Repetitive and 2-D Systems Theory Approach for Modeling in Gas Networks

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Abstract— The paper introduces some classes of differential repetitive processes and discrete 2-D control system for modeling gas distribution networks. These models are suitable for handling problems of optimal control of pressure and flow in gas transport pipeline units and in a pipeline networks. The focus is on the development of a comprehensive optimization theory based on a constructive approach.

Keywords— networks modeling; gas transportation; repetitive processes; 2-d system; control and optimization theory

I. INTRODUCTION

Gas transportation networks are well known to constitute complex and large scale distributed parameter system of great practical interest. Modeling approaches, numerical methods and optimization of operating modes of gas transport networks have, therefore, been of permanent interest for researchers in the last decades, and a large number of papers were published both in civil engineering and the mathematical community, see e.g.[1]. However, optimization and control of complicated gas networks still remains a challenging problem. The general model of a gas transportation network typically includes a large number of nonlinear elements such as pipelines, gasholders, compressor stations and others. In this paper the mathematical model and corresponding optimization problem of gas network units are introduced on the basis of the so-called repetitive processes [2] and the 2-D system theory setting [3], which is a starting point for the further investigation of complex networks and which provide a fairly well-established mathematical framework. Some aspects of control theory for multidimensional systems are investigated in [4,5] and application of it to gas networks have been considered in [6]. In this paper we develop substantial new results on optimal control of differential linear repetitive processes with constraints which we motivate from the introduced linearization gas pipeline model in the presence of constraints The analysis is based on generalizing the well known maximum and ε - maximum principles. A sensitivity analysis of the resulting optimal control is also undertaken.

II. GASFLOW MODELS IN PIPELINES UNITS

This section gives an approach for modeling gas-flow in pipelines. The proposed method generates special classes of differential repetitive processes and discrete 2-D control optimization problems for studying of which the suitable mathematical tools are developed.

A. Linearized model for pipeline units

The purpose of the modeling presented here is to guarantee a predefined regime for each pipeline unit. The aim of this section is to use the linear differential models for studying control problems in gas pipeline units. The state space parameters are gas pressure p and mass flow Q at the points of the pipe. For calculating the state space parameters for isothermal gas flow in a long pipeline the following system of non-linear differential equations from the theory of gas dynamics can be used (see, e.g. [7]):

$$\begin{aligned} \frac{\partial Q(t,x)}{\partial t} &= -S\frac{\partial p(t,x)}{\partial x} - \frac{\lambda c^2}{2DS}\frac{Q^2(t,x)}{p(x,t)}, \\ \frac{\partial p(t,x)}{\partial t} &= -\frac{c^2}{S}\frac{\partial Q(t,x)}{\partial x}, \end{aligned} \qquad (1)$$

where x denotes the space variable, t the time variable, S the cross sectional area, D the pipeline diameter, c the isothermal speed of sound and λ the friction factor. It is known that some important dynamic characteristics of the processes can be evaluated from the linearized model of the processes. The most accurate linear model can be realized in some neighborhood of the known basic regime $\overline{Q}(x,t), \overline{p}(x,t)$ of the considered process. It can be shown [4] that the linearized model in the neighborhood of the known (pre-assigned/basic) regime $(\overline{Q}, \overline{p})$ has the following form

$$\frac{\partial Q}{\partial t} = -S\frac{\partial p}{\partial x} - \delta Q + \beta p, \frac{\partial p}{\partial t} = -\alpha\frac{\partial Q}{\partial x}, \qquad (2)$$

where

$$\delta = 2\gamma\frac{\overline{Q}}{\overline{p}}, \beta = \gamma\frac{\overline{Q}^2}{\overline{p}^2}, \gamma = \frac{\lambda c^2}{2DS}, \alpha = \frac{c^2}{S}.$$

Note that a basic/pre-assigned regime $(\overline{Q}, \overline{p})$ can be obtained by different methods, see e.g. [4,7]. Hence, the problem has to be first settled for the linearized model of single pipe.

