Identification and Modeling of the Nonlinear Dynamic Open Loop Systems

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***Abstract*— The problems of structural and parametric identification of nonlinear systems on the set of continuous block-oriented models, elements of which are different modification of Hammerstein and Wiener models, at the system's deterministic input influences are considered. The problem of the structural identification is considered according to the classical definition of identification of Zadeh. The structure definition criterion of the model is developed. Parameters' estimations are received by the method of the least squares by using Fourier approximation.**

Keywords*— *structural identification; parametric identification; nonlinear system; block-oriented models; forced oscillatiion

I. INTRODUCTION

The systems identification is connected with the solution of different problems depending on the a priori information about the system. The construction of the system's optimal model in many respects is determined by the structural and parametric identification problems problem solving.

The majority of real systems are nonlinear. They are generally represented by block-oriented (for example [1-3]) or general models, in particular, the Volterra's functional series (for example [4-5]) and continuous and the Kolmogorov-Gabor's discrete polynomials [6-7].

When representing nonlinear systems by block-oriented models the basic results in the sphere of structural identification are obtained at the identification by discrete models. The determination of the structure of the model for continuous nonlinear systems is usually carried out on the certain sets of block-oriented models consisting of different modifications of the Hammerstein and Wiener models.

In this work the problem of structural identification of nonlinear continuous stationary systems is considered on the set of continuous block-oriented models which is "greater" than the one considered earlier. This problem is brought in correspondence with the L. Zadeh classical determination of identification, i.e. it is implied that the classes of models and input signals are set. A criterion determining the model structure from the class of models is needed to be developed.

To the problem of parametrical identification of nonlinear systems is devoted a considerable quantity of scientific works in which that problem is solved on the basis of different approaches and methods. At the representation of nonlinear systems by the block-oriented models, most of the methods of parametric identification is developed for the simple Hammerstein model (for example [8-9]). Comparatively small quantity of works is devoted to the identification of parameters of the simple Wiener model (for example [10]). As to the identification of parameters of other block-oriented models it is possible to name only several works from this area. This can be explained by the fact that estimation of the model parameters is difficult, because the majority of these models are nonlinear in parameters, and also because of the big number of estimated parameters.

In this work the problem of parametrical identification of the second order nonlinear systems is considered at their representation by the simple and generalized Hammerstein and Wiener models. The problem of parametrical identification is solved by the Fourier approximation and by the least squares method.

The developed methods and algorithms of structural and parametric identifications are investigated by means of both, the theoretical analysis and the computer modeling.

II. CLASSES OF MODELS AND INPUT SIGNAL

The model structure is determined by the following class of continuous block-oriented models

$$L = \{s_i | i = 1,2,...,9\}, \tag{1}$$

where s_1 and s_2 - the simple and generalized Hammerstein models, s_3 and s_4 - the simple and generalized Wiener models, s_5 - the simple Wiener- Hammerstein cascade model, s_6 - the expanded Wiener model, s_7 and s_8 - the generalized and expanded Wiener-Hammerstein cascade models, s_9 - the simple Hammerstein- Wiener cascade model (Fig. 1).

The nonlinear static elements which are included in the models are described by *n* degree polynomial functions:

$$f(u) = f_1(u) = \sum_{i=1}^{n} c_i u^i(t), \; f_2(x) = \sum_{j=1}^{m} d_j x^j(t), \tag{2}$$

where $c_i \, (i = 0,1,\cdots,n)$, $d\text{j} \, (j = 0,1,\cdots,m)$ - constant coefficients.

