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Modeling of a Generalized Brownian Motion Based on Wavelet – Computer Technology

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Abstract— **In the present work the simple way of modeling of the generalized Brownian motion on the basis of Wavelet technologies is offered. It is realized in the program Mathcad environment; using concept of the generalized Brownian motion, it is possible to define nature of researched process. It can possess persistentny or antipersistentny properties; the indicator of H characterizes dimension (crenation) temporary series: antipersistentnost possesses has high dimension (H<0.5), while (H>0.5) has low dimension; the signals for which H>0.5 should be related to group chaotically – determined signals (Fractal Signals), which are the carriers both determined and random properties.**

Keywords— modeling; antipersistentny and persistentny process; generalized Brownian motion; Wavelet computer technology.

I. Introduction

The chance is inherent in all natural phenomena, which behaviour is described temporary processes or series. For estimation of a share of chance in temporary series use parameter Herst (H) [1], on the basis of which, is judged: if H=0.5, that the process is described classical Brownian motion, if 0<H<0.5 is that processe antipersistentny, when rising tendency is replaced descending and on the contrary. At process 0.5<H<1 is persistentny, when the tendency is observed rising, in the future she will continue the growth. When H>0.5 increase the stability becomes more appreciable [2].

Known that the simulation of generalized Brownian motion is a compex task [3].

II. Problem Statement

In this paper the simple way of modeling Generalized Brownian motion is offered on the basis of Wavelet's technologies, which is realized in the program MathCad environment.

The orthonormal wavelet transform is used to modeling Generalized Brownian Motion (fBm).

Let's receive process, when H=0.1. H - is the fBm scaling parameter and is called the Hurst coefficient.

sd=0.5 sd - is the standard deviation.
len=1024 len is the length of the fBm time series to be generated. This length must be a power of two.

$noise := rnorm(len, 0, 1)$ "noise" - vector of "len" random numbers, having the normal distribution with mean=0 and standard deviation sigma=1.

The following function creates the approximate fBm from white noise in the wavelet transform domain by scaling the noise deviates at the level i by $(2^i)^{H+\frac{1}{2}}$ times the standard deviation (In MathCad program).

```
fBm(noise, H, sd) := | len ← rows(noise)
                     | numLev ← log(len, 2)
                     | v_0 ← 0
                     | for i ∈ numLev..1
                     |     v ← stack[v, submatrix(noise, len/2^i, len/2^(i-1) − 1, 0, 0)·sd·(2^i)^(H+1/2)]
                     | v
```

$v := fBm(noise, H, sd)$

We have an approximate Generalized Brownian Motion in the Wavelet transform domain. To get the representation in the time domain, simply perform the inverse wavelet transform:

$filter = symmlet(8)$ symmlet(8) - returns the low-pass filter of the 8-coefficient least asymmetric Daubechies wavelet.

$w := idwt(v, \log(rows(v), 2), filter)$ idwt - returns the inverse wavelet transform.

$k = 0..len - 1$

The result of inverse wavelet transform is given on Fig.1

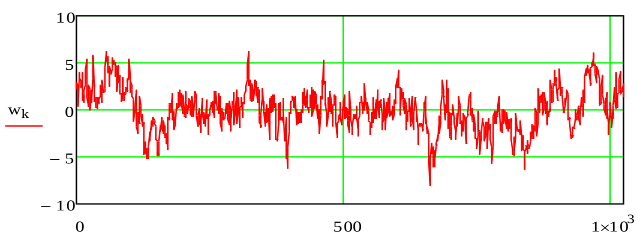

Figure 1. The result of modeling Generalized Brownian Motion, when H=0.1

The first difference of an fBm series is known as fractional Gaussian noise.

$$wd_{k+1} = w_{k+1} - w_k$$

The result of difference of an fBm is given on Fig.2

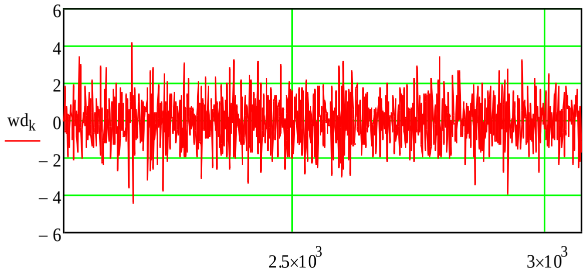

Figure 2. The result of modeling fractional Gaussian noise, when H=0.1

It should be noted that process modeling at different H is carried out simply by substitution in expression of fBmScale of the corresponding value H.

Note that, when H=0.5, a Generalized Brownian Motion becomes ordinary Brownian motion.

The result of modeling is given on Fig. 3

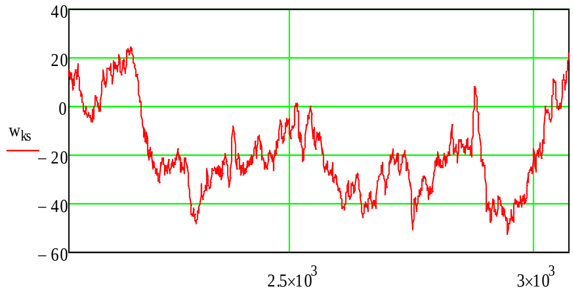

Figure 3. The result modeling classical Brownian Motion when H=0.5

Let's consider a case, when H=0,99 The result of modeling is presented on Fig. 4

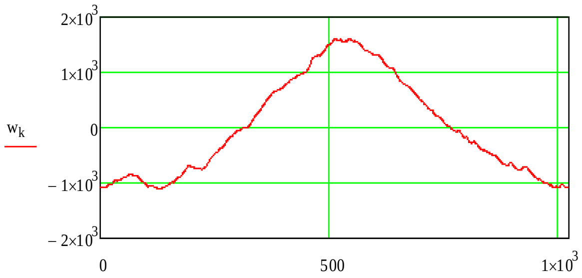

Figure 4. The result of modeling Generalized Brownian Motion, when H=0.99

III. Conclusion

In the results we will note the following:

1) In the present work the way of modeling of the Generalized Brownian Motion on the basis of Wavelet transform is realized in the program Mathcad environment.

2) Using concept of the Generalized Brownian motion, it is possible to define nature of researched process. It can possess persistentny or antipersistentny properties.

3) The indicator of H characterizes dimension (crenation) temporary series: antipersistentnost possesses more high dimension, while H>0.5 has low dimension.

4) The signals for which H>0.5 should be related to group chaotically – determined, which are the carriers both determined and random properties.

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