Detection of Hopf Bifurcation Using Eigenvalue Identification

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***Abstract*— In this research paper, we propose a new method which maps the identified complex eigenvalues to a set whose elements demonstrate the effect of each complex eigenvalue in oscillatory instability of the system. Combining beneficial properties of identification methods and the processing advantages of the proposed index, we project a new algorithm for power system monitoring. The algorithm is easy and straight-forward. We conduct several test conditions for the proposed materials using 2-area 4-machine system. Simulation result expresses good performance of proposed index in comparison to former methods. The proposed method has fairly linear behavior, without discontinuities with respect to increases of system load. What's more, it shows very good behavior when we apply it for online monitoring of power systems.**

***Keywords*— identification; eigenvalue; bifurcation; power system**

I. INTRODUCTION

Along with the continual extension of the scale of power systems, the dynamic characteristics of power systems become further complicated. Moreover, economic and environmental burdens are causing the power systems to be operated close to their stability limits [1, 2]. As a result, some nonlinear singularity phenomena are detected regularly in modern power systems. Therefore, it is necessary to monitor dynamic behavior of power system to improve its performance [3].

Because of the high non-linearity level of the electric power systems, it is possible, from a linearization at a pre-defined operative condition, to assess the local behavior of the system under study by eigen-analysis. This methodology identifies the natural frequencies, at which the oscillations between mechanical and electrical systems become self-sustained and grow up (asymptotically unstable). That behavior is denoted by the existence of, at least, a pair of complex conjugate eigenvalues, with positive real part, associated to the system linearized state space matrix. The behavior is also called a kind of bifurcation.

There are different types of bifurcations known as cause of instability in power systems; saddle-node, limit-induced, and Hopf bifurcations (HBs). The first two types of bifurcations may lead to voltage collapse. HBs, moreover, may lead to oscillatory instability of power system [4, 5].

In this research paper, we are interested in investigation of HB in power systems. Previous researches devoted to HBs are usually trying to reveal the presence of HBs and HB prediction is of less concern [5-7]. Using system identification techniques, it is valuable to provide an index to forecast system HB point with respect to system load variations. However, there has been some effort done in this field; A predictable index, such as what presented in [6], is useful in prognostic challenging load levels, for a given generation and load directions. Such an index has quasi-linear profiles. However, its prediction is still a challenging problem, specially, for large scale power systems.

The value of an index is for easiness of its estimation and assessment. Therefore, application of identification algorithms can significantly affect the results of monitoring process. SSI methods have worthy and wonder properties which may be included in the following sections. SSI methods have become more attractive in recent years, since they provide us with beneficial stuffs. SSI has also found many applications in power systems [8]. SSI methods are tools of our work when we estimate proposed HB index.

We arrange different sections of the paper as follows; power system modeling and hopf bifurcation definition and a review of power system HB indices is presented in the next section. The proposed HB index is included in section III and then there is an introduction of SSI methods in section IV. The innovated algorithm for estimation of proposed HB index is presented in section V. Simulation results and comparison of different indices is provided in the section VI using different test power system.

II. POWER SYSTEM MODEL AND HOPF BIFURCATION

In order to introduce an index for Hopf bifurcation, we begin with the following model of power system;

$$\begin{cases} \dot{x} = f(x, z, \mu, \rho) \\ 0 = g(x, z, \mu, \rho) \end{cases} \qquad (1)$$

$$x \in \Re^n, z \in \Re^m, \mu \in \Re^l, \rho \in \Re^a$$

where n is the dynamic rank of power system, x is the power system state vector which contains generator states (angle, angular speed …), dynamic load states and states of controllers; z is a vector containing algebraic variables of power system steady state such as voltage magnitudes, loads … which usually may arise when neglecting fast dynamics of angle and magnitude of voltage vector of some loads; μ is a set of uncontrollable parameters such as variations of active and reactive loads; ρ stands for controllable parameters such as auto-voltage regulator set points; f is a vector function expressing nonlinear dynamics of states differential equations; g is a nonlinear vector function expressing the load flow nonlinear algebraic equations.

It is obvious that we can provide a linear model of system as following:

$$\Delta\dot{x} = A\,\Delta x \,, \qquad (2)$$

where A is state matrix of system which is related to x_o, z_o, μ and ρ.

If all the eigenvalues of state matrix have negative real part, it is said that operating point o is exponentially stable. Variations in μ and/or ρ may cause a pair of complex eigen values move toward imaginary axis and probably it cause them to cross the imaginary axis. In this case, it is said that the operating point (x_o, z_o, μ_o, ρ_o) is oscillatory instable and a Hopf Bifurcation may occur.