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III. LINEAR DIFFERENTIAL REPETITIVE MODEL FOR GAS PIPELINES

The linearization procedure, as a first order approximation, reduces the accuracy of the mathematical description of the real processes in the gas units. Thus, in order to reduce this losses, some 'controlled' inputs $r(t, x)$ and/or $q(t, x)$ can be added into the linear model (2) as follows

$$\frac{\partial Q}{\partial t} = -S \frac{\partial p}{\partial x} - \delta Q - \beta p + \mu r,$$
$$\frac{\partial p}{\partial t} = -\alpha \frac{\partial Q}{\partial x} + \nu q \tag{3}$$

where μ, ν are some normalizing coefficients. On the other side, the physical meaning of such control functions $r(t, x)$ can be treated, for example, as a correcting pressure generated by compressor station and gasholders to increase the velocity $Q'_t(t, x)$ of the running gas volume for the considered gas unit. Analogously, the control variable $q(t, x)$ can be interpreted as a an additional flow (supply/offtake) to change the velocity $p'_t(t, x)$ of the pressure $p(t, x)$. Approximate the spatial derivatives in (3) by the backward differences

$$\partial Q(t,x)/\partial x \approx Q(t,x) - Q(t,x-h)/h, \; \partial p(t,x)/\partial x \approx p(t,x) - p(t,x-h)/h$$

and introduce the following notations

$$Q_k(t) = Q(t, kh), \; p_k(t) = p(t, kh),$$
$$r_k(t) = r(t, kh), \; q_k(t) = q(t, kh) \tag{4}$$

for the values of the unknown functions $Q(t, x)$ and $p(t, x)$ calculated in the nodes of integer lattice $\{(kh)\}$, $k = 1, 2, \ldots, N$ (where $N = \left[\frac{L}{h}\right]$, L is the length of the pipe, h is a discretization step). Then the system (3) can be rewritten as follows

$$\dot{Q}_k(t) = \frac{S}{h} p_{k-1}(t) - \delta Q_k(t) - (\frac{S}{h} + \beta) p_k(t) + \mu r_k(t),$$
$$\dot{p}_k(t) = \frac{-\alpha}{h} Q_k(t) + \frac{\alpha}{h} Q_{k-1}(t) + \nu q_k(t). \tag{5}$$

Introduce the following (2×2) are matrixes and corresponding vectors:

$$A = \begin{pmatrix} -\delta & -(\beta + \frac{S}{h}) \\ \frac{-\alpha}{h} & 0 \end{pmatrix}, D = \begin{pmatrix} 0 & \frac{S}{h} \\ \frac{\alpha}{h} & 0 \end{pmatrix},$$

$$B = \begin{pmatrix} \mu & 0 \\ 0 & \nu \end{pmatrix}, x_k(t) = \begin{bmatrix} Q_k(t) \\ p_k(t) \end{bmatrix}, u_k(t) = \begin{bmatrix} r_k(t) \\ q_k(t) \end{bmatrix}$$

Then the dynamical model for the pipeline unit is defined by the linear differential repetitive state space model of the form

$$\dot{x}_k(t) = Ax_k(t) + Dx_{k-1}(t) + Bu_k(t),$$
$$k = 1, 2, \ldots, N, t \in [0, T] \tag{6}$$

The dynamics of (6) generates the output vector, or pass profile vector $x_k(t), t \in [0, T]$ produced on passing $(k-1)$ acts as a forcing function on the next pass k, and hence contributes to the dynamics of the new pass profile $x_k(t)$, $t \in [0, T]$, $k = 1, 2, \ldots, N$. Here we assume that the time duration period T of the process is finite. In order to complete the description of the process for a single pipeline model, it is necessary to specify the boundary and initial conditions, i.e.

$$x_0(t), t \in T \text{ and } x_k(0), k = 1, \ldots, N$$

on each pass. The boundary condition $x_0(t), t \in T$ can be treated as a standard pumping regime. The initial conditions $x_k(0), k = 1, \ldots, N$ describe the values of this standard regime calculated in the starting moment $t = 0$ at the discrete grid points $x = kh$ of the pipe. In order to formulate the optimization problem we need to specify a cost function. For this purpose the task of keeping the preassigned regime at the chosen points of the pipe appears reasonable. In particular, the total gas volume needed to guarantee some technically approved pressure values $p_j(T) = g_j$, $j = 1, 2, \ldots, M$ at the pre-assigned points of the pipe appears to be an appropriate choice. Also, we can admit that there exists several gas off-takes, points which, for simplicity, coincide with some points of the grid. Introduce the following (1×2) are vectors:

$$e = \begin{bmatrix} 1 & 0 \end{bmatrix}, l = \begin{bmatrix} 0 & 1 \end{bmatrix}.$$

