Graph Representation of Locked Numbers and Sequences

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***Abstract*— Locked numbers and sequences are a generalization of the Kaprekar constant discovered in 1946 by the Indian mathematician D. R. Kaprekar for different bases and number of digits. In this study, the notion of locked numbers and sequences are explained and graphs of locked numbers and sequences are prepared.**

Keywords*— *Kaprekar's constant; locked numbers; sequences; graph

I. Introduction

Notion of "Locked Number" was defined by Dj. A. Babayev firstly in 2004 [1, 2]. This notion is a generalization of the Kaprekar constant (6174) discovered in 1946 by the Indian mathematician D. R. Kaprekar for different bases and number of digits [3]. Let "*N*", be a four-digit integer, with not all digits same. Rearrange its digits into descending and ascending order smaller one is called S(N) and larger is L(N). Small number is subtracted from the larger one and the result is called *R(N)*, (*R(N)=L(N)-S(N)*), and a constant number is reached when repeating same ordering and subtracting processes for result.

Two cases occur at the end of ordering and subtracting processes for integer "*N*" given. First situation ends with a repetition sequence at the end of ordering and subtracting processes of integer "*N*". Second situation ends with a constant as also stated above in ordering and subtracting processes of integer "*N*".

II. Locked Numbers and Sequences

2.1. Formulation and Definitions

2.1.1. Formulation

N : An integer with not all digits are same (at least one of its digits is different).

L=L(N) : The integer obtained by ordering digits of N in descending order.

S=S(N) : The integer obtained by ordering digits of N in ascending order. *L(N)* and *S(N)* are the largest and smallest numbers having the same set of digits.

R=R(N) : *L(N)-S(N)*, remainder.

2.1.2. Definitions

Ordering and subtracting processes consists of defining of *L(N)*, *S(N)* and *R(N)* for a given *N*.

1. Given *N* define $N^1=N$ and apply ordering and subtracting processes to N^1
Define $L(N^1)$, $S(N^1)$, and $R(N^1)=L(N^1)-S(N^1)$

2. $N^2=R(N^1)$ and ordering and subtracting processes are applied for N^2.

1) In general ordering and subtracting processes consists of consecutive repetitions of N^i, i=1,2,.... $L(N^i)$, $S(N^i)$, $R(N^i)=L(N^i)-S(N^i)$ and $N^{i+1}=R(N^i)$ are defined. If the last digit of a given L(N) equal to zero, S(N) starts with zero.

2) E.g., L(N)=32100 and S(N)=00123 for a 10-based number N=21003. To keep the number of digits unchanged during ordering and subtracting processes these zeros are not removed although unnecessary.

3) Integers with numbers of digits less than the selected N may appear also in the ordering and subtracting processes. The remainder $R(N^i)$ may have number of digits less than (L)N.

4) E.g., A 10-based N=2122 implies L(N)=2221, S(N)=1222 and R(N)=999. Similarly to the above mentioned, this number will be written as 0999. This allows all numbers generated in the ordering and subtracting processes to have the same number of digits.

2.2. Sequences and Locked Numbers

2.2.1. Sequences

In ordering and subtracting processes, given number starts with $N^1=N$ and generates a sequence as $N^2=R(N^1)$, $N^3=R(N^2)$, …, $N^{i+1}=R(N^i)$.. All generated numbers have the same number of digits. There is *n*-digit number on finite number generated in ordering and subtracting processes for given *n*-digit number. N^i generated in ordering and subtracting processes will produce $N^m=N^i$ by repeating after *m* step. It implies that this number sequence will be repeated starting from N^i to N^m. Length of this sequence generated in ordering and subtracting; equals to *m-i*.

E.g., 2.2.1.1.

To show the sequence generated on 10-base 5-digit;

Given *N*=70605

1. Step. N^1=70605; *L*=76500, *S*=00567, *L-S*=75933,
2. Step. N^2=75933; *L*=97533, *S*=33579, *L-S*=63594,

3. Step. N^3=63594; L=96543, S=34569, L-S=61974,
4. Step. N^4=61974; L=97641, S=14679, L-S=82962,
5. Step. N^5=82962; L=98622, S=22689, L-S=75933,
6. Step N^6=75933; L=97533, S=33579, L-S=63594.

