On a Class of Inverse Problems for Loaded Equations with Nonlocal Conditions

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***Abstract*— We investigate problems of restoring the parameters of an object the state of which is described by a non-autonomous system of ordinary loaded differential equations with nonseparated point and integral conditions. To restore the parameters, additional conditions are given. We propose an approach to the numerical solution to the problem. The approach is based on the operation that convolves given integral conditions into point conditions. This approach allows reducing the solution to the initial problem to a Cauchy problem with respect to systems of ordinary differential and of linear algebraic equations. The approach is extended to a class of one-dimensional inverse problems for parabolic equations.**

Keywords*— *loaded differential equations; nonlocal conditions; system parameters; integral conditions; method of lines

I. INTRODUCTION

In the work, we investigate problems of restoring the parameters of an object described by a non-autonomous system of ordinary loaded differential equations with nonseparated point and integral conditions. To restore the parameters, additional conditions are given. We propose an approach to the numerical solution to the problems considered. This approach is based on the operation that convolves the integral conditions into point conditions [1]. The approach allows reducing the solution to the initial problem to a Cauchy problem with respect to systems of ordinary differential and of linear algebraic equations. By using method of lines, the approach can be extended to the class of one-dimensional inverse problems with respect to a loaded parabolic equation.

II. PROBLEM STATEMENT

Consider the following problem of restoring the parameters of the loaded differential equations

$$\dot{x}(t) = A(t)x(t) + \sum_{s=1}^{l_3} B^s(t)x(\breve{t}_s) + C(t)\lambda + K(t), \quad t \in (t_0, T), \tag{1}$$

$$\sum_{i=1}^{l_1} \int_{\bar{t}_i}^{\bar{t}_i+\Delta_i} \overline{D}_i(\tau)x(\tau)d\tau + \sum_{j=1}^{l_2} \widetilde{D}_j x(\widetilde{t}_j) + \sum_{s=1}^{l_3} \breve{D}_s x(\breve{t}_s) = L_0. \tag{2}$$

Here $x(t)$ is the unknown n-dimensional vector function; given matrices and vectors: $A(t)$, $B^s(t)$, $s = 1,...,l_3$, of the dimension $(n \times n)$, $C(t) - (n \times m)$, $K(t) - n$; $\bar{t}_i$, $\widetilde{t}_j$, $\breve{t}_s$, $i = 1,...,l_1$, $j = 1,...,l_2$, $s = 1,...,l_3$ are given ordered points of time from $[t_0, T]$; Δ_i are given lengths of the intervals of the state measurement; the $((n+m)\times n)$ matrix functions $\overline{D}_i(\tau)$ and numerical matrices $\widetilde{D}_j, \breve{D}_s$, as well as the $(n+m)$-dimensional vector L_0, l_1, l_2, l_3 are given. λ is the m-dimensional vector of the restorable parameters of the problem.

Note that many boundary-value problems involving partial derivatives along with nonlocal initial and boundary conditions are reduced to this kind of problem by using method of lines [2].

III. METHOD OF SOLUTION

Introduce the following $((n+m)\times n)$ matrix function:

$$D(t) = \sum_{i=1}^{l_1} [\chi(\bar{t}_i + \Delta_i) - \chi(\bar{t}_i)] \overline{\overline{D}}_i(t) + \sum_{j=1}^{l_2} \widetilde{D}_j \delta(t - \widetilde{t}_j) + \sum_{s=1}^{l_3} \breve{D}_s \delta(t - \breve{t}_s)$$

where $\chi(t)$ is the Heaviside function, $\delta(t)$ is the delta function,

$$\overline{\overline{D}}_i(t) = \begin{cases} \overline{D}_i(t), & t \in [\bar{t}_i, \bar{t}_i + \Delta_i], \\ 0, & t \notin [\bar{t}_i, \bar{t}_i + \Delta_i] \end{cases}.$$

