Identification of Heterogeneous Stratum Parameters in Gas-Condensate Mixture Filtration

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***Abstract*— Modeling of strata heterogeneity during gas-condensate mixture filtration has been carried out on the basis of binary model whereby the problem of determination of vertical stratum permeability change has generally been solved in variational formation.**

***Keywords*— gaz-condensate mixture; variation problem; pressure; porosity; permeability**

I. Introduction

The productive horizons of the hydrocarbon accumulations as a rule have a complex structure but stratum reservoir properties vary as through the section so in area of its strike. Technological parameters of the reservoir development in their prediction depend strongly on the heterogeneity of strata permeability. Therefore questions of their possible definitions are an important problem in the theory and practice of hydrocarbon fields development.

Assume that the heterogenous stratum with thickness H and with the radius R_κ is bounded by two impermeable surfaces and exposed by the well with radius r_c which works with bottom-hole pressure $p_c(z,t)$. It is necessary to determine: pressure distribution $p(r,z,t)$ and some physical reservoir characteristics.

II. Statement of the Problem

Mathematically the problem reduces to solution of the system of equations [1]:

$$\frac{1}{r}\frac{\partial}{\partial r}\left\{rk(r,z)\left(\frac{F_g(\rho_k)p\beta\left[1-c(p)\bar{\gamma}(p)\right]}{\mu_g(p)z(p)p_{atm}}+\frac{F_k(\rho_k)S_k(p)}{\mu_k(p)a_k(p)}\right)\frac{\partial p}{\partial r}\right\}+$$
$$+\frac{\partial}{\partial z}\left\{k(r,z)\left(\frac{F_g(\rho_k)p\beta\left[1-c(p)\bar{\gamma}(p)\right]}{\mu_g(p)z(p)p_{atm}}+\frac{F_k(\rho_k)S_k(p)}{\mu_k(p)a_k(p)}\right)\frac{\partial p}{\partial z}\right\}=$$
$$=\frac{\partial}{\partial t}\left\{m\left(\frac{(1-\rho_k)p\beta}{z(p)p_{atm}}\left[1-c(p)\bar{\gamma}(p)\right]+\rho_k\frac{S_k(p)}{a_k(p)}\right)\right\},$$
$$(r,z)\in D,\quad t\in(0,T), \tag{1}$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left\{rk(r,z)\left(\frac{F_k(\rho_k)}{\mu_k(p)a_k(p)}+\frac{F_g(\rho_k)c(p)\beta}{\mu_g(p)z(p)p_{atm}}\right)\frac{\partial p}{\partial r}\right\}+$$
$$+\frac{\partial}{\partial z}\left\{k(r,z)\left(\frac{F_k(\rho_k)}{\mu_k(p)a_k(p)}+\frac{F_g(\rho_k)pc(p)\beta}{\mu_g(p)z(p)p_{atm}}\right)\frac{\partial p}{\partial z}\right\}=$$
$$=\frac{\partial}{\partial t}\left\{m\left(\frac{\rho_k}{a_k(p)}+(1-\rho_k)\frac{p\beta c(p)}{z(p)p_{atm}}\right)\right\},$$
$$(r,z)\in D,\quad t\in(0,T). \tag{2}$$

The initial and boundary conditions are as follows:

$$p(r,z,t)\big|_{t=0}=p_0,\ \rho_k(r,z,t)\big|_{t=0}=0,\ (r,z)\in D, \tag{3}$$

$$p(r,z,t)\big|_{r=r_w}=p_w(z,t), \tag{4}$$

$$\frac{\partial p(r,z,t)}{\partial r}\bigg|_{r=R_k}=0,\ \frac{\partial p(r,z,t)}{\partial z}\bigg|_{z=0;H}=0,\ t\in(0,T), \tag{5}$$

where $p(r,z,t)$ is the pressure; $\rho_\kappa(r,z,t)$ - is the condensate saturation, $F_g(\rho_k)$ and $F_\kappa(\rho_k)$ - are the is relative phase permeability respectively for gas and liquid phases; $c(p)$ - content of the condensate in the gas phase $\bar{\gamma}(p)$ - is the ratio of the condensate gravity in the liquid and gas phase under normal conditions; $S_\kappa(p)$ - amount of dissolved gas in a liquid; $a_\kappa(p)$ - volumetric factor of the liquid phase; m - porosity; k - permeability; t - time; p_{atm} - atmospheric pressure, β and $z(p)$ - coefficients according to temperature correction and compressibility for gas phase, $\mu_g(p)$ and $\mu_k(p)$ - viscosity according to gas and liquid phases; D - filtration area.

