About One Forecast Model of Stochastic Programming Based on Time Series and Genetic Algorithms

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Abstract – **We consider the problem of forecasting of complex objects' state, which characteristics are functions of time, that reduces to the solution of stochastic programming with probabilistic constraints. We suggest an approach in which stochastic programming is analyzed by time series and genetic algorithms.**

Key words– dynamic object; casual process; interval of anticipation; exponential smoothing; mutation rate; genetic algorithm with the prediction.

I. INTRODUCTION

Let the set

$$\{X_1, X_2, ..., X_m\} \quad (1)$$

represents the final set of some complex, dynamic objects, which states are described by

$$x(t) = (x_1(t), x_2(t), \ldots, x_n(t)). \quad (2)$$

The set of objects is considered as admissible if signs (2) are defined by areas of corresponding values G_i, that is

$$x_{ui}(t_k) \in G_i, i = \overline{1, n} \quad (3)$$

for all $u = 1,2,\ldots m$ and for each moment of time

$$0 < t_0 < t_1 < \ldots t_\delta = T + \Delta t. \quad (4)$$

If changing process of objects' conditions (1) is considered as casual process, then signs (2) represent functions of the valid parameter $t \in [t_0, t_\tau]$ which values at everyone t are random variables, and their set can be considered as some realisation of observable casual process [7]. In the paper we consider approach to object's classifications.

I. METHODS OF SOLUTION

For training sample [4,5] we accept a set of objects, $\{X_\nu, \nu = \overline{1, N}\} \subseteq \{X_1, X_2, \ldots, X_m\}, \quad N \le m,$ to which in signs' space set of trajectories

$$\{L_\nu, \nu = \overline{1, N}\}, \quad (5)$$

defined at all time part $[t_0, t_\tau]$, corresponds. Training sample (5) is considered broken into a final set of classes $K_j,\ j = \overline{1, l},\ l \ge 2$, so

$$L_\nu \in k_j,\ N_{j-1} + 1 \le \nu \le N_j,\ N_0 = 0,\ N_l = N, \quad (6)$$

thus

$$\{L_1, L_2, \ldots, L_\nu\} = \bigcup_{j-1}^{l} K_j,$$
$$K_j \cap K_{j_2} = \theta,\ j_1 \ne j_2. \quad (7)$$

Similarly, signs' values (2) of objects $\{X'_1, X'_2, ..., X'_q\}$, which turn out by tests, realised during the discrete moments of time

$$0 = t_0 < t_1 < \ldots < t_k < \ldots < t_p = T, \quad (8)$$

also are represented in the form of matrixes of dimension $(p+1) \times n$, that is

$$\left\| x'_{\mu 1}(t_k) \ldots x'_{\mu i}(t_k) \ldots x'_{\mu n}(t_k) \right\|, \quad (9)$$

where $\mu = \overline{1, q}$; $k = \overline{0, p}$

Signs' space of objects

$$\{L_1', L_2', \ldots, L_q'\}, \qquad (10)$$

defined on piece $[0,T]$, which continuations on an anticipation interval remain unknown, is corresponded to X'_μ, $\mu = 1,2,\ldots,q$. It is required to construct continuations of sample on an interval of anticipation $[T, T+\Delta t]$ concerning a characteristic, with due regard for restrictions at the moment $T+\Delta t$ [6].

Let it is given $I_0(K_1, K_2, \ldots, K_l; L'_1, L'_2, \ldots, L'_q)$ as the initial information [6]. $0 \le A_{\mu j} \le 1, \mu = \overline{1,q}$; $j = \overline{1,l}$ are given quantities. We will consider function $f'(t)$, which characterises trajectories L'_μ of object number X'_μ, that is time- and signs- dependent resource function,

$$f'(t) = f'(t, x'_1(t), x'_2(t)........, x'_n(t)). \qquad (11)$$

We will offer, that function $f'(t)$ is provided as

$$f'(t_k) = \sum_{i=1}^{n} c_i x'_i(t_k) + \varepsilon(t_k),$$

$$t_k \in [0,T], k = \overline{0,p}$$

where $c_{i,} i = \overline{1,n}$ are some unknown constants which are must be defined. $f'(t_k)$ and $x'_i(t_k)$ are known corresponding values of $f'(t)$ and $x'_i(t)$ in points $t_k, k = \overline{0,p}$. $\varepsilon(t_k)$ - casual deviations in the same points

$t_{p+1}, t_{p+2}, \ldots., t_\tau$. Thus, the calculation of values of function $f'(t, \tilde{x}(t))$ on forestalling interval (i.e. forecasting) can be executed by the formula

$$\hat{f}(t_k) = \hat{f}(t_k, \hat{x}_1(t_k), \hat{x}_2(t_k), \ldots., \hat{x}_n(t_k)) = \sum_{i=1}^{n} C_i^* \hat{x}_i(t_k)$$

$$k = \overline{p+1, \tau} \qquad (12)$$

where $\hat{x}_1(t_k), \hat{x}_2(t_k), \ldots\ldots, \hat{x}_n(t_k), t_k \in [T, T+\Delta t]$ are values of the signs' forecast in points $t_{p+1}, t_{p+2}, \ldots., t_\tau$.

