use.
4) The existence of a real-life situations multiphase applications require their inclusion in the SMS. And this in turn requires the expansion of the definitions by J.Kendall with additional elements, taking into account the variety of applications, including multi-phase.

With these properties it is possible to vary the intensity of channels in the MS and the distribution of applications, total vacancies in the buffers for a certain period of time T between the SMS and the various nodes of the CN. The distribution is generally carried out sequentially with differentiation both in time and over the set of nodes of the CN. This sequential differentiation contributes to the initial one-level representation of the CN in the form of a multilevel hierarchical structure. And this in turn contributes to simplification of the MS in the CN.

The task for each MS level, perhaps with the exception of the first, in view of these observations is how the dynamic type of distribution. Each task in the general case consists of the integral and local criteria and constraints, difference equations of state change buffers, the intensities of the different types of service requests, etc. As a criterion might be the cost of servicing orders, revenue, profits, number served or not served requests, etc. The solution of the optimal MS at each level ℓ, $\ell=\overline{2,L}$ requires a prediction of the intensities of the different types of applications received for the corresponding period T ℓ. This prediction is usually carried out from the bottom up starting with the first level with consistent aggregation of information. But solution of optimal MS usually carried out top down, starting from the top L level. From the solution of the optimal MS for each ℓ -level $\ell=\overline{2,L}$ change is determined by the limits of a number of integrated functions and performance incentives are used to form the criteria for Interval ℓ-1 level.

For the formation of this problem at every level will take the following notation: N, k is the number and the number of discrete intervals of the period T of this level; S1(k), S2(k) - number of own and leased equipment at time k; $\mu_{1i\alpha}^{v}$, $\mu_{2i\alpha}^{v}$ - the intensity of the i-th own and leased equipment for service α application v-th species at time k; k - the number of applications; V1, V2 - total volume of own and leased buffers; $y_1^v(k)$, $y_2^v(k)$ - the length of the queue of the application v-th species in its own and leased buffers. at time k; v '(k) - the total free volume of buffer SMS - nodes at time k; $v_j^{'}(k)$ - free volume buffer of j-th node at time k; $G_j^{\alpha,v}(k)$ - the average cost of service α-th application v-th species in the i-th channel and j-th node; $\lambda^v(k)$ - the intensity of applications v-th species; $\lambda(k)$ - the intensity of all types of applications; $\mu_j^{v,i}(k)$ - the intensity service of applications v-th species in the i-th channel and j-th node of the SMS; yj(k), $\overline{y}_j(k)$, - the total length of the queue in a buffer SMS j-th node and the allowable limit of its variation at time k.

Using the notation adopted on the solution of the optimal MS 1 can be written as:

$$Q=\sum_{k=1}^{N}\sum_{j=1}^{J}\sum_{v=1}^{K}\sum_{i=1}^{I}C_j^{v,i}(k)\mu_j^v(k)\Delta_k \to \min_{\mu(k)\in G_\mu(k)} \qquad (1)$$

under the conditions

$$y_j^{\nu}(k+1) = y_j^{\nu}(k) + \left(\lambda_j^{\nu}(k) - \mu_j^{\nu}(k)\right)\Delta_k,$$
$$0 \le y_j^{\nu}(k+1) \le \bar{y}_j^{\nu}(k+1) = V, \quad (2)$$

$$y_j(k+1) = y_j(k) + \sum_{\nu=1}^{K}\left(\lambda_j^{\nu}(k) - \mu_j^{\nu}(k)\right)\Delta_k,$$
$$0 \le y_j(k+1) \le \bar{y}_j(k+1),$$
$$y(k+1) = y(k) + \sum_{j=1}^{J}\sum_{\nu=1}^{K}\left(\lambda_j^{\nu}(k) - \mu_j^{\nu}(k)\right)\Delta_k,$$
$$0 \le y(k+1) \le v(k),$$
$$\sum_{k=1}^{N}\sum_{j=1}^{J}\sum_{\nu=1}^{K}\sum_{i=1}^{I}\alpha_{j\beta}^{\nu,i}(k)\mu_j^{\nu,i}(k)\Delta_k \le b_\beta$$
$$\frac{1}{N}\sum_{k=1}^{N}\mu_j^{\nu,i}(k) = \tilde{\mu}_j^{\nu,i}(k), \quad \frac{1}{N}\sum_{k=1}^{N}\sum_{i=1}^{I}\mu_j^{\nu,i}(k) = \tilde{\mu}_j^{\nu},$$
$$\frac{1}{N}\sum_{k=1}^{N}\sum_{j=1}^{J}\mu_j^{\nu,i}(k) = \tilde{\mu}^{\nu,i}, \quad \frac{1}{N}\sum_{k=1}^{N}\sum_{j=1}^{J}\sum_{i=1}^{I}\mu_j^{\nu,i}(k) = \tilde{\mu}^{\nu}, \quad (3)$$
$$\frac{1}{N}\sum_{k=1}^{N}\lambda^{\nu}(k) = \tilde{\lambda}^{\nu}.$$