III. Solition to the Problem

Differential solutions algorithm $p(x,z,t)$, $\rho_k(x,z,t)$ at given $k(r,z)$ under the conditions (1) - (5) takes the form:

$$e^{-2x_i}\left\{\frac{1}{\Delta x_i}\left[k^n_{i+1/2,j}\cdot\Psi^{n+1}_{i+1/2,j}\frac{p^{n+1}_{i+1,j}-p^{n+1}_{i,j}}{\Delta x_{i+1/2}}-k^n_{i-1/2,j}\cdot\Psi^{n+1}_{i-1/2,j}\cdot\frac{p^{n+1}_{i,j}-p^{n+1}_{i-1,j}}{\Delta x_{i-1/2}}\right]\right\}+$$
$$+\frac{1}{\Delta z_j}\left[k^n_{i,j+1/2}\cdot\Psi^{n+1}_{i,j+1/2}\frac{p^{n+1}_{i,j+1}-p^{n+1}_{i,j}}{\Delta z_{j+1/2}}-k^n_{i,j-1/2}\cdot\Psi^{n+1}_{i,j-1/2}\cdot\frac{p^{n+1}_{i,j}-p^{n+1}_{i,j-1}}{\Delta z_{j-1/2}}\right]-$$
$$-A^n_{i,j}\left\{e^{-2x_i}\left[\frac{1}{\Delta x_i}\left(k^n_{i+1/2,j}\cdot\Phi^{n+1}_{i+1/2,j}\frac{p^{n+1}_{i+1,j}-p^{n+1}_{i,j}}{\Delta x_{i+1/2}}-k^n_{i-1/2,j}\cdot\Phi^{n+1}_{i-1/2}\cdot\frac{p^{n+1}_{i,j}-p^{n+1}_{i-1,j}}{\Delta x_{i-1/2}}\right)\right]+\right.$$
$$\left.+\frac{1}{\Delta z_j}\left[k^n_{i,j+1/2}\cdot\Phi^{n+1}_{i,j+1/2}\frac{p^{n+1}_{i,j+1}-p^{n+1}_{i,j}}{\Delta z_{j+1/2}}-k^n_{i,j-1/2}\cdot\Phi^{n+1}_{i,j-1/2}\cdot\frac{p^{n+1}_{i,j}-p^{n+1}_{i,j-1}}{\Delta z_{j-1/2}}\right]\right\}=$$
$$=\left[\left(Q'^n_{i,j}-A^n_{i,j}M'^n_{i,j}\right)+\rho_{ki,j}\left(N'^n_{i,j}-A^n_{i,j}B'^n_{i,j}\right)\right]\frac{p^{n+1}_{i,j}-p^n_{i,j}}{\Delta\tau},\qquad(6)$$

$$e^{-2x_i}\left\{\frac{1}{\Delta x_i}\left[k^n_{i+1/2,j}\cdot\Phi^{n+1}_{i+1/2,j}\frac{p^{n+1}_{i+1,j}-p^{n+1}_{i,j}}{\Delta x_{i+1/2}}-k^n_{i-1/2,j}\cdot\Phi_{i-1/2}\cdot\frac{p^{n+1}_{i,j}-p^{n+1}_{i-1,j}}{\Delta x_{i-1/2}}\right]\right\}+$$
$$+\frac{1}{\Delta z_j}\left[k^n_{i,j+1/2}\cdot\Phi^{n+1}_{i,j+1/2}\frac{p^{n+1}_{i,j+1}-p^{n+1}_{i,j}}{\Delta z_{j+1/2}}-k^n_{i,j-1/2}\cdot\Phi^{n+1}_{i,j-1/2}\cdot\frac{p^{n+1}_{i,j}-p^{n+1}_{i,j-1}}{\Delta z_{j-1/2}}\right]-$$
$$-\left[M^n_{i,j}+\rho^n_{ki,j}B'^n_{i,j}\right]\frac{p^{n+1}_{i,j}-p^n_{i,j}}{\Delta\tau}=B^n_{i,j}\frac{\rho^{n+1}_{ki,j}-\rho^n_{ki,j}}{\Delta\tau},\qquad(7)$$