Then the optimal control problem is to find the admissible controls $u_k(t)$ such that the corresponding solutions of the control system (6) with initial data keep a preassigned regime along the pipe at the final moment:

$$l_k x_k(T) = g_k, \quad k = 1, \ldots, N \tag{7}$$

and maximize the cost functional (maximize the total output flow at the final moment T)

$$\max_{u_k} J(u), J(u) = \sum_{k=1}^{N} e_k x_k(T), \qquad (8)$$

where $e_k = \begin{bmatrix} 1 & 0 \end{bmatrix}$, $l_k = \begin{bmatrix} 0 & 1 \end{bmatrix}$ is the nontrivial entry of which can be equal zero if the corresponding grid points do not belong to the list of pre-signed offtakes and/or pressure controlled points of the pipe. Obviously, the introduced control function u can not be taken arbitrarily. We assume that they, together with an ability to scale by the corresponding coefficients μ, ν can be defined as follows: For each pass number $k, k = 1, \ldots, N$ the piecewise continuous function $u_k : T \to R^2$ is termed an admissible control for this pass if its components $r_k(t)$, $q_k(t)$ satisfies conditions:

$$|r_k(t)| \le 1, \quad |q_k(t)| \le 1, t \in T.$$

The resulting control problem (6)—(8) gives a motivation for development of an adequate optimization method for the special classes of repetitive processes. In this paper we develop substantial new results on the optimal control of differential linear repetitive processes with constraints. The results themselves are obtained by a from of extending the maximum principle and the ε-maximum principle to them. A sensitivity analysis of the resulting optimal control is also undertaken and some relevant differentiation properties are established. Finally, a numerical example is given.

IV.   2-D SYSTEM SETTING FOR GAS PIPELINES

The aim of this section is to use the 2-D control theory setting for studying control problems in gas pipeline units. For the partial differential model (3) introduce the following combined discretization scheme (with steps h_1, h_2 on t, x, respectively):

$$\begin{aligned}
\partial Q(t,x) \partial t &= Q(t+h_1,x) - Q(t,x)h_1, \\
\partial Q(t,x) \partial x &= Q(t,x+h_2) - Q(t,x-h_2)2h_2, \\
\partial p(t,x) \partial t &= p(t+h_1,x) - p(t,x)h_1, \\
\partial p(t,x) \partial t &= p(t,x+h_2) - p(t,x-h_2)2h_2.
\end{aligned}$$

From (3) follows, that the discrete values $Q(k_1h_1, k_2h_2)$ and $p(k_1h_1, k_2h_2)$ of the function $Q(x,t)$ and $p(x,t)$ calculated in the nodes of integer lattice $\{(k_1h_1, k_2h_2)\}$ satisfy the following equations

$$Q((k_1+1)h_1, k_2h_2) = Q(k_1h_1, k_2h_2) -$$
$$\frac{h_1}{2h_2} S(p(k_1h_1, (k_2+1)h_2) - p(k_1h_1, (k_2-1)h_2)) - \qquad (9)$$
$$- h_1 \rho Q(k_1h_1, k_2h_2) - h_1 \beta p(k_1h_1, k_2h_2),$$

$$p((k_1+1)h_1, k_2h_2) = p(k_1h_1, k_2h_2) +$$
$$+ \frac{h_1}{2h_2} \alpha(Q(k_1h_1, (k_2+1)h_2) - Q(k_1h_1, (k_2-1)h_2))$$

Denote $x_1(t,s) = Q(th_1, sh_2), x_2(t,s) = p(th_1, sh_2)$, where t, s are integers. Then, the system (9) can be rewritten as follows

$$x(t+1,s) = A_0 x(t,s) + A_1 x(t,s+1) + A_2 x(t,s-1) \quad (10)$$

where

$$x(t,s) = \begin{bmatrix} x_1(t,s) \\ x_2(t,s) \end{bmatrix}; A_0 = \begin{bmatrix} 1-\rho & -\beta \\ 0 & 1 \end{bmatrix};$$

$$A_1 = \begin{bmatrix} 0 & -\dfrac{h_1 S}{2h_2} \\ \dfrac{h_1 \alpha}{2h_2} & 0 \end{bmatrix}; A_2 = \begin{bmatrix} 0 & -\dfrac{h_1 S}{2h_2} \\ -\dfrac{h_1 \alpha}{2h_2} & 0 \end{bmatrix};$$