The problem of the lower-level interval represented in the form of

$$Q_k = \sum_{j=1}^{J}\sum_{\nu=1}^{K}\sum_{i=1}^{I}C_j^{\nu,i}(k,\eta)\theta_j^{\nu}(k) \to \min, \quad (4)$$

$$q_k = \sum_{j=1}^{J}\sum_{\nu=1}^{K}\sum_{i=1}^{I}b_j^{\nu,i}(k)\mu_j^{\nu,i}(k) \to \min(\max), \quad (5)$$

under the conditions

$$\sum_{j=1}^{J}\sum_{\nu=1}^{K}\sum_{i=1}^{I}a_{j\beta}^{\nu,i}(k\mu^{\nu,i}(k) \le b_\beta(k), \quad \sum_{i=1}^{I}\mu_j^{\nu,i}(k) = \tilde{\mu}_j^{\nu}(k),$$
$$\sum_{j=1}^{J}\mu_j^{\nu,i}(k) = \mu^{\nu,i}(k), \quad \sum_{j=1}^{J}\sum_{i=1}^{I}\mu_j^{\nu,i}(k) = \tilde{\mu}^{\nu}(k), \quad (6)$$
$$\mu_{-j}^{\nu,i}(k) \le \mu_j^{\nu,i}(k) \le \bar{\mu}_j^{\nu,i}(k); \quad \nu = \overline{1,k}; i = \overline{1,I}; j = \overline{1,J}; k = \overline{1,N},$$

where I ishe number of service channels of this type of application for this type node; θ is the vector of parameters stimulation, determined from the solution of (1) - (3) of this level, which is used to form the criteria for interval (4), taking into account its strategy; qк. - local (own) criteria for interval the existence of which the upper level can be a little informed; $\tilde{\mu}$ is he average intensity of the corresponding figure for N intervals;

β, bβ - the kind of resource to be used for MS in the CN; $\alpha_{j\beta}^{\nu,i}$ - costs coefficients of β-th type of resource; μ - generalized representation of the intensity of service.

III. Solution Algorithm

Solution algorithm (1), (2), (3) and (4), (5), (6) together account for two-level cluster problem where the solution of the first tasks is used to coordinate interval of optimal MS.

The problem (1), (2), (3) is variation, and (4), (5), (6) is two-criterion problems of mathematical programming, for solutions that one can attract the more effective methods of appropriate.

From these methods for the solution of the upper level, you can specify a discrete maximum principle and dynamic programming method by F.Krotov, where the first method is based on a necessary condition, and the second at a sufficient condition for optimality. To use the first method for the problem presented by the Hamilton function is compiled, and the second by the Krotov function, which has more opportunities.

Here is the solution scheme presented problems with the use of a discrete maximum principle. To do this, the introduction of new variables, the criterion (1) and the constraint function in the last four constraints in (3) gives the appropriate form - the form of the difference equations.

