Necessary Optimality Conditions in One Discrete Problem of Optimal Control Involving Inequality Type Functional Constraints on the Right End of the Trajectory

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***Abstract*— An optimal control problem described by difference analogy of Volterra integro-differential equation is studied. Necessary optimality conditions are obtained.**

***Keywords*— optimal control; necessary conditions; functional constraints; convex set**

I. Introduction

In this paper, the case of presence of functional constraints of inequality type on the right side of the trajectory is studied for control problems described by a system of difference equations of Volterra type. Necessary optimality conditions of first order are obtained.

II. Statement of Problem

Consider a problem on minimum of the functional

$$S_0(u)=\varphi_0(x(t_1)), \qquad (1)$$

under constraints

$$S_i(u)=\varphi_i(x(t_1))\le 0,\ i=\overline{1,p}, \qquad (2)$$

$$u(t)\in U\subset R^r,\ t\in T=\{t_0,t_0+1,\dots,t_1-1\}, \qquad (3)$$

$$\begin{gathered} x(t+1)=\sum_{\tau=t_0}^{t} f(t,\tau,x(\tau),u(\tau)),\ t\in T, \\ x(t_0)=x_0. \end{gathered} \qquad (4)$$

Here $\varphi_i(x),\ i=\overline{0,p}$ are the given continuously differentiable functions, $x(t)=(x_1(t),x_2(t),\dots,x_n(t))'$ is a state vector, t_0, t_1, x_0 are given, moreover t_1-t_0 is an integer, $f(t,\tau,x,u)$ is a giver $n-$dimensional vector function continuous by totality in variables together with partial derivatives with respect to x.

The control function $u(t)$ is called an admissible control if its appropriate solution $x(t)$ of system (4) satisfies constraints (2).

III. Solving the Problem

Assume that $u^\circ(t)$ is a fixed admissible control and along it the set

$$\begin{gathered} f(t,\tau,x^\circ(\tau),U)= \\ =\{\alpha:\alpha=f(t,\tau,x^\circ(\tau),v),\quad v\in U\} \end{gathered} \qquad (5)$$

is convex.

Introduce the denotation

$$H(t,x,u(t),\psi_i^o(t))=\sum_{\tau=t}^{t_1-1}\psi_i^{o'}(\tau)f(\tau,t,x,u(t)).$$

Here $\psi_i^o(t)$ is a solution of the conjugated system

$$\psi_i^o(t-1)=\frac{\partial H(t,x^\circ(t),u^\circ(t),\psi_i^o(t))}{\partial x},$$

$$\psi_i^\circ(t_1-1)=-\frac{\partial\varphi_i(x^\circ(t_1))}{\partial x}.$$

Let us define as follows:

$$I(u^\circ)=\{i:S_i(u^\circ(t))=0,\ i=\overline{1,p}\},$$

$$J(u)=\{0\}\cup I(u).$$

Thus, $I(u^\circ)$ is a set of active constraints at the «point $u^\circ(t)$».

Theorem 1. If along the process $(u^\circ(t),x^\circ(t))$ the set (5) is convex, then for optimality of the admissible control $u^\circ(t)$ in problem (1)-(4) it is necessary that the inequality

$$\min_{i\in J(u^\circ)}\sum_{t=t_0}^{t_1-1}\Delta_{v(t)}H(t,x^\circ(t),u^\circ(t),\psi_i^o(t))\le 0$$

be fulfilled for all $v(t)\in U,\ t\in T$.

Theorem 2. Let the set U be convex, the vector-function $f(t,\tau,x,u)$ be continuous in totality of variables with respect to (x,u) to second order inclusively. Then for optimality of the admissible control $u(t)$ it is necessary that the inequality

$$\min_{i \in J(u^0)} \sum_{t=t_0}^{t_1-1} H'_u\left(t, x^0(t), u^0(t), \psi_i^0(t)\right)\left(v(t) - u(t)\right) \le 0$$

be fulfilled for all $v(t) \in U$, $t \in T$.

IV. CONCLUSION

An optimal control problem described by a system of Volterra type difference equations with moving right and of the trajectory is considered. Necessary optimality condition is proved.

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