There are many literatures which are devoted to Hopf Bifurcation Indices [2, 4, 6, 7, 9]. There is an introduction for different HB indices in [6], we reintroduce one of them here for further investigation;

Let's define the system matrix as A and then provide A_m such as;

$$A_m = \begin{bmatrix} A & +\beta_c I_n \\ -\beta_c I_n & A \end{bmatrix}, \qquad (3)$$

where I_n is an identity matrix. since minimum singular value can be a measure for determinant distance to zero, another Hopf bifurcation index can be detected using minimum singular value of A_m;

$$HBI1\left(A, \beta_c\right) = \sigma_{min}\left(A_m\right), \qquad (4)$$

where σ_{min} is the minimum singular value. This is called First Hopf Bifurcation Index (*HBI1*).

There is a difficulty in application of above index; it is necessary to monitor critical eigenvalue of power system which is itself a challenging problem [10]. *HBI1* is also more problematic to evaluate, since it should be provided with a matrix which its dimension is *2n*.

III. Proposed Hopf Bifurcation Index

Hopf Bifurcation (HB) may happen when a pair of eigenvalue that touches imaginary axis before other [6]. All conventional HB indices are based on critical complex eigenvalues pairs. In these indices, critical eigenvalues pairs are recognized based on their proximity to imaginary axis. The closer the pair to the imaginary axis, the more critical it is. The distance to imaginary axis is not the only parameter that causes a pair of eigenvalues reaches to imaginary axis sooner than the others. Although some pairs are initially close to imaginary axis, their movement towards imaginary axis due to load changes may be insignificant, therefore critical eigenvalue is not appropriate for HB prediction. Consequently, considering speed and direction of the pair's movement toward the imaginary axis, in addition to distance, play a pivotal role in HB prediction. In order to calculate a new HB index, the eigenvalues are ranked. Next for better linear behavior, this eigenvalue is used instead of critical eigenvalue in conventional indices. In this study, the following equation is exploited to rank eigenvalues [16].

$$\sigma_k = -\frac{\alpha_k}{d\alpha_k / d\mu}, \qquad (5)$$

where σ_k is a Hopf Bifurcation Rank for an eigenvalue whose real part is α_k and $d\alpha_k / d\mu$ is the rate of α_k changes due to load changes. In order to rank eigenvalues, σ_k is calculated for a definite number of eigenvalue pairs with biggest real parts. Positive σ_k means that eigenvalue pair is moving toward the imaginary axis. The smaller the σ_k, the more critical the eigenvalue pair is. In contrast, negative σ_k shows that the distance of eigenvalue to imaginary axis is increasing. Therefore, that eigenvalue pair whose corresponding σ_k has the smallest positive ones is considered as critical.

It is important that some eigenvalues may revolve and reverse movement direction due to load changes; it causes some error in σ_k prediction. The maximum limit for σ_k should be defined in an offline assessment to eliminate this problem. But in online prediction, it is not possible to consider this phenomenon. However the reverse movement of the critical eigenvalues almost happen near the no load condition and this condition is not critical in HB prediction.

We are supposed to find the most critical eigenvalues and rank them using (5). Therefore, we need to identify a reduced order A matrix for the power system. System identification techniques are helpful in this context. We propose to use Subspace System Identification (SSI) methods to provide the state matrix.

Stochastic SSI provides us with a reduced state space matrix using output only measurements. We can rank the eigenvalues of the identified state matrix using (5) and provide the ranks in a set called Σ. Then we can provide a new HB index using the following geometric mean of the eigenvalue ranks;

$$GMI = \sqrt[n]{\prod_{k=1}^{n} \left|\sigma_k\right|} \;\; ; \;\; \forall \sigma_k \in \Sigma \mid \sigma_k \le 0, \qquad (6)$$

The geometric mean is a useful tool in this case, since the ranks are normalized version of real part of eigenvalues.

IV. System Identification

In order to estimate the proposed HB index, we need to provide system matrix; A. System matrix can be extracted from state space model of power system as depicted in (2). Therefore, we need to identify a state space model of power system. Subspace System Identification (SSI) is one of the most attractive identification approaches which provide us with the system matrix.