Namely, the introduction of variables:

$$Q = \sum_{j=1}^{J}\sum_{\nu=1}^{K}\sum_{i=1}^{I}C_j^{\nu,i}(k)\mu_j^{\nu}(k)\Delta_k,$$
$$\tilde{y}_\beta = \sum_{j=1}^{J}\sum_{\nu=1}^{K}\sum_{i=1}^{I}\alpha_{j\beta}^{\nu,i}(k)\mu_j^{k,\nu}(k)\Delta_k, \quad (7)$$
$$\tilde{y}_j^{\nu,i} = \mu_j^{\nu,i}(k), \quad \tilde{y}_j^{\nu} = \sum_{i=1}^{I}\mu_j^{\nu,i}(k),$$
$$\tilde{y}^{\nu,i} = \sum_{j=1}^{J}\mu_j^{\nu,i}(k), \quad \tilde{y}^{\nu} = \sum_{j=1}^{J}\sum_{i=1}^{I}\mu_j^{\nu}(k),,$$
$$\tilde{y}^{\nu}(k) = \lambda^{\nu}(k).$$

These functions are written as difference equations:

$$y(k+1) = y(k) + \frac{1}{N}A(k), \quad \tilde{y}(N) = Q,$$
$$\tilde{y}_\beta(k+1) = \tilde{y}(k) + \frac{1}{N}A_\beta(k), \quad \tilde{y}_\beta(N) = b_b, \quad (8)$$
$$\tilde{y}_j^{\nu,i}(k+1) = \tilde{y}_j^{\nu,i}(k) + \frac{1}{N}A_j^{\nu,i}(k), \quad \tilde{y}_j^{\nu,i}(N) = \tilde{\mu}_j^{\nu,i},$$

$$\tilde{y}^{v}(k+1)=\tilde{y}^{v}(k)+\frac{1}{N}A_j^{v}(k)\,,\ \tilde{y}^{v}(N)=\tilde{\mu}_j^{v}\,,$$

$$\tilde{y}_j^{v,i}=\tilde{y}_j^{v,i}(k)+\frac{1}{N}A_j^{v,i}(k)\,,\ \tilde{y}_j^{v,i}(N)=\tilde{\mu}^{v,i}\,,$$

$$y^{v}(k+1)=y^{v}(k)+\frac{1}{N}A^{v}(k)\,,\ \tilde{y}^{v}(N)=\tilde{\mu}_j^{v}\,,$$

where A(k) - the corresponding indices express the right sides entered into (4) variables.

In view of equations (8) the original problem can be written as:

$$y(N)\to\min_{\mu} \tag{9}$$

under conditions (2), (3).

To solve this problem using the conjugate variables and Lagrange multipliers of the index corresponding to the indices of (8), a distribution is made:

$$J(x,\psi,\lambda)=\sum_{k=1}^{N}(y(k+1)-y(k)-A(k))\psi(k+1)+$$
$$+\sum_{k=1}^{N}\ell_k(y(k)-\bar{y}(k))+\ell(y(T)-y(T-1)) \tag{10}$$

and consider the problem:

$$J(x,\psi,\lambda)\to\min_{\mu}\ \max_{\psi,\ell,\ell(k)} \tag{11}$$

under conditions where $\psi,\ell,\ell(k)$ - are the vector representation of the conjugate variables and the Lagrange multipliers corresponding to the indices of (8), and $\ell(k)\ge 0,\ k=\overline{0,N-1}$.

From the problem (11) it is easy to obtained Hamiltonian function:

$$H(x,\psi,\lambda)=-A(k)\psi(k)\to\min_{\mu}\ \max_{\psi,\ell,\ell(k)} \tag{12}$$

$$H(x,\psi,\lambda)=A(k)\psi(k)\to\max\,, \tag{12$'$}$$

where $\psi(k)$ is the vector of conjugate variables determined from the condition $\frac{\partial J}{\partial y(k)}=0$, i.e. from the equation:

$$\psi(k)=\psi(k+1)+A(k)u(k),\quad k=\overline{0,N-1}\ ,\psi(N)=\lambda \tag{13}$$

From the solution of the problem (12) with the values $\mu(k)$ found in the previous step, the values $\psi,\ell,\ell(k)$, act as incentives (coordination) of the parameters. With the known values of $\psi,\ell,\ell(k)$ as a criterion Q_k on the lower level serves the Hamiltonian (12).

We can find from the solution of the multi-criteria problem (4) - (5) is the optimal value.

IV. Conclusions

A critical analysis of the current issue of the problem MS was conducted and proposed a generalized problem MS in networks as a dynamic multi-level (variation) the optimal control problem in the reporting period of time. Solution of the problem with a certain projection of incoming orders allow optimally distribute the orders over time and levels of the network.

The challenge for the time interval at each level can be expressed as a static multi-criteria problem.

References

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