where

$$\Psi(p,\rho_k)=\left[\frac{F_g(\rho_k)p\beta[1-C(p)\gamma(p)]}{\mu_g(p)z(p)p_{atm}}+\frac{F_k(p_k)S_k(p)}{\mu_k(p)a_k(p)}\right],$$
$$\Phi(p,\rho_k)=\left[\frac{F_g(\rho_k)pC(p)\beta}{\mu_z(p)z(p)p_{atm}}+\frac{F_k(\rho_k)}{\mu_k(p)a_k(p)}\right],$$
$$B(p)=m\left[\frac{1}{a_k(p)}-\frac{p\beta c(p)}{z(p)p_{atm}}\right],\ M(p)=m\frac{p\beta c(p)}{z(p)p_{atm}},$$
$$N(p)=m\left[\frac{s_k(p)}{a_k(p)}-\frac{p\beta[1-c(p)\bar{\gamma}(p)]}{z(p)p_{atm}}\right],$$
$$Q(p)=m\frac{p\beta[1-c(p)\bar{\gamma}(p)]}{z(p)p_{atm}},\ A(p)=\frac{N(p)}{B(p)},$$
$$x=\ln r,\ x_w=\ln r_w,\ x_k=\ln R_k,\ \left(x_i,z_j,t_n\right)=$$
$$=(x_i,\ 0=x_0<x_1<...<x_{N_X}=L_x,$$
$$z_j,\ 0=z_0<z_1<...<z_{N_Z}=L_z,\ t_n,\ 0=t_0<t_1<...<t_{N_\tau}=T),$$

L_x, L_z - correspond to absolute reservoir elevation,

$$\Delta\tau=\frac{t_n}{n},\quad n=1,2,3,.....,N_\tau,\quad x_{i+1/2}=x_i+\frac{1}{2}\Delta x_{i+1/2},$$

$$\Delta x_{i+1/2}=x_{i+1}-x_i,\ i=\overline{1,N_x},\quad z_{j+1/2}=z_j+\frac{1}{2}\Delta z_{j+1/2},$$
$$\Delta z_{j+1/2}=z_{j+1}-z_j,\ j=\overline{1,N_z},$$
$$\Delta x_i=\frac{1}{2}\left(\Delta x_{i+1/2}+\Delta x_{i-1/2}\right),\quad i=\overline{0,N_x};$$
$$x_1=0,x_{N_{x1}}=x_k,\ N_{x1}=N_x+1,$$
$$\Delta z_j=\frac{1}{2}\left(\Delta z_{j+1/2}+\Delta z_{j-1/2}\right),\ j=\overline{0,N_z};$$
$$z_1=0,z_{N_{z1}}=H,\ N_{z1}=N_z+1,$$
$$\Phi^{n+1}_{i,j}=C^{n+1}_{1i,j}F^n_{2i,j}+C^{n+1}_{2i,j}F^n_{ki,j},,$$
$$\Psi^{n+1}_{i,j}=C^{n+1}_{3i,j}F^n_{\tilde{a}ij}+C^{n+1}_{4i,j}F^n_{ki,j},$$
$$C^{n+1}_{1i,j}=\frac{p^{n+1}_{i,j}c(p^{n+1}_{i,j})\beta}{\mu_z(p^{n+1}_{i,j})z(p^{n+1}_{i,j})},$$
$$C^{n+1}_{2i,j}=\frac{1}{\mu_k(p^{n+1}_{i,j})a_k(p^{n+1}_{i,j})},$$
$$C^{n+1}_{3i,j}=\frac{p^{n+1}_{i,j}[1-c(p^{n+1}_{i,j})\bar{\gamma}(p^{n+1}_{i,j})]\beta}{\mu_z(p^{n+1}_{i,j})z(p^{n+1}_{i,j})},$$
$$C^{n+1}_{4i,j}=\frac{S_k(p^{n+1}_{i,j})}{\mu_k(p^{n+1}_{i,j})a_k(p^{n+1}_{i,j})}.$$

Discrete analogues of the initial and boundary conditions (3) - (5) take the form

$$p^0_{i,j}=p_{0i,j},\rho^0_{ki,j}=\rho_{k_0}=0,i=\overline{0,N_x},j=\overline{0,N_z},\qquad(8)$$

$$p^n_{0,j}=p^n_{cj}\ ,j=\overline{0,N_z},\qquad(9)$$

$$p^n_{N_x,j}=p^n_{N_{x1},j};\quad p^n_{i,0}=p^n_{i,1};$$
$$p^n_{i,N_z}=p^n_{i,N_{z1}};\quad i=\overline{1,N_x}\qquad(10)$$

The difference scheme (6)-(7) according to pressure is implicit and non-linear but according to saturation it is explicit. Therefore according to pressure $p^{n+1}_{i,j}$ it can be solved only by iterative method. For this purpose we linearize equation (6) according to the method of simple iteration [2] and give the final form of the linearized system of equations closed to pressure $p^{\nu+1}_{i,j}$:

$$g^\nu_{i,j}p^{\nu+1}_{i,j-1}+c^\nu_{i,j}p^{\nu+1}_{i-1,j}+a^\nu_{i,j}p^{\nu+1}_{i,j}+b^\nu_{i,j}p^{\nu+1}_{i+1,j}+f^\nu_{i,j}p^{\nu+1}_{i,j+1}=d_{i,j}\ .\qquad(11)$$