Then (2) can be rewritten in the following form

$$\int_{t_0}^{T} D(\tau)x(\tau)d\tau = L_0. \tag{3}$$

Introduce the following $(n+m)$-dimensional vector functions

$$\overline{L}(t) = \int_{t_0}^{t} D(\tau)x(\tau)d\tau, \quad \underline{L}(t) = \int_{t}^{T} D(\tau)x(\tau)d\tau, \tag{4}$$

for which there holds true

$$\overline{L}(t_0) = \underline{L}(T) = 0, \quad \underline{L}(T) = \overline{L}(t_0) = L_0. \tag{5}$$

Definition. The $(n+m)\times n$ matrix functions $\overline{\alpha}(t), \underline{\alpha}(t)$, $\overline{\beta}^s(t), \underline{\beta}^s(t)$, $s = 1,...,l_3$, $(n+m)\times m$ functions $\overline{\xi}(t), \underline{\xi}(t)$, and $(n+m)$-dimensional vector functions $\overline{\gamma}(t), \underline{\gamma}(t)$ convolve integral conditions into point conditions if for $x(t)$, $t \in [t_0, T]$, the solution to the system (1), there hold true the following equalities

$$\int_{t_0}^{t} D(\tau)x(\tau)d\tau = \overline{\alpha}(t)x(t) + \sum_{s=1}^{l_3} \overline{\beta}^s(t)x(\breve{t}_s) + \overline{\xi}(t)\lambda + \overline{\gamma}(t), \tag{6}$$

$$\int_t^T D(\tau)x(\tau)d\tau = \underline{\alpha}(t)x(t) + \sum_{s=1}^{l_3} \underline{\beta}^s(t)x(\breve{t}_s) + \underline{\xi}(t)\lambda + \underline{\gamma}(t), \quad (7)$$

It is clear that there hold true the equalities

$$\bar{\alpha}(T)x(T) + \sum_{s=1}^{l_3} \bar{\beta}^s(T)x(\breve{t}_s) + \bar{\xi}(T)\lambda + \bar{\gamma}(T) = L_0, \quad (8)$$

$$\underline{\alpha}(t_0)x(t_0) + \sum_{s=1}^{l_3} \underline{\beta}^s(t_0)x(\breve{t}_s) + \underline{\xi}(t_0)\lambda + \underline{\gamma}(t_0) = L_0, \quad (9)$$

Each of the conditions (8) and (9) represents a nonlocal point boundary condition. Let $\bar{\alpha}(t), \bar{\beta}^s(t), s = 1,...,l_3, \bar{\xi}(t), \bar{\gamma}(t)$ and $\underline{\alpha}(t), \underline{\beta}^s(t), s = 1,...,l_3$, $\underline{\xi}(t)$, $\underline{\gamma}(t)$ be two pairs of functions that convolve the integral conditions (3) into point conditions from left to right and vice versa, respectively.

The functions $\bar{\alpha}(t), \bar{\beta}^s(t), s = 1,...,l_3, \bar{\xi}(t), \bar{\gamma}(t)$, and $\underline{\alpha}(t)$, $\underline{\beta}^s(t), s = 1,...,l_3$, $\underline{\xi}(t)$, $\underline{\gamma}(t)$, which convolve the integral conditions (3), are not uniquely determined. Let $0_{(n+m)\times n}$ be a $((n+m)\times n)$ null matrix, and $0_{(n+m)}$ be a $(n\times m)$ -dimensional null vector. The following theorem holds.

Theorem 1. If the functions $\bar{\alpha}(t)$, $\bar{\beta}^s(t)$, $s = 1,...,l_3$, $\bar{\xi}(t)$, $\bar{\gamma}(t)$ are the solutions to the following Cauchy problems:

$$\bar{\alpha}'(t) = -\bar{\alpha}(t)A(t) + D(t), \qquad \bar{\alpha}(t_0) = 0_{(n+m)\times n}, \quad (10)$$

$$\bar{\beta}'^s(t) = -\bar{\alpha}(t)B^s(t), \quad s = 1,...,l_3, \quad \bar{\beta}^s(t_0) = 0_{(n+m)\times n}, \quad (11)$$

$$\bar{\xi}'(t) = -\bar{\alpha}(t)C(t), \quad \bar{\xi}(t_0) = 0_{(n+m)\times m}, \quad (12)$$

$$\bar{\gamma}'(t) = -\bar{\alpha}(t)K(t), \quad \bar{\gamma}(t_0) = 0_{(n+m)}, \quad (13)$$

then these functions convolve the integral conditions (3) into the point conditions (8) from left to right.