There are several different algorithms available for SSI. Generally, we can arrange SSI methods into two categories from the measurement view; stochastic and deterministic SSI algorithms. If the SSI algorithm uses exogenous input measurements in its raw identification data set, it is called as deterministic SSI algorithm. Otherwise, it is called a Stochastic Subspace System Identification (SSSI) method [11].

It is obvious that we are not allowed to disturb an online power system for monitoring purposes. Therefore in order to estimate the proposed HB index, we are interested in stochastic subspace system identification (SSSI) methods in this paper.

SSI algorithms are slightly different; they use different measurements data set, usually the same block Hankel matrices, different types of projections, SVD of different matrices, the same method for extraction of system order and different extended observability matrices. Among SSI algorithms, MOESP (Multivariable Output Error State sPace) does not need to estimate future states of system, but N4SID (Numerical algorithms for Subspace State Space System) provides future state vectors by using a weighting matrix. MOESP uses extended observability matrix to extract system matrices but N4SID uses future states and through a least square problem estimates system matrices.

Investigating above mentioned SSI algorithms, we can express the following advantages for subspace system identification algorithms:

- SSI Algorithms are the only system identification methods that can easily and extensively be applied to all MIMO and SISO systems.
- Estimation of system order is one of the steps of SSI algorithms. This advantage reduces amount of time, cost and calculations.
- SSI methods can handle big packages of data.
- Additive noise does not affect SSI results [11].
- Online operations of SSI methods are easier and they can easily be applied to MIMO systems.
- SSI methods use robust mathematical tools such as SVD, LQ decomposition, least square and QR decomposition. They also don't need nonlinear optimization.
- Some SSI algorithms only use output data to identify a model. This is a considerable advantage.

V. Estimation of Proposed HB Index

If the power system is engaged in small and stochastic oscillations of load and/or other internal phenomena, one can provide a linear model of power system using measurements of some system signals and application of stochastic subspace system identification (SSSI) algorithms.

We are interested in using SSSI algorithm advantages for estimation of proposed GMI. Thus, we present the following algorithm for Hopf Bifurcation monitoring of power system using advantages of SSI methods;

Algorithm:

1. Provide a block of data.
2. Provide A using an SSI algorithm.
3. Find all the complex eigenvalues of A.
4. Calculate (5) for each eigenvalue.
5. Calculate (6) as an index for HB index.

The algorithm does not need all the products of SSSI method. It provides an A matrix which contain most dominant eigenvalues of system, since SVD of Hankel matrix removes noise and non-dominant dynamics. Therefore, finding the most critical eigenvalue may not be an issue. In addition, the computation time and load will be reasonable.

The proposed algorithm can be implemented online in two ways; i) Implementing the algorithm using a block of measured data which will be updated at the end of each algorithm run. ii) Updating data whenever a new sample data is provided. The first method is superior in economical view but the second method provides a more accurate and softer result.

Since electro-mechanical modes of power system are affected by angles and speeds of different machines, it is obvious that we should use measurements of these two signal variations and electro-mechanical power or torque variations for identification of a state space model of power system. Therefore, the model may content those modes which are more affected by the mentioned signals [12].

VI. Tests; 2-Area 4-Machine System

We apply the proposed index for several power system tests. We assess its performance, advantages and possibility of its application as an index for HB monitoring of power systems.

Figure-1 shows a single line diagram of two area test system. The system includes two areas which are connected by a weak line.

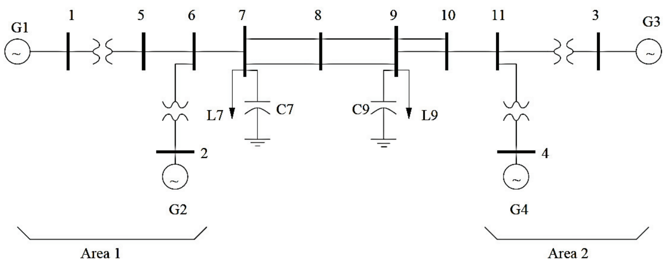

Figure-1. Single Line Diagram of Two-Area System

In [13], it was used for investigation of small signal analysis and inter-area oscillations. Two-area power system is proper for investigation of proposed index, since we have to extract some properties of power system from small signal variations.

We use computer simulation for investigation of small signal behavior of two-area power system. There are loads on 7^{th} and 9^{th} bus of system. In computer simulations, in order to insert disturbances into system, we increase the loads by a constant factor to the extent which instability occurs. Computer simulations show that the system has three basic mode; an inter-area mode and two local modes. Damping factors of other modes are more than 60 percent. Therefore, we can ignore their effects when we are interested in small signal analysis. We increase both of the loads in bus 7 and 9 by a constant factor, gradually. Figure-2 illustrates the results. Local mode of area one and inter-area mode affect stability, both together. However, local mode of area one crosses the imaginary axis sooner than inter-area mode.