The coefficients of this system are defined as follows:

$$g_{i,j}^{v} = e^{-2x_i} k_{i-1/2,j}^{n} \left(A_{i,j}^{n} \Phi_{i-1/2,j}^{v} - \Psi_{i-1/2,j}^{v} \right) \cdot \frac{1}{\Delta x_i \Delta x_{i-1/2}},$$

$$c_{i,j}^{v} = k_{i,j-1/2}^{n} \left(A_{i,j}^{n} \Phi_{i,j-1/2}^{v} - \Psi_{i,j-1/2}^{v} \right) \cdot \frac{1}{\Delta z_j \Delta z_{j-1/2}},$$

$$f_{i,j}^{v} = e^{-2x_i} k_{i+1/2,j}^{n} \left(A_{i,j}^{n} \Phi_{i+1/2,j}^{v} - \Psi_{i+1/2,j}^{v} \right) \cdot \frac{1}{\Delta x_i \Delta x_{i+1/2}},$$

$$b_{i,j}^{v} = e^{-2x_i} k_{i,j+1/2}^{n} \left(A_{i,j}^{n} \Phi_{i,j+1/2}^{v} - \Psi_{i,j+1/2}^{v} \right) \cdot \frac{1}{\Delta z_j \Delta z_{j+1/2}},$$

$$a_{i,j}^{v} = -\left(g_{i,j}^{v} + c_{i,j}^{v} + f_{i,j}^{v} + b_{i,j}^{v} \right) + \varphi_{i,j}^{v},$$

$$\varphi_{i,j}^{v} = \frac{1}{\Delta\tau} \left[\left(A_{i,j}^{n} M_{i,j}^{\prime n} - Q_{i,j}^{\prime n} \right) + \rho_{ki,j}^{n} \left(A_{i,j}^{n} B_{i,j}^{\prime n} - N_{i,j}^{\prime n} \right) \right],$$

$$d_{i,j}^{v} = \varphi_{i,j}^{v} \cdot p_{i,j}^{v}.$$

It is seen that in the system of equations (11) the following relations are carried out regardless of boundary conditions,:

$$g^{v}{}_{1,j} = f^{v}{}_{N_{x+1},j} = 0, \quad j = \overline{1, N_y},$$
$$c^{v}{}_{i,1} = b^{v}{}_{i,N_{y+1}} = 0, \quad i = \overline{1, N_x} \qquad (12)$$

Taking into consideration (12), it is possible to write system (11) in matrix form:

$$E\vec{P} = J, \qquad (13)$$

where solution $\vec{P}$ - is the unknown vector, E and J - are the matrices.

The matrix of the system (13) has pentadiagonal structure. The iterative pointwise Jacobi method is used for solution system [2]. At each iteration the equation for the pressure is solved according to nonlinearity and $p_{i,j}^{v+1}$ is found according to known $\rho_{ki,j}^{n}, p_{i,j}^{n}$. According to the required accuracy achievement of the unknown functions value $p_{i,j}^{n+1}$ is solved at the last iteration. Then according to explicit formulas $\rho_{ki,j}^{n+1}$ is determined by values $p_{i,j}^{n+1}$ and $\rho_{ki,j}^{n}$ then analogical calculations are continued on the next temporal layer.

Determination of parameter $k(r,z)$ requires the solution to a problem of the parametric identification. For this purpose it is necessary to add an additional condition to system (1)- (5)

$$2\pi r_w \int_0^{H_2} \left(V_{gr} + V_{kr} \right)\Big|_{r=r_w} dz = q_w(t), \; t \in (0,T), \qquad (14)$$

where
$$V_{gr} = k(r,z) \left(\frac{F_g(\rho_k) p\beta \left[1 - c(\rho)\bar{\gamma}(p) \right]}{\mu_g(p) z(p) p_{atm}} + \frac{F_k(\rho_k) S_k(p)}{\mu_k(p) a_k(p)} \right) \frac{\partial p}{\partial r}$$

$$V_{kr} = k(r,z) \left(\frac{F_g(\rho_k) p\beta c(\rho)}{\mu_g(p) z(p) p_{atm}} + \frac{F_k(\rho_k)}{\mu_k(p) a_k(p)} \right) \frac{\partial p}{\partial r},$$

here $V_{gr}, V_{\kappa r}$ are the rate of filtration of fluids in the direction of the axis r ; r_w is well radius; $q_w(t)$ is the production rate.