Proof. Assume that there exists the following dependence

$$\bar{L}(t) = \bar{\alpha}(t)x(t) + \sum_{s=1}^{l_3} \bar{\beta}^s(t)x(\breve{t}_s) + \bar{\xi}(t)\lambda + \bar{\gamma}(t), \ t \in [t_0, T]. \quad (14)$$

Let $\bar{\alpha}(t), \bar{\beta}^s(t), s = 1,...,l_3, \bar{\xi}(t), \bar{\gamma}(t)$ be yet arbitrary matrix and vector functions of the dimensions $((n+m)\times n)$, $((n+m)\times n)$, $((n+m)\times n)$, and $(n+m)$, respectively, which satisfy the condition (5). Then there holds true the following equalities:

$$\bar{\alpha}(t_0) = 0_{(n+m)\times n}, \ \bar{\beta}^s(t_0) = 0_{(n+m)\times n}, \ s = 1,...,l_3, \\ \bar{\xi}(t_0) = 0_{(n+m)\times m}, \ \bar{\gamma}(t_0) = 0_{(n+m)}. \quad (15)$$

Differentiating (14) and taking (1) and (4) into account, we have

$$\left[\bar{\alpha}'(t) + \bar{\alpha}(t)A(t) - D(t)\right]x(t) + \\ + \sum_{s=1}^{l_3}\left[\bar{\beta}'^s(t) + \bar{\alpha}(t)B^s(t)\right]x(\breve{t}_s) + \\ + \left[\bar{\xi}'(t) + \bar{\alpha}(t)C(t)\right] + \left[\bar{\gamma}'(t) + \bar{\alpha}(t)K(t)\right] = 0. \quad (16)$$

Having regard to the arbitrariness of the functions $\bar{\alpha}(t), \bar{\beta}^s(t)$, $s = 1,...,l_3, \bar{\xi}(t), \bar{\gamma}(t)$, and to the fact that (16) must be satisfied for all $x(t)$, the solutions to the system (1), then it is necessary that each of the expressions in the brackets in (16) vanish, i.e. the conditions (10)-(13) of the theorem be satisfied.

The same proof remains valid for the following theorem.

Theorem 2. If the functions $\underline{\alpha}(t)$, $\underline{\beta}^s(t)$, $s = 1,...,l_3$, $\underline{\xi}(t)$, $\underline{\gamma}(t)$ are the solutions to the following Cauchy problems:

$$\underline{\alpha}'(t) = -\underline{\alpha}(t)A(t) - D(t), \qquad \underline{\alpha}(T) = 0_{(n+m)\times n}, \quad (17)$$

$$\underline{\beta}'^s(t) = -\underline{\alpha}(t)B^s(t), \quad s = 1,...,l_3, \quad \underline{\beta}^s(T) = 0_{(n+m)\times n}, \quad (18)$$

$$\underline{\xi}'(t) = -\underline{\alpha}(t)\ C(t), \quad \underline{\xi}(T) = 0_{(n+m)\times m}, \quad (19)$$

$$\underline{\gamma}'(t) = -\underline{\alpha}(t)\ K(t), \quad \underline{\gamma}'(T) = 0_{(n+m)}, \quad (20)$$

then these functions convolve the integral conditions (3) into the point conditions (9) from right to left.

The choice of the convolution scheme, which is to be applied to the condition (2), depends on the properties of the matrix $A(t)$, namely on its eigenvalues. If they are all positive, then the system (10)-(13) is stable; if they are all negative, then the system (17)-(20) is stable. If the matrix $A(t)$ has both positive and negative eigenvalues, and they are sufficiently large in modulus, then both of the systems have fast increasing solutions, and therefore unstable, and their numerical solution may result in a poor accuracy. In this case it is recommended to make use of the convolving functions proposed in the following theorem, which have a linear growth with respect to time.