It is assumed, that the linear dynamic parts, with transfer functions $W(p)$ and $W_i(p) (i = 1,2,\ldots,4)$ in the operational

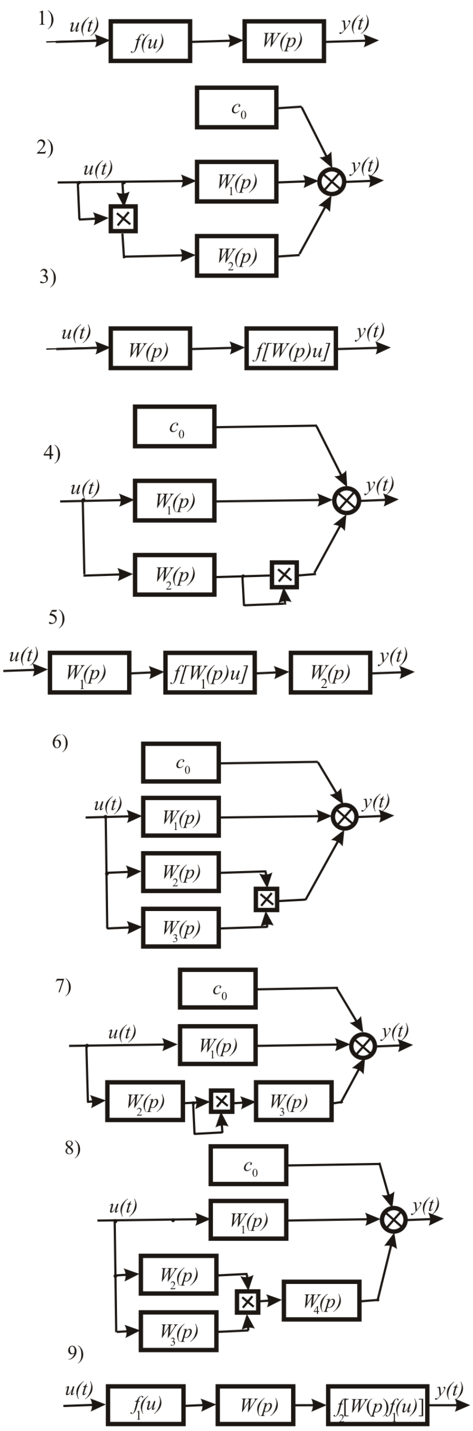


Figure 1. The block-oriented models: 1) simple Hammerstein model; 2) generalized Hammerstein model; 3) simple Wiener model; 4) generalized Wiener model; 5) simple Wiener-Hammerstein cascade model; 6) expanded Wiener model; 7) generalized Wiener- Hammerstein cascade model; 8) expanded Wiener- Hammerstein cascade model; 9) simple Hammerstein - Wiener cascade model

form - p denotes the operation of differentiation $p \equiv d/dt$, which are included in the class of block-oriented models are steady.

To solve the problems of structural and parametric identification of nonlinear systems in the steady state, on the basis of active experiment, it is supposed, that input variable of system $u(t)$ is sinusoidal function:

$$u(t) = A\cos\omega t \,. \qquad (3)$$

III. Structural Identification

When on the input of nonlinear system acts the harmonious influence, after the termination of transient process in the steady state on output of the system forced oscillation is obtained, having features for different models from set (1).

For example below it is given analytical expressions of the forced oscillations, obtained on the output of generalized Hammerstein model and of Wiener simple model:

$$y(t) = c_0 + \frac{1}{2}A^2|W(0)| + |W_1(j\omega)|A\cos[\omega t + \varphi(\omega)] + $$
$$+ \frac{1}{2}|W_2(2j\omega)|A^2\cos[2\omega t + 2\varphi(\omega)]; \qquad (4)$$

$$y(t) = c_0 + \sum_{i=1}^{k}\frac{C_{2i}^{i}}{2^{2i}}c_{2i}A^{2i}|W(j\omega)|^{2i} +$$
$$+\left(\sum_{i=1}^{k}\frac{C_{2i-1}^{i-1}}{2^{2(i-1)}}c_{2i-1}A^{2i-1}|W(j\omega)|^{2i-1}\right)\cos[\omega t + \varphi(\omega)] +$$
$$+\left(\sum_{i=1}^{k}\frac{C_{2i}^{i-1}}{2^{2i-1}}c_{2i}A^{2i}|W(j\omega)|^{2i}\right)\cos 2[\omega t + \varphi(\omega)] +$$
$$+\cdots+\frac{1}{2^{2(k-1)}}c_{2k-1}A^{2k-1}|W[j\omega]|^{2k-1}\cos(2k-1)+$$
$$+\frac{1}{2^{2k}}c_{2k}A^{2k}|W(j\omega)|^{2k}\cos 2k[\omega t + \varphi(\omega)]. \qquad (5)$$

where C_k^i is binomial coefficient, $W(j\omega) = U(\omega) + jV(\omega)$, $|W(\omega)| = \sqrt{U^2(\omega) + V^2(\omega)}$, $\varphi(\omega) = arctg[V(\omega)/U(\omega)]$.

The analysis of output variables models allow defining the criteria of a choice model structure on set (1):

- Hammerstein simple model - constant component of periodic output signal does not depend on change of frequency of input influence. Output periodic function is presented in the form of the sum of n harmonics;
- Hammerstein Generalized model - constant component of output signal does not depend on the change of input influence frequency. The output periodic signal contains only the first and second harmonic;
- Wiener simple model - constant component of output periodic signal depends on the change of input influence frequency;
- Wiener generalized model - the difference between the constant component and amplitude of the second

harmonic does not depend on frequency, and the ratio of a square of amplitude of the first harmonic to amplitude of the second harmonic depends on frequency;

- Wiener-Hammerstein simple cascade model – the difference between the constant component and amplitude of the second harmonic depends on frequency;
- Wiener expanded model - all above listed values depend on frequency, however the constant component and the ratio of constant components' difference at different amplitudes of input influence to amplitude of the second harmonic, represents trigonometrical functions of frequency;
- Wiener-Hammerstein generalized cascade model - constant component and the ratio of difference between constant components at different amplitudes of input influence to amplitude of the second harmonic, depend on frequency, however these dependences are not trigonometrical functions of frequency;
- Wiener-Hammerstein expanded cascade model - constant component represents trigonometrical function of frequency, however the ratio of a difference of constant components at different amplitudes of input influence to amplitude of the second harmonic depends on frequency, however this dependence is not trigonometrical function of frequency;
- Simple Hammerstein - Wiener cascade model - constant component of output periodic signal depends on the change of input influence frequency. Periodic signal contains *n*+*m* harmonics.

The above listed values - constant components, amplitudes of harmonics of the output forced oscillation of the system can be defined by means of the numerical harmonic analysis [11].

IV. Parametric Identification

For the parametric identification of nonlinear systems at their representation by Hammerstein and Wiener simple and generalized models, analytical expressions of the forced oscillation for structural identification are used. With the Fourier approximation [12] the problem of parameter identification is reduced to the solution of algebraic equations system. Parameter estimations are determined by least squares method.

It is assumed, that the nonlinear static elements, which are included in the models are described by the second degree polynomial function and transfer functions of linear dynamic parts of models are defined by expression:

$$W_i(p)=\frac{1}{T_{0i}p^2+T_ip+1}\quad(i=1,2),\qquad(6)$$

where $T_{0i}>0\,(i=1,2)$ has dimension of square of time, and $T_i>0\,(i=1,2)$ - dimension of time.

For example below are given the expressions of some parameter estimations for Hammerstain generalized model:

$$\hat{T}_{01}=\frac{\left(\sum_{i=1}^{n}\omega_i^2\right)\left(\sum_{i=1}^{n}\frac{a_{1i}^2}{b_{1i}^2}\omega_i^2\right)-\left(\sum_{i=1}^{n}\frac{a_{1i}}{b_{1i}}\omega_i\right)\left(\sum_{i=1}^{n}\frac{a_{1i}}{b_{1i}}\omega_i^3\right)}{\left(\sum_{i=1}^{n}\omega_i^4\right)\left(\sum_{i=1}^{n}\frac{a_{1i}^2}{b_{1i}^2}\omega_i^2\right)-\left(\sum_{i=1}^{n}\frac{a_{1i}}{b_{1i}}\omega_i^3\right)^2},\qquad(7)$$

$$T_2=\frac{1}{2}\frac{\left(\sum_{i=1}^{n}\omega_i^4\right)\left(\sum_{i=1}^{n}\frac{a_{2i}}{b_{2i}}\omega_i\right)-\left(\sum_{i=1}^{n}\omega_i^2\right)\left(\sum_{i=1}^{n}\frac{a_{2i}}{b_{2i}}\omega_i^3\right)}{\left(\sum_{i=1}^{n}\omega_i^4\right)\left(\sum_{i=1}^{n}\frac{a_{2i}^2}{b_{2i}^2}\omega_i^2\right)-\left(\sum_{i=1}^{n}\frac{a_{2i}}{b_{2i}}\omega_i^3\right)^2}.\qquad(8)$$

where a_{1i}, b_{1i}, a_{2i}, b_{2i} $(i=1,2,\ldots,n)$ - values of Fourier coefficients during ω_i frequency.

V. Accuracy of the Received Results

The reliability of the received results at the identification of nonlinear systems in the industrial conditions under noise and disturbances depends on the measurement accuracy of the system's output signals and on the mathematical processing of the experimental data. When running experiments it is recommended to use in the system registering apparatuses, the inertance of which is much less than the one of the object. When using various schemes of the numerical harmonic analysis, for example, the Runge's schemes [11], it is recommended to accept

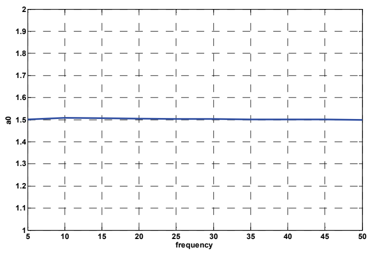
$$\bar{y}=\frac{1}{n}\sum_{i=1}^{n}y_i\ ,$$

as the value of the output signal at the certain time moment, where $\bar{y}$ is an estimation of the mathematical expectation of the value of the output function at the present time moment, n is a number of the experiments or the processing periods. In the industrial conditions it is possible to use the calculators realizing the algorithm of the Furrier coefficients calculation using the integration operation (for example [12]) for carrying out the numerical harmonic analysis. It is well-known that, theoretically, increasing the integration time, we can decrease the interference effect to the minimum. Besides this the method of least squares used for parametric identification gives reliable results with respect to noise immunity.

VI. Computer Modelling

The investigation of the algorithms of the structural and parametric identification of nonlinear systems with the feedback was carried out by means of the computer modelling based on using MATLAB at input sinusoidal influences of the system.

a)



b)

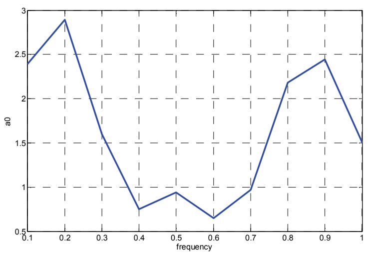


Figure 2. The dependences $a_0 = f(\omega)$: (a) for Hammerstein generalized model, (b) for Wiener simple model

We used both, the tool of package Simulink-toolbox for the system modeling and tool Symbolic Math Toolbox for the solution of the equations.

Upward, the dependences of the constant component of the output periodic signal on the frequency of the input sinusoidal signal $a_0 = f(\omega)$ (Fig. 2) for some nonlinear models of the set (1) is given as an example.

When investigating the algorithms of parametric identification the programs corresponding to such algorithms were made. Using such programs the estimations of unknown parameters have been obtained. Below, as an example the results obtained for the Hammerstein generalized model are given. In the experiment the following values of the parameters $c_0 = 2, T_{01} = 1{,}5, T_1 = 2, T_{02} = 0{,}5, T_2 = 1$ were used at the input sinusoidal signal with amplitude $A = 3$ and with frequency $\omega = 2$ rad/sec . Following estimations of the unknown parameters are obtained: $c_0 = 2, T_{01} = 1{,}4709