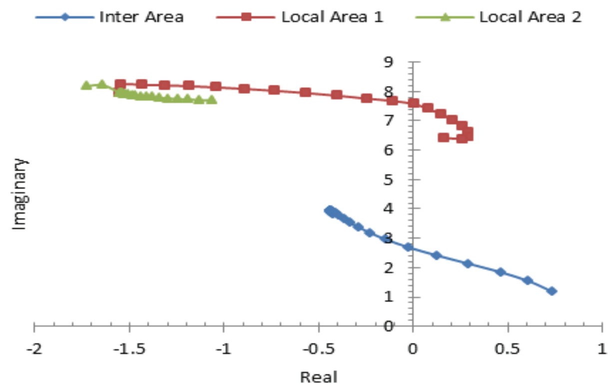

Figure-2. Two- Area System; Effect of total load increase on modes; Modes travels to right as total load increases gradually.

In order to find instability margin for the test system, one can investigate Figure-3. It shows variation of damping factors in accordance to total load variation. The test system is instable when damping factor of at least one of its mode becomes non-positive.

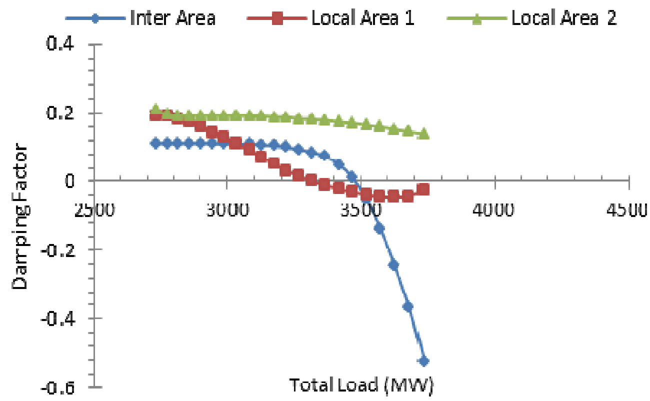

Figure-3. Two- Area System; decrease of mode damping factors in accordance to increase of total load.

Now, we try to find HB point using the proposed method. Therefore, we simulate two-area test system based on the mentioned total load increase, and then we sample the proper identification signals. We also add measurement noises to the output signals. However, measurement noise does not affect SSI results.

We use HBI1 and GMI in order to monitor two-area test system. Figure-4 illustrates online application of the proposed algorithm. It is also comparison of HBI1 and GMI. Both indices estimated 3280MW as marginal loading which is identical to analysis result in Figure-3.

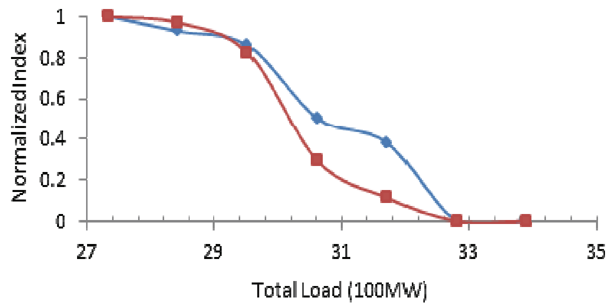

Figure-4. Two- Area System; Online Application of HBI1 (square) and GMI (diamond).

VII. Conclusions

In this work, we demonstrated a new Hopf Bifurcation index which is called GMI. It does not need to track an especial critical eigenvalue and in contrast to former HB indices, it does not depond on the frequency of the critical eigenvalue. Therefore, its estimation is easy and straight forward.

We also proposed Stochastic Subspace System Identification (SSSI) for estimation and prediction of GMI. SSI is attractive in system identification because of many advantages such as robust and consistent linear algebraic tools, large data handling, indefinite system order, and easy application for MIMO systems.

As a result of combining stochastic version of SSI methods and GMI, we presented an algorithm which easily and robustly can predict HB occurrence in power systems. It uses accessible signals such as electro-mechanical torques, speeds and angles of synchronous machines. Therefore, it is very easy to apply GMI for monitoring and prediction of abnormalities. GMI also has a fairly linear behavior without discontinuities with respect to system load changes. In comparison to former HB indices, the proposed index expresses a very successful online behavior when using SSSI algorithms.

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