Theorem 3. If the n-dimensional vector functions $g_\nu^1(t)$, $r_\nu(t)$, $q_\nu^s(t)$, $s = 1,...,l_3$, and scalar functions $g_\nu^2(t)$, $m_\nu(t)$ are the solutions to the following nonlinear Cauchy problems:

$$g_\nu'^1(t) = S(t)g_\nu^1(t) - A^*(t)g_\nu^1(t) + m_\nu(t)D_\nu^*(t), \ g_\nu^1(t_0) = 0, \ (21)$$

$$q_\nu'^s(t) = S(t)q_\nu^s(t) - B^{s*}(t)g_\nu^1(t), \ s = \overline{1, l_3}, \ q_\nu^s(t_0) = 0, \ (22)$$

$$r_\nu'(t) = S(t)r_\nu(t) - C^*(t)g_\nu^1(t), \quad r_\nu(t_0) = 0, \quad (23)$$

$$g_\nu'^2(t) = S(t)g_\nu^2(t) - K^*(t)g_\nu^1(t), \quad g_\nu^2(t_0) = 0. \quad (24)$$

$$\dot{m}_\nu(t) = S(t)m_\nu(t), \qquad m_\nu(t_0) = 1, \quad (25)$$

$$S(t) = \frac{\left[\dfrac{1}{2(T-t_0)} + g_\nu^{1*}(t)A(t)g_\nu^1(t) + m_\nu(t)D_\nu(t)g_\nu^1(t)\right.}{\left[g_\nu^{1*}(t)g_\nu^1(t) + (g_\nu^2(t))^2\right]} - \\ - \frac{\left.K^*(t)g_\nu^1(t)g_\nu^2(t)\ \right]}{\left[g_\nu^{1*}(t)g_\nu^1(t) + (g_\nu^2(t))^2\right]}, \quad (26)$$

then the functions $g_\nu^1(t), g_\nu^2(t)$ convolve the ν th integral condition (11) from left to right, and there holds true

$$g_\nu^{1*}(t)g_\nu^1(t) + \left(g_\nu^2(t)\right)^2 = (t - t_0)/(T - t_0), \ t \in [t_0, T]. \quad (27)$$

Proof. Multiplying the ν th equality from (14) by a yet arbitrary function $m_\nu(t)$ which satisfies the condition

$$m_\nu(t_0) = 1, \quad (28)$$

we obtain

$$m_\nu(t)\bar{L}(t) = m_\nu(t)\bar{\alpha}_\nu^*(t)x(t) +$$

$$+ m_\nu(t)\sum_{s=1}^{l_3}\bar{\beta}_\nu^{\,s}(t)x(\breve{t}_s) + m_\nu(t)\bar{\xi}_\nu(t)\lambda + m_\nu(t)\bar{\gamma}_\nu(t),\ t\in[t_0,T].$$

Here the $n-$dimensional vector $\bar{\alpha}_\nu(t)$ is the ν^{th} row of the matrix $\bar{\alpha}(t)$.

Introduce the notations

$$g_\nu^1(t) = m_\nu(t)\bar{\alpha}_\nu^*(t),\ \ g_\nu^2(t) = m^\nu(t)\bar{\gamma}_\nu(t),$$
$$q_\nu^s(t) = m^\nu(t)\bar{\beta}_\nu^{\,s}(t), s=\overline{1,l_3},\ \ r_\nu(t) = m_\nu(t)\bar{\xi}_\nu(t), \tag{29}$$

and it is clear that

$g_\nu^1(t_0)=0$, $g_\nu^2(t_0)=0$, $q_\nu^s(t_0)=0$, $r_\nu(t_0)=0$.

Ensuring the fulfillment of the condition (27), i.e. linear growth of the sum of squares of the convolving functions, is required of the function $m_\nu(t)$.

