

Algorithms for Solving the Optimal Stabilization Problem in Gaslift Process

Fikret Aliev¹, Mutallim Mutallimov², Naila Velieva³
Institute of Applied Mathematics of Baku State University, Baku, Azerbaijan
¹f_aliev@yahoo.com, ²mmutallimov@bsu.az

Abstract— A problem of optimal stabilization of the injected gas and production rate of oil wells by gas lift method is considered. Under certain natural assumptions, the general problem is reduced to the linear quadratic control problem (LQCP) that allows one to find program controls and trajectories on which optimal regulator is constructed over the whole or a part (by production rate) of phase coordinates. For the specific case, numerical examples are illustrated demonstrating possibility of use of this method.

Keywords— gaslift; program controls and trajectories; optimal stabilization

I. MATHEMATICAL MODEL, CONSTRUCTION THE PROGRAM TRAJECTORIES AND CONTROLS

The mathematical model describing the gas lift process in the form of the system of partial differential equations [1-3] that under certain natural assumptions is reduced to the following linearized system

$$\begin{cases} -F_0 \frac{\partial p}{\partial x} = \frac{\partial Q}{\partial t} + 2aQ + (1 + \beta)w_0 \frac{\partial Q}{\partial x} \\ -F_0 \frac{\partial p}{\partial t} = c^2 \frac{\partial Q}{\partial x} \end{cases} \quad (1)$$

Usually, in such problems, it is required to find minimal volume of gas in order to receive the given production rate of oil. Further, we could accept this volume and production rate as program control and trajectory, respectively [4,5]. In the present case, the control law should be determined so that the production rate could be stabilized near program trajectory [2, 6].

Description of this process by the system of partial differential equations is very complicated problem [7]. Thus the given problem is reduced to the following LQCP [4, 8, 9]

$$\dot{x} = Ax + Gu, \quad x(0) = x^0, \quad (2)$$

$$J = \frac{1}{2} (x(T) - \bar{x})^T N (x(T) - \bar{x}) + \frac{1}{2} \int_0^T u^T R u dt \rightarrow \min. \quad (3)$$

Choosing a symmetric matrix N from (3) in the appropriate way, we can give to problem (2), (3) the following

sense [2, 6]: for the minimal discharge of injected gas one can get desired production rate whose value is determined by the variable \bar{x} .

From problem (2), (3) we can determine optimal trajectories $x_{op}(t)$ and controls $u_{op}(t)$.

Notice that for above-mentioned data, optimal program trajectories and controls obtained on the segment $[0, T]$ are given in fig.1

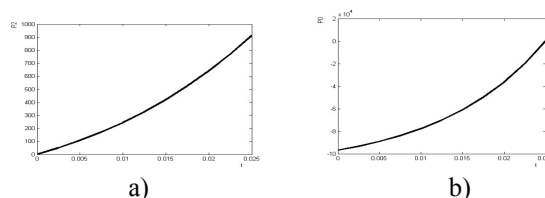


Figure.1. Optimal program trajectory a) and control b)

Thus, in spite of the fact that for $n = 2$ system (2) very roughly approximates equations (1), the obtained results are close to with the character of experimental results for some taken well [1, 2, 6].

Another algorithms from reviews papers [10, 11] may be used for large values of n .

Notice that to study the consideration optimal stabilization problem we should have program trajectories and controls on $[0, \infty)$. Since we want to maintain the obtained result in certain time, it is expedient to determine program trajectories and controls

$$\begin{aligned} x_{pr}(t) &= \begin{cases} x_{op}(t), & t < T, \\ x_{op}(T), & t \geq T, \end{cases} \\ u_{pr}(t) &= \begin{cases} u_{op}(t), & t \leq T, \\ u_{op}(T), & t > T \end{cases}, \end{aligned} \quad (4)$$

that will be used in constructing appropriate controllers.

II. STABILIZATION PROBLEM

Now, let's consider the stabilization of injected gas and production rate near the found program controls and trajectories $u_{pr}(t)$ and $x_{pr}(t)$. Really, as in [4], denoting by $\varepsilon(t) = x(t) - x_{pr}(t)$, $v(t) = u(t) - u_{pr}(t)$ for the stabilized object we have the following system of equations

$$\dot{\varepsilon} = A\varepsilon + Gv, \quad (5)$$

where it is required to determine such control law

$$v = K\varepsilon, \quad (6)$$

that minimizes the quadratic functional

$$J = \frac{1}{2} \int_0^T (\varepsilon^T Q \varepsilon + v^T R v) dx. \quad (7)$$

It is known that [4, 12], for determination of K we have the following formula

$$K = -R^{-1}G^T S,$$

where S is a positive – defined solution of the following matrix algebraic Riccati equation

$$-A^T S - SA + SGR^{-1}G^T S - Q = 0. \quad (8)$$

For the solution of (8) there are well known computing algorithms [13,14] that will be used in for construction the of controls (6). From (5),(6) for determination of current values of $x(t), u(t)$ we have

$$\dot{x} = Ax + Gu - Ax_{pr} - Gu_{pr}, \quad (9)$$

$$u = Kx + u_{pr} - Kx_{pr}, \quad (10)$$

For the above mentioned values of the parameters for stabilization of the production rate $Q_2(t)$ we get fig.2

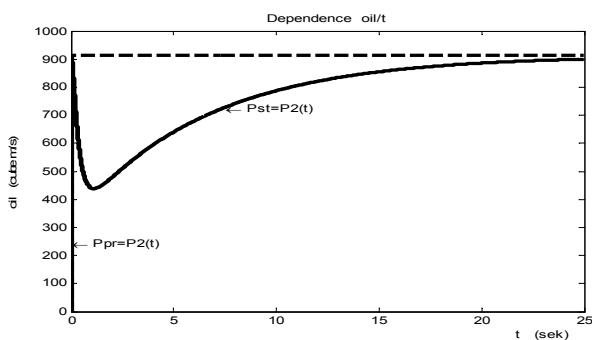


Figure.2. Stabilization in all phase coordinates.