For effective and sustainable method development of the inverse solution of (1) - (5), (14) it can be transformed to equivalent variational formulation [3,4]. This approach involves the reduction of the corresponding inverse problem to a conditional extremum, in particular, to the minimization of the following functional

$$J(u) = \int_0^T \left[\left\{ \int_0^H 2\pi r (V_{\varepsilon r} + V_{\kappa r})\Big|_{r=r_c} \right\} dz - q_c(t) \right]^2 dt, \qquad (15)$$

under conditions (1) - (5).

Taking parameter $k(r,z)$, you can use the theoretical law [3]:

$$k(r,z) = k_0 e^{bz/H}, \qquad (16)$$

where H is total thickness of the productive stratum b is logarithm of the maximal permeability $k_{\max}$ to its minimum value k_0. Here coefficients b and k_0 are subjected to determination.

An expression (16) is used for their determination. The conjugate gradient method [4] is proposed to use for minimization (16).

The numerical calculations were carried out according to above-mentioned design scheme and algorithm. Its approbation on some model solution is necessary and sufficient for reliability testing of the obtained solution. In this case it was considered forward modeling about bringing gas-condensate well into production with constant bottom-hole pressure according to their results the inverse problem was solved.

The direct problem in the case of gas-condensate mixture filtration was solved with the following initial data:

$$r_w = 0{,}1\, m,\; R_\kappa = 750\, m,\; H = 20\, m,\; p_0 = 40 \cdot 10^6\, Pa,$$
$$p_w(t) = 38 \cdot 10^6\, Pa, \quad m = 0{,}2, \quad b = 3{,}12, \quad k_0 = 0{,}05 \cdot 10^{-12}\, m^2.$$

Dependences of gas and condensate parameters on pressure and relative phase permeabilities on saturation have been taken from [5].

The results of calculations have been presented in Table I. The dependence $q_w(t)$ under the condition $p_w = const$ given in Table I includes the effect of permeable stratum heterogeneity, given as (16) with definitely accepted values b and k_0. Therefore the purpose of identification was the recovery of the recorded coefficients values according to data $q_w(t)$ and $p_w = const$.

TABLE I. PRODUCTIVITY CHANGE OVER TIME

time, s	1000	4000	8000	15000	30000
q_w, 10^4 m^3/s	87.81	86.43	85.05	83.32	81.6

The results of the conducted identification have been presented in Table II, which have been obtained at the appropriate initial approximations $b = 3.6$ and $k_0 = 0.07 \cdot 10^{-12} m^2$. At determined calculated values b and k_0 the direct problem with the previous initial data has been resolved and results have been presented in Table. III.

TABLE II. IDENTIFIABLE PARAMETERS OF b AND k_0

Parameters	Exact value	Calculated value
b	3.12	3.091
k₀	0.05	0.051

TABLE III. PRODUCTIVITY CHANGE OVER TIME AT DIFFERENT VALUES OF PARAMETERS b AND k_0

Time, s	q_{gw}, 10^4 m^3/s	
	k_0=0.05; b=3.12	k_0=0.05; b=3.09
1000	87.81	87.9
4000	86.43	86.5
8000	85.05	85.15
15000	83.32	83.37
30000	81.6	81.675

As can be seen that obtained heterogeneous stratum parameters as a result of identification have provided very high accuracy at prediction of the temporal change in well productivity which indicates on the high accuracy of the proposed method.

On the basis of the variational approach the inverse problem of the gas-condensate mixture filtration was solved and the technique of the sufficiently accurate determination of the heterogeneous stratum parameters was developed.

REFERENCE

[1] M. T. Abasov, and F.G. Orujaliyev, Gashydrodinamics and development of gas-condensate fields. Moscow: Nedra, 1989, 262 p.

[2] Kh. Aziz, and E. Sattari, Mathematical modeling of the reservoir systems. Trans. from English, Moscow: Nedra, 1982, 407 p.

[3] G. I. Jalalov, and T.M. Ibrahimov, Identification methods of the filtration and capacitive parameters of the deformed strata at the unsteady fluids filtration. Baku: Elm,1989, 48 p.

[4] F. P. Vasiliyev, Methods of solution to extreme problems. Moscow:Nauka, 1980, 520 p.

[5] M. T. Abasov, G. I. Jalalov, and Kh. A. Feyzullayev, "Identification of the hydrodynamic model parameters of the gas-condensate deposit," Izv. of ANAS, Earth Sciences series, 2008, № 2, pp.78-90.