N^2 was repeated in 6 steps; in this case, length of sequence is 6-2=4.

2.2.2. Locked Numbers

The sequences recurring themselves in ordering and subtracting processes, on the other hand, their lengths equal to 1 are called locked numbers. Shown as follows:

$R(N^i)=R(N^{i+1})$.

E.g., 2.2.2.1.

Given N=578

1. Step. N^1=578; L=875, S=578, L-S=297,
2. Step. N^2=297; L=972, S=279, L-S=693,
3. Step. N^3=693; L=963, S=369, L-S=594,
4. Step. N^4=594; L=954, S=459, L-S=495,
5. Step. N^5=495; L=954, S=459, L-S=495.

As it is seen, $R(N^4)=R(N^5)$. 495 is locked number in 3-digit numbers.

III. GRAPH REPRESENTATION OF LOCKED NUMBERS AND SEQUENCES

Numbers can be classified after determining the path of each number reached to locked numbers and sequences [4].

Number : An integer with any n-digit.

Basic Number : An integer arranged in order of descending the digits.

Order Number : Basic numbers sharing the same path to reach to locked number or sequence.

Class : Order numbers including the ordering and subtracting processes in same step.

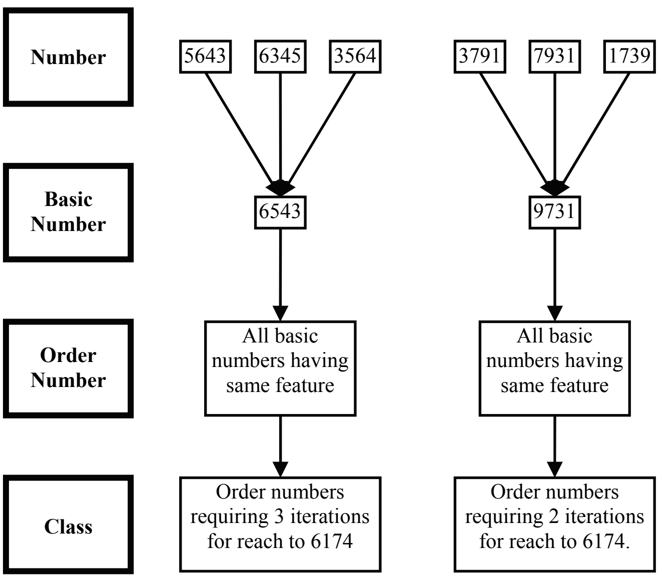

Figure 3.1. Classification of numbers

The graph of the path followed by a number while reaching to the locked numbers and sequences can be drawn. Numbers encountered after ordering and subtracting for order number are equal to each other. For vertices on graph, it will be more meaningful that using a formulation of order numbers instead of using all number in same digits. As a result, each vertex shows a special representation of an order number.

3.1. Formulation: We can classify all basic numbers in n-digit with same order as follows;

Including $k=\lfloor n/2 \rfloor$ and $L(n)= l_1l_2l_3....l_{n-1}l_n$, digits of number k are respectively as follow:

$k=\{l_i-l_j | i+j=n+1,\ i=1,2,...,\lfloor n/2 \rfloor,\ \text{and}\ j=n,(n-1),...(\lceil n/2 \rceil+1)\ \}$

A_k, is a set of all order numbers that obtained same number k.

3.2. Representation of Two-Digit Numbers as Graph

A_1={10, 21, 32, 43, 54, 65, 76, 87, 98}

A_2={20, 31, 42, 53, 64, 75, 86, 97}

A_3={30, 41, 52, 63, 74, 85, 96}

A_4={40, 51, 62, 73, 84, 95}

A_5={50, 61, 72, 83, 94}

A_6={60, 71, 82, 93}

A_7={70, 81, 92}

A_8={80, 91}

A_9={90}

For 10-Based 2-Digit Numbers;

Locked Number →None

Sequence →9→81→63→27→45
=A_9→A_7→A_3→A_5→A_1

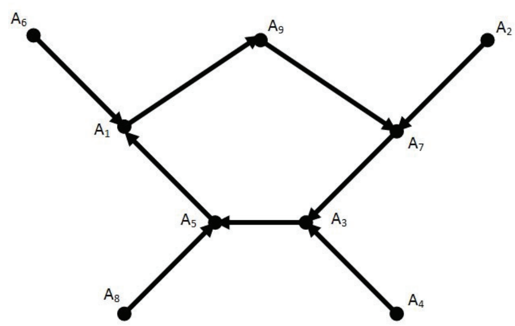

Figure 3.2. Graph of 10-based 2-digit numbers (A_9→A_7→A_3→A_5→A_1)