The considered model is a discrete version of the gas transport network problem along the single pipe. Since each gas pipe is long, then it is reasonable to set $s \in Z_+$, where Z_+ is the set of nonnegative integers which images the fact that upon discrete approximation the amount of discrete values s can be huge. Then an actual problem is to find the suitable control program for the gas pressure and gas flow on the pre-defined time period for each gas network unit. Thus, we have the following $2-D$ control optimization problem: minimize

$$J(u) = \sum_{t=1}^{T} \sum_{s \in Z_+} (Gx(t,s), x(t,s)) + (Ru(t), u(t)) \qquad (11)$$

over the solution of the system (10) with initial and boundary control condition of the form

$$\begin{aligned}
x(0,s) &= \varphi(s), s \in Z_+ \setminus \{0\}, \\
x(t,0) &= \psi(t) = u(t), t = 0,1,2,...T.
\end{aligned} \qquad (12)$$

where $G \ge 0$ and $R > 0$ are given matrices. The parameters $u(t)$ can be interpreted as the controlled factors: gas pressure and gas flow at the pre-assigned time moments needed to keep the desired regime how to "pump in -pump out" through time. The initial data $x(0,s) = \varphi(s), s \in Z_+$, can be treated as an preassigned starting pumping regime given at the initial moment $t = 0$. Also note that the quadratic cost functional is based on the estimate of the deviation from the pre-assigned regime $\overline{Q(x,t)}, \overline{p(x,t)}$ determined by the first-step simulation, for example. The obtained model gives a motivation to start the investigation of a general class of two-dimensional optimization control systems for gas networks.

A. Boundary optimal control

An essential problem in optimization area is to establish optimality conditions in the feedback form that is of interest in both systems theory and applications. The aim of this paragraph is to obtain the representation of optimal control function for the optimization problem (10)—(12) by means of an additional variables $z(t,s)$ that present the state variables of the so-called conjugate system connected with initial one. The following result is hold.

Theorem. The optimal control $u^0 = (u_1^0, u_2^0, ..., u_T^0)$ of the problem (10)—(12 is given as

$$u_t^0 = -R^{-1} A_2^* z^0(t,0), \quad t = 0,1,...,T-1$$

where $z(t,s)$ is determined by the following system of equations

$$z(t,s) = A_0^* z(t+1,s) + A_1^* z(t+1,s+1) +$$
$$+ A_2^* z(t+1,s-1) + Qx(t+1,s), s \in Z_+$$
$$x(t+1,s) = A_0 x(t,s) + A_1 x(t,s+1) +$$
$$+ A_2 x(t,s-1) - A_2 R^{-1} A_2^* z(t,0), t = 0,1,...,T-1$$

with the boundary conditions

$$x(0,s) = \varphi(s), z(T,s) = 0, s \in Z_+ \setminus \{0\} .$$

Here $A_i^*, i = 0,1,2$ denote the conjugate matrixes for the matrixes $A_i, i = 0,1,2$, respectively.

V. CONLUSIONS

This paper presents some results of mathematical description of the distributed gas networks in framework of multistage modeling. The first stage modeling is based on the simplest graph model which gives a good tool to express potentially critical flow/pressure values within the given margins of inflows, outflows and setting of active components such as storage capacity gasholders, compressor stations and others in order tosatisfy/optimize the demand distributed over different nodes. It is assumed that on the second stage of modeling the detalization of single units will realized if it is needed.

In particular, for this purpose this paper presents some modeling and mathematical analysis of distributed gas networks in the framework of a repetitive processes and 2-D system theory.

The first part of the paper is connected with modeling the pipe units of the network based on the notion of stationary repetitive differential linear systems which gives a good tool to express potentially critical flow/pressure values within the given margins in order to satisfy/optimize the demand distributed over single pipe units. In this paper the supporting control functions approach has been applied to study the optimal control problems for stationary differential linear repetitive processes. The main contribution there is the development of constructive necessary and sufficient optimality conditions in forms which can be effectively used for the design of numerical algorithms. The second part of the paper uses the linear-quadratic optimization approach for 2-D control system. There we have been developed a method to establish optimality conditions in the feedback form that is of interest in both systems theory and applications. These results are first in this general area and work is currently proceeding in a number of follow up areas. One such area is sensitivity analysis of optimal control in the presence of disturbances. Also, the extension of the obtained results for the case of complex and large scale distributed gas networks is an object of the forthcoming research.

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