As criterion of the best forecast on an interval of forestalling $[T, T+\Delta t]$ let's consider following functional

$$\tilde{J}(\tilde{C}_1, \tilde{C}_2, \ldots, \tilde{C}_n) =$$

$$= M\left\{ \sum_{k=p+1}^{\tau} \left[\hat{f}(t_k) - \sum_{i=1}^{n} \tilde{C}_i X_i(t_k) \right]^2 \right\}, \qquad (13)$$

where M is expectation value by all $X_i(t_k), i = \overline{1,n}$; $t_k \in [T, T+\Delta t]$.

The mathematical formulation of prolongation problem of trajectory L_i' of object $Q_i' \in K_j$ is reduced to the following:

- According to $f^j(t_\tau), j = \overline{1,l}$, characterising values of resource function of each class $K_u, u = \overline{1,l}$, $f^j(t_\tau)$ usually is defined from (4)-(6) as an average of value of function $f(t)$ for the objects belonging to class K_j at the moment of $t_\tau = T + \Delta t$;
- Set $A_u, u = \overline{1,l}$ for object $Q_i' \in K_j$ and $\tilde{\varepsilon}_u, u = \overline{1,l}$;

It is required to minimise functional

$$J(\tilde{C}_1, \tilde{C}_2, \ldots.., \tilde{C}_n) =$$

$$= M\left\{ \sum_{k=p+1}^{\tau} \left[\hat{f}(t_k) - \sum_{i=1}^{n} \tilde{C}_i X_i(t_k) \right]^2 \right\} \qquad (14)$$

with restrictions

$$\left. \begin{array}{l} P\left\{ (\sum_{i=1}^{n} \tilde{C}_i x_i(t_\tau) - f^j(t_\tau))^2 < \tilde{\varepsilon}_j^{\,2} \right\} \ge A_j \\ P\left\{ (\sum_{i=1}^{n} \tilde{C}_i x_i(t_\tau) - f^u(t_\tau))^2 < \tilde{\varepsilon}_u^{\,2} \right\} \le A_u \end{array} \right\} \qquad (15)$$

u = 1,2, ..., j - 1, j + 1, ..., l where $\tilde{\varepsilon}_u, u = \overline{1,l}$ are defined as:

$$\tilde{\varepsilon}_u = \frac{1}{2} \max_{\alpha,\beta} \left| f^i_\alpha(t_\tau) - f^j_\beta(t_\tau) \right|, u = \overline{1,l}.$$

Here $f^j_\alpha(t_\tau), f^j_\beta(t_\tau)$ accordingly characterises value of function $f(t)$ of trajectories L_α, L_β from class $K_j, j = \overline{1,l}$.

The problem (14) and (15) is a problem of stochastic programming with probabilistic restrictions [3,5]. In works [7, 8, 9], the definition of optimum values $\widetilde{C}_1^*, \widetilde{C}_2^*, ..., \widetilde{C}_n^*$ (for each object $X_i^{'} \in \{X_1^{'}, X_2^{'}, ..., X_q^{'}\}$) was reduced to the decision of a problem of stochastic programming (14) and (15).

In the given work, for the decision of the problem (14) and (15), pursuant to works [3], the method which is based on algorithm of J. Lamarck's evolutionary principle is offered.

In this paper we propose an approach that differs from the above approaches for fuzzy time series. For sufficiently reliable calculation requires a minimum of 20-30 parameters - members of time series of the studied process, which are laid along the axis y [4]. We present the scheme of the proposed approach:

1. The time sampling of the studied parameter of the process is produced by method of hierarchical cluster analysis.

2. It is made two-dimensional data matrix $n \times 2$, where n is the length of time series. Thus the first row - the time vector of parameter, the second row is time.

3. We construct the trend using the exponential smoothing. If there is no need to take into account the trend of the process, the median is constructed

$$M = \frac{1}{n}\sum x_i \ .$$

Calculate the standard deviation of the mean values:

$$\sigma = \frac{1}{n-1}\sum_{i=1}^{n} x_i^{\ 2} \ .$$

If the time series of gene is statistically unmanageable, then mutation rate abruptly increase to $M \times K$, which is calculated by the following formula

$$M_{t+1} = M_t + \frac{M_0(K-1)}{n}, M = \lim_{t \to \infty} M_t \ .$$

Otherwise, mutation rate dynamically decreases to the value

$$M_{t+1} = M_t + \frac{M_0\left(\frac{1}{K}-1\right)}{n}, M = \lim_{t \to \infty} M_t \ .$$

Usually, K is empirically calculated [2] and is equal to 2, n - the length of time series, M_t is the coefficient of mutations at the time. Thus, control of rate of mutation is carried out by aforementioned scheme and the process is carried out once in n generation. Further, based on the control of mutation rate it is proposed the genetic algorithm with the prediction. For example, the best representatives of the population have already been identified under the above scheme, and the task of selecting the optimal values of the genome can be reduced to the problem of statistical forecasting of time series. GA with the prediction performed as follows:

If the time series of i -th gene of the best representative of population is managed in the last L generations by above scheme, then add the individual in the population, which genome is composed by predicted values of genes for K generations, and where the values L and K are chosen by the user and satisfy the following conditions:

$$L > 15, \quad 5 < K < 15 \, .$$

II. Conclusion

In this paper as a procedure of prediction it is used the method of exponential smoothing, where values of exponential smoothing parameter α are separately computed for each genome [1].

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