3.3. Formulation of Three-Digit Numbers as Graph

A_1={1x0, 2x1, 3x2, 4x3, 5x4, 6x5, 7x6, 8x7, 9x8}

A_2={2x0, 3x1, 4x2, 5x3, 6x4, 7x5, 8x6, 9x7}

A_3={3*x*0, 4*x*1, 5*x*2, 6*x*3, 7*x*4, 8*x*5, 9*x*6}

A_4={4*x*0, 5*x*1, 6*x*2, 7*x*3, 8*x*4, 9*x*5}

A_5={5*x*0, 6*x*1, 7*x*2, 8*x*3, 9*x*4}

A_6={6*x*0, 7*x*1, 8*x*2, 9*x*3}

A_7={7*x*0, 8*x*1, 9*x*2}

A_8={8*x*0, 9*x*1}

A_9={9*x*0}

Note: In this formulation, middle digit was displayed as *x* for 10-based 3-digit basic numbers because of insignificant.

For 10-Based 3-Digit Numbers;

Locked Number ➔495 $=A_5$

Sequence ➔ None

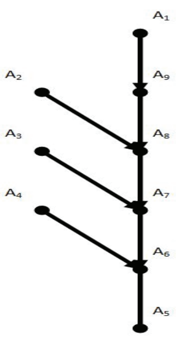

Figure 3.3. Graph of 10-based 3-digit numbers (A_5)

3.4. Formulation of Four-Digit Numbers as Graph

A_{10}={1000, 1110, 2111, 2221, 3222, …, 9998}

A_{11}={1100, 2211, 3322, 4433, …, 9988}

⋮

A_{99}={9900}

For 10-Based 4-Digit Numbers;

Locked Number ➔6174 $=A_{62}$

Sequence ➔ None

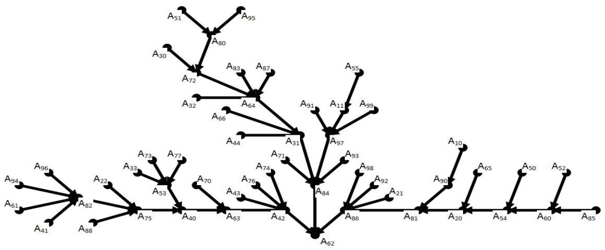

Figure 3.4. Graph of 10-based 4-digit numbers (A_{62})

3.5. Formulation of Five-Digit Numbers as Graph

A_{10}={10*x*00, 11*x*10, 21*x*11, 22*x*21, 32*x*22, …, 99*x*98}

A_{11}={11*x*00, 22*x*11, 33*x*22, 44*x*33, …, 99*x*88}

⋮

A_{99}={99*x*00}

For 10-Based 5-Digit Numbers;

Locked Number ➔ None

Sequence ➔ 53955 → 59994
$= A_{60} \rightarrow A_{54}$

➔63954 → 61974 → 82962 → 75933
$=A_{62} \rightarrow A_{83} \rightarrow A_{76} \rightarrow A_{64}$

➔74943 → 62964 → 71973 → 83952
$= A_{63} \rightarrow A_{72} \rightarrow A_{84} \rightarrow A_{75}$

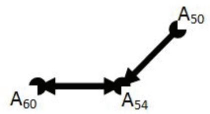

Figure 3.5. Graph of 10-based 5-digit numbers ($A_{60} \rightarrow A_{54}$)

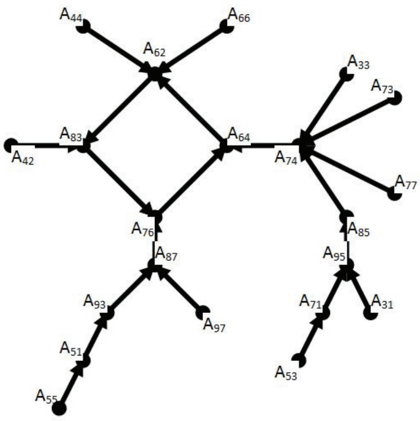

Figure 3.6. Graph of 10-based 5-digit numbers ($A_{62} \rightarrow A_{83} \rightarrow A_{76} \rightarrow A_{64}$)