As shows the calculating experiment, $x(t)$ approaches to $x_{pr}(t)$ sufficiently close after $t > 25$.

The present stabilization problem considers all the coordinates of the object. However, we are interested only in production rate $Q_2(T)$, therefore, we can simplify the stabilization problem, i.e. we observe $y = Cx$, where $C = [0 \ 0 \ 0 \ 1]$. Then instead of (10) we look for an appropriate control in the form [15]:

$$u = WQ_2(t) + u_{pr} - WQ_{2pr}(t)$$

and the solution of problem (5)-(7) is reduced to definition of W from the following nonlinear system of matrix equations

$$\begin{aligned} W &= R^{-1}G^T SUC^T (CUC^T) \\ (A + GWC)^T S + S(A + GWC) + Q + C^T KRKC &= 0, \\ (A + GUC)U + U(A + GUC)^T &= E, \end{aligned} \quad (11)$$

where the unknown symmetric matrices S and U must be defined.

Here we assume that the initial value x_0 is a random variable with zero mathematical expectation and dispersion $\langle x_0, x_0^T \rangle = E$. To solve the system of nonlinear equations (11), there exist various algorithms. Applying them [14, 15], we get the following value

$$W = \begin{bmatrix} -13035 \cdot 10^{-3} & 2438.8 \\ 2.824 \cdot 10^{-6} & 0.0161 \end{bmatrix}.$$

Stabilization on incomplete observation is represented in fig.3

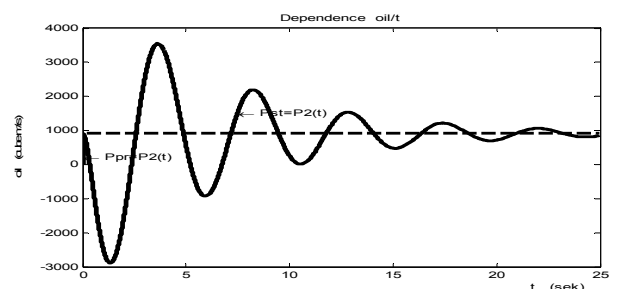


Figure.3. Stabilization on incomplete observation.

III. CONCLUSION

In the paper, calculating algorithms for finding program trajectories and algorithms on the base of well known LQCP for gas lift process are cited. Further, an optimal control on the whole and a part (by production rate) of phase coordinate is

structured. Computing experiments affirm coordination of the considered theory with field experimental results.

REFERENCES

- [1] N.A.Charniy. Unsteady flow of real fluid in pipes. M., Nedra, 1975. (in Russian)
- [2] V.I.Shurov. Technology and techniques of oil recovery. M., Nedra, 1983. (in Russian)
- [3] F.A.Aliev, M.Kh.Ilyasov, M.A.Jamalbekov. Simulation of gas-lift well operation // Dokl. NAS of Azerbaijan, № 4, 2008, pp.30-41. (in Russian).
- [4] V.B.Larin. Control of walking devices. Kiev, Naukova Dumka, 1988.
- [5] F.L.Chernousko. Optimization of progressive motions for multibody systems // Appl. Comput. Math. 7 (2008), No.2, pp.168-178.
- [6] A.Kh.Mirzajanzadeh, I.M.Akhmedov, A.M.Khasayev, V.I.Gusev. Technology and techniques of oil recovery. Editor: prof. A.Kh.Mirzajanzadeh. M., Nedra, 1986, 382 p. (in Russian).
- [7] K.A.Lourve Optimal control in mathematical physics problems. M., Nauka, 1975. 480 p. (in Russian).
- [8] Kh.Kvakernaak, R.Sivan. Linear optimal control systems M.,Mir, 1977, 650p. (in Russian).
- [9] R.Gabasov, F.M.Kirillova, N.S.Paulianok. Optimal control of linear systems on quadratic performance index // Appl. Comput. Math. 7 (2008), No.1, pp.4-20.
- [10] Y.Ruth Misener and A.Floudas Christodoulos. A review of advances for the pooling problem: modeling, global optimization, and computational studies // Appl. Comput. Math. 8 (2009), No.1, p.3-22.
- [11] M.P.Pardalos, E.Kundakcioglu. Classification via mathematical programming // Appl. Comput. Math. 8 (2009), No.1, pp.23-35.
- [12] S.K.Korovin, A.V.Kudritskii, A.S.Fursov. Simultaneous stabilization of arbitrary-order linear objects by a controller with a given structure // DAN, 2008, vol.423, №2, pp.173-177. (in Russian).
- [13] D.Petrova, A.Cheremensky. Linear control systems: feedback and separation principles. // Appl. Comput. Math. 8 (2009), No.1, p.114-129.
- [14] V.B.Larin. High-accuracy algorithms for solution of discrete periodic Riccati equations // Appl. Comput. Math. 6 (2007), No.1, pp.10-17.
- [15] El-Sayed M.E.Mostafa. First-order penalty methods for computing suboptimal output feedback controllers // Appl. Comput. Math. 6 (2007), No.1, pp.66-83.