Differentiating (27), we obtain

$$2(\dot{g}_1^{\,\nu}(t), g_1^{\,\nu}(t)) + 2\dot{g}_2^{\,\nu}(t)g_2^{\,\nu}(t) = 1/(T-t_0). \tag{30}$$

Differentiating (29) and taking (10)–(13) into account, it is not difficult to obtain the following set of equations

$$g_\nu'^{1}(t) = \frac{m_\nu'(t)}{m_\nu(t)}g_\nu^{\,1}(t) - A^*(t)g_\nu^{\,1}(t) + m_\nu(t)D_\nu^*(t) \tag{31}$$

$$q_\nu'^{s}(t) = \frac{m_\nu'(t)}{m_\nu(t)}q_\nu^s(t) - B^{s*}(t)g_\nu^1(t),\ \ s=\overline{1,l_3}, \tag{32}$$

$$r_\nu'(t) = \frac{m_\nu'(t)}{m_\nu(t)}r_\nu(t) - C^*(t)g_\nu^1(t), \tag{33}$$

$$g_\nu'^{2}(t) = \frac{\dot{m}^\nu(t)}{m^\nu(t)}g_\nu^{\,2}(t) - K^*(t)g_\nu^{\,1}(t). \tag{34}$$

Substituting the derivatives obtained into (29), we have

$$\Big(\frac{m_\nu'(t)}{m_\nu(t)}g_\nu^1(t) - A^*(t)g_\nu^1(t) + m_\nu(t)D_\nu^*(t), g_\nu^1(t)\Big) +$$
$$+\frac{m_\nu'(t)}{m_\nu(t)}\left(g_\nu^2(t)\right)^2 - K^*(t)g_\nu^1(t)g_\nu^2(t) = \frac{1}{2(T-t_0)}.$$

From here, it is not difficult to obtain the equation (25) by using the notation (26). Substituting (25) into (31)-(34), we obtain the equations (21)-(24).

Let, for example, the convolution of the condition (2) to the right have been used, and the problem (1), (8) of determining $x(t)\in R^n$ and $\lambda\in R^m$ been obtained. Next, we shift the condition (8) to the left with the use of the $(n\times n)$ functions $Q(t)$, $G^s(t)$, $s=1,...,l_3$, $(n\times m)$ function $V(t)$, and n-dimensional vector function $\mathrm{W}(t)$, for which there hold true [3, 4]:

$$Q(t)x(t) + \sum_{s=1}^{l_3}G^s(t)x(\breve{t}_s) + V(t)\lambda = \mathrm{W}(t),\ \ \ t\in[T;\breve{t}_{l_3}), \tag{35}$$

$$Q(T) = \bar{\alpha}(T), G^s(T) = \bar{\beta}^s(T), s=1,2...,l_3, \tag{36}$$

$$V(T) = \bar{\xi}(T),\ \mathrm{W}(T) = L_0 - \bar{\gamma}(T). \tag{37}$$

Implementing this shift procedure successively for all the intervals $t\in(\breve{t}_{s+1},\breve{t}_s]$, we obtain a system of linear algebraic equations of the order $(m+l_3n+n)$ with respect to unknowns $x(T)$, $x(\breve{t}_s)\in R^n$, $s=1,2...,l_3$, $\lambda\in R^m$. Having solved this system and substituting the solution obtained into (1), (2), the desired solution is determined from the solution to the Cauchy problem ensued.

The similar procedure is possible for shifting the condition (7) successively to the right.

The functions $\bar{\alpha}(t), \underline{\alpha}(t)$, $\bar{\beta}^s(t), \underline{\beta}^s(t)$, $s=1,...,l_3$, $\bar{\xi}(t),\underline{\xi}(t)$, $\bar{\gamma}(t),\underline{\gamma}(t)$ convolving the integral conditions, and the functions $Q(t)$, $G^s(t)$, $s=1,...,l_3$, $V(t)$, $\mathrm{W}(t)$, shifting the conditions (6), (7), are not uniquely defined.

IV. Partial Differential Equations

The approach proposed in the work, by using method of lines as described in [5], can be extended to partial differential equations.

In particular, let us consider the following parametric identification problem with respect to a one-dimensional loaded parabolic equation:

$$u_t(x,t) = u_{xx}(x,t) + \sum_{s=1}^{l_3}B^s(x,t)u(\breve{x}_s,t) + H(t)E(x,t) + f(x,t),$$
$$(x,t)\in\Omega = \{(x,t): 0<x<l,\ \ 0<t\le T\}, \tag{38}$$

under initial condition

$$u(x,0) = \varphi(x),\ 0\le x\le l, \tag{39}$$

and nonlocal point and integral nonseparated conditions:

$$\sum_{i=1}^{l_1}\int_{\bar{x}_i}^{\bar{x}_i+\Delta_i}\bar{D}_i(x,t)u(x,t)dx +$$
$$+\sum_{j=1}^{l_2}\tilde{D}_j(t)u(\tilde{x}_j,t) + \sum_{s=1}^{l_3}\breve{D}_s(t)u(\breve{x}_s,t) = L_0(t), \tag{40}$$

where t and x are time and spatial coordinates, respectively; $u(x,t)$ is the process state at the point x at the point of time t; $\bar{x}_i,\tilde{x}_j,\breve{x}_s$, $i=1,...,l_1$, $j=1,2,...l_2$, $s=1,2,...,l_3$ are given points of the interval $(0,l)$, at that $\bar{x}_{i+1}>\bar{x}_i$ and $\bar{x}_l+\Delta_l\in[0,l]$; $\min(\bar{x}_1,\tilde{x}_1)=0$, $\max(\bar{x}_{l_1}+\Delta_1,\tilde{x}_{l_2})=l$, and for all $i=1,...,l_1$, $j=1,...,l_2$ there holds the condition $\tilde{x}_j \,\bar{\in}\, [\bar{x}_i,\bar{x}_i+\Delta_i]$; $f(x,t)$, $\varphi(x)$ are given continuous functions for $0\le x\le l$ and $0\le t\le T$; $\bar{D}_i(x,t)$, $\tilde{D}_j(t)$ are the three-dimensional vector functions continuous with respect to all their arguments.

The inverse problem (38)-(40) consists in determining the unknown continuous function $H(t)$ ***and the corresponding solution to the nonlocal problem*** $u(x,t)$ ***satisfying the conditions (39)-(40).***

To apply method of lines at the segment $[0,T]$, take the points $t_k = k\tau$, $\tau = T/n$ and draw straight lines $t = t_k$, $k = 0,1,...,n$. Introducing the notations $U^{(k)}(x) = u(x,t_k)$, $f^{(k)}(x) = f(x,t_k)$, $E^{(k)}(x) = E(x,t_k)$, $H^{(k)} = H(t_k)$, and replacing $\frac{\partial u(x,t)}{\partial t}\Big|_{t=t_k}$ with a difference relation, we obtain the following second order ordinary differential equations:

$$U''^{(k)}(x) = \frac{1}{\tau}U^{(k)}(x) - \sum_{s=1}^{l_3} B_s^{(k)}(x)U^{(k)}(\breve{x}_s) - \\ - E^{(k)}(x)H^{(k)} - \frac{1}{\tau}U^{(k-1)}(x) - f^{(k)}(x), \quad k = \overline{1,n}, \\ U^{(0)}(x) = \varphi(x), \tag{41}$$

solved sequentially from $k = 1$ to $k = n$.
Under (40), there are the following nonlocal conditions for these equations:

$$\sum_{i=1}^{l_1} \int_{\bar{x}_i}^{\bar{x}_i+\Delta_i} \overline{D}_i^{(k)}(x)U^{(k)}(x)dx + \\ \sum_{j=1}^{l_2} \widetilde{D}_j^{(k)}U^{(k)}(\widetilde{x}_j) + \sum_{s=1}^{l_3} \breve{D}_s^{(k)}U^{(k)}(\breve{x}_j) = L_0^{(k)}, \tag{42}$$

where we used the notations $\overline{D}_i^{(k)}(x) = \overline{D}_i(x,t_k)$, $\widetilde{D}_j^{(k)} = \widetilde{D}_j(t_k)$, $\breve{D}_s^{(k)} = \breve{D}_s(t_k)$.

The problem (41)-(42) can be reduced to the following problem with respect to the system of two first order differential equations:

$$U_1'^{(k)}(x) = U_2^{(k)}(x), \\ U_2'^{(k)}(x) = \frac{1}{\tau}U_1^{(k)}(x) - -\sum_{s=1}^{l_3} B_s^{(k)}(x)U_1^{(k)}(\breve{x}_s) - \\ -\frac{1}{\tau}U_1^{(k-1)}(x) - E^{(k)}(x)H^{(k)} - f^{(k)}(x), \quad k = 1,2,...,n, \tag{43}$$

at that $U_1^{(0)}(x) = \varphi(x)$, with nonseparated integral and point conditions:

$$\sum_{i=1}^{l_1} \int_{\bar{x}_i}^{\bar{x}_i+\Delta_i} \overline{D}_i^{(k)}(x)U_1^{(k)}(x)dx + \\ + \sum_{j=1}^{l_2} \widetilde{D}_j^{(k)}(t)U_1^{(k)}(\widetilde{x}_j) + \sum_{s=1}^{l_3} \breve{D}_s^{(k)}U_1^{(k)}(\breve{x}_s) = L_0^{(k)}. \tag{44}$$

Next, to solve the system of differential equations (43) with nonlocal conditions (44), one can apply the above mentioned scheme.

Note another case; let

$$u_t(x,t) = u_{xx}(x,t) + \sum_{s=1}^{l_3} B^s(x,t)u(x,\breve{t}_s) + H(x)E(x,t) + f(x,t), \\ (x,t) \in \Omega = \{(x,t): 0 < x < l, \ 0 < t \le T\}, \tag{45}$$

and the initial condition (39) can be given in the form of nonseparated point and integral conditions. Assume that instead of the initial condition (39), we have another condition in the following form:

$$\sum_{i=1}^{l_1} \int_{\bar{t}_i}^{\bar{t}_i+\Delta_i} \overline{D}_i(x,t)u(x,t)dt + \\ + \sum_{j=1}^{l_2} \widetilde{D}_j(x)u(x,\widetilde{t}_j) + \sum_{s=1}^{l_3} \breve{D}_s(x)u(x,\breve{t}_s) = L_0(x), \tag{45}$$

and boundary conditions have the classical form:

$$u(0,t) = \psi_1(t), \ u(l,t) = \psi_2(t), \quad 0 \le x \le T. \tag{46}$$

Here $\psi_1(t)$, $\psi_2(t)$ are given continuous functions; functions $\overline{D}_i(x,t)$, $\widetilde{D}_j(x)$, $\breve{D}_s(x)$, $L_0(x)$ are continuous with respect to x, t, $i = 1,...,l_1$, $j = 1,...,l_2$, $s = 1,...,l_3$.

The inverse problem (38),(45),(46) consists in determining the unknown continuous function $H(x)$ and the corresponding solution to the nonlocal problem $u(x,t)$ satisfying the conditions (39)-(40).

To apply method of lines at the segment $[0,l]$, take the points $x_k = kh$, $h = l/n$ and draw straight lines $x = x_k$, $k = 0,1,...,n$. Introducing the notations $U^{(k)}(t) = u(x_k,t)$, $E^{(k)}(t) = E(x_k,t)$, $H^{(k)} = H(x_k)$, and replacing $\frac{\partial^2 u(x,t)}{\partial x^2}\Big|_{x=x_k}$ with a difference relation, we obtain the following system of n first order ordinary differential equations:

$$U'^{(k)}(t) = \frac{\left(U^{(k+1)}(t) - 2U^{(k)}(t) + U^{(k-1)}(t)\right)}{h^2} + \\ + \sum_{s=1}^{l_3} B_s^{(k)}(x)U^{(k)}(\breve{t}_s) + E^{(k)}(t)H^{(k)} - f^{(k)}(t), \ k = \overline{1,n}, \\ U^{(0)}(t) = \psi_1(t), \ U^{(n+1)}(t) = \psi_2(t), \tag{47}$$

with nonseparated multipoint and integral conditions

$$\sum_{i=1}^{l_1} \int_{\bar{t}_i}^{\bar{t}_i+\Delta_i} \overline{D}_i^{(k)}(t)U^{(k)}(t)dt + \sum_{j=1}^{l_2} \widetilde{D}_j^{(k)}(t)U^{(k)}(\widetilde{t}_j) + \\ + \sum_{s=1}^{l_3} \breve{D}_s^{(k)}(t)U^{(k)}(\breve{t}_s) = L_0^{(k)}. \tag{48}$$

It is easy to see that the problem (47), (48) coincides with the problem (1), (2), therefore, to its numerical solution one can apply the results of section 3.

Results of the numerical experiments carried out and their analysis will be reported at the presentation.

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