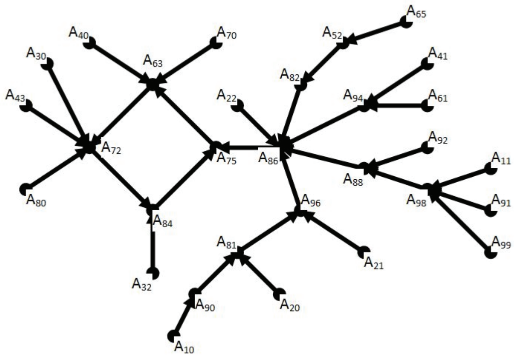

Figure 3.7. Graph of 10-based 5-digit numbers ($A_{63} \rightarrow A_{72} \rightarrow A_{84} \rightarrow A_{75}$)

3.6. Formulation of Six-Digit Numbers as Graph

For 10-Based 6-Digit Numbers;

Locked Number ➔549945 $=A_{550}$

➔631764 $=A_{632}$

Sequence ➔ 851742 → 750843 → 840852→860832 → 862632 → 642654 → 420876 $=A_{651}$→A_{841}→A_{861}→A_{863}→A_{643}→A_{421}→A_{852}

Figure 3.8. Graph of 10-based 6-digit numbers (A_{550})

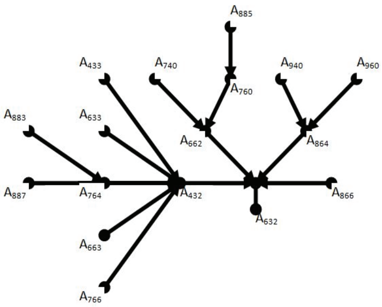

Figure 3.9. Graph of 10-based 6-digit numbers (A_{632})

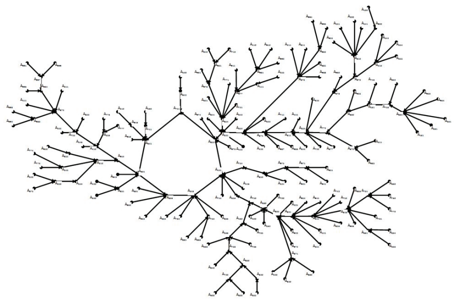

Figure 3.10. Graph of 10-based 6-digit numbers (A_{651}→A_{841}→A_{861}→A_{863}→A_{643}→A_{421}→A_{852})

3.7. Formulation of Seven-Digit Numbers as Graph

For 10-Based 7-Digit Numbers;

Locked Number ➔ None

Sequence ➔ 8429652→ 7619733→ 8439552→ 7509843→ 9529641→ 8719722→ 8649432→7519743 $=A_{762}$→ A_{844}→ A_{751}→ A_{953}→ A_{872}→ A_{865}→ A_{752}→ A_{843}

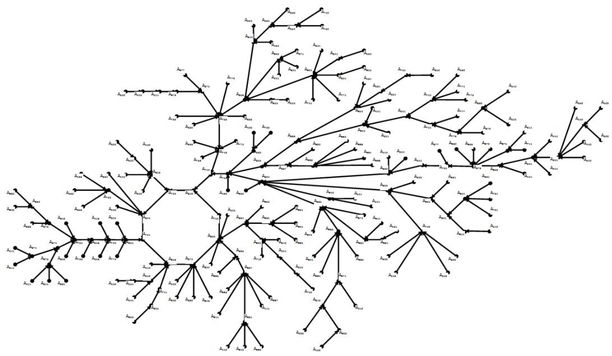

Figure 3.11. Graph of 10-based 7-digit numbers (A_{762}→ A_{844}→ A_{751}→ A_{953}→ A_{872}→ A_{865}→ A_{752}→ A_{843})

IV. CONCLUSION

In this study, graph representation of 10-based locked numbers and sequences were examined. It is thought that locked numbers and sequences will be used in information technology (cryptology, computer games, and so on) [5].

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