Strong Laws of Large Numbers for Hilbert Space-valued Dependent Random Fields

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***Abstract*— We consider the random fields with values in a separable Hilbert space. We give a strong law of large numbers for Hilbert space-valued random fields which is valid under some moment conditions and conditions on the covariance's of random elements.**

Keywords*— random variables, Hilbert space, Banach spaces of Rademacher** type *p*, ***strong law of large numbers.

I. Introduction

This paper was motivated by the results of the papers [1],[2] and monograph [3]. In [1] the authors prove theorems on a.s. convergence of double sums of independent random variables with values in Banach spaces of Rademacher type *p*. Recall the definition of Rademacher type *p* Banach spaces.

We say that a separable Banach space *B* (with a norm $\|\cdot\|$) is of Rademacher type *p* if there exists a constant $0 < C < \infty$ depending only on *B*, such that

$$E\left\|\sum_{i=1}^{n} V_i\right\|^p \le C\sum_{j=1}^{n} E\left\|V_j\right\|^p$$

for every finite collection $\{V_1, V_2, \ldots, V_n\}$ of independent mean zero random variables with values in *B* and $E\left\|V_j\right\|^p < \infty \quad j = 1, 2, \ldots$

The following theorem is one of the main results of [1].

Theorem 1. Let $1 \le p \le 2$ and let *B* be a real separable Banach space. Then the following two statements are equivalent:

(i) The Banach space *B* is of Rademacher type *p*.

(ii) For every double array $\{V_{mn}, m \ge 1, n \ge 1\}$ of independent mean zero random variables with values in *B*, the condition

$$\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{E\left\|V_{mn}\right\|^p}{m^p n^p} < \infty$$

implies that the following strong law of large numbers holds:

$$\lim_{m\vee n\to\infty}\frac{\sum_{i=1}^{m}\sum_{j=1}^{n} V_{ij}}{mn} = 0 \quad a.s. \qquad (1)$$

where $m \vee n = \max(m, n)$.

In [2]and [3] the authors proved the strong laws of large numbers for Hilbert and Banach space-valued dependent random variables respectively. Our main aim is to extend the results of [2] and [3] for random fields and to generalize the results of [1] in particular case of Hilbert space-valued random variables. Note that Hilbert space is Rademacher type 2 space.

II. Main Results

Let $\{X(i, j), (i, j) \in Z^2\}$ be a random field with values in separable Hilbert space H (with inner product $(\cdot,\cdot)$ and a norm $\|\cdot\|$). We are interested in the strong laws of large numbers for $\{X(i, j), (i, j) \in Z^2\}$.

We will assume that $X(i, j)$ satisfies the following conditions

$$EX(i, j) = 0,\ \sup_{i,j} E\left\|X(i, j)\right\|^2 < M, \qquad (2)$$

$$\sup_{i,j}\left|E\left(X(i, j), X(i+m, j+l)\right)\right| \le \varphi\left(\left\|(m, l)\right\|_1\right) \qquad (3)$$

for some non-increasing function $\phi(\cdot)$, M>0 and a norm $\|\cdot\|_1$ in Z^2.

The following theorem is one of our main results.

Theorem 2. Let $X(i, j)$ be a random field with values in *H* satisfying the conditions (2),(3) and:

$$\sum_{i=1}^{n}\sum_{j=1}^{m}\varphi\left(\left\|(i, j)\right\|_1\right) = o(nm) \text{ as } m \vee n \to \infty$$

Then as $m \vee n \to \infty$, for some $\gamma \in [1,2), \beta > \frac{1}{2}$

$$\frac{(mn)^{\frac{2-\gamma}{4}}}{(\log mn)^{\beta}} \frac{1}{mn} \sum_{i=1}^{n} \sum_{j=1}^{m} X(i,j) \to 0 \quad \text{a.s.} \qquad (4)$$

Corollary. Let $\{X(i,j),(i,j) \in Z^2\}$ satisfy the conditions of Theorem 2. Then as $n \to \infty$

$$\frac{1}{n^2} \sum_{i=1}^{n} \sum_{j=1}^{n} X(i,j) \to 0 \quad \text{a.s.}$$

Above corollary implies Theorem 1 of [2].

Note that since (4) implies (1) the statement of the Theorem 2 is slightly better than of the statement of Theorem 1(ii).

As a corollary of Theorem 2 we can formulate the strong laws of large numbers for the mixing random fields with values in Hilbert space.

REFERENCES

[1] A. Rosalsky, L. V. Thanh, "Strong and Week Laws of Large Numbers for Double Sums of Independent Random Elements in Rademacher Type p Banach Spaces," Stochastic Analysis and Applications, 2006, 24, 1097-1117.

[2] T.M. Zuparov, K. B. Mamadaliev K.B. "Strong laws of large numbers for quasistationary sequences in Hilbert spaces," Izvestiya Akademii Nauk UzSSR, Ser. Fiz.-Math., 1982, N1, 15-19.(In Russian)

[1] D.Bosq. Linear Processes in Function Spaces. Theory and applications. Lecture notes in Statistics, 149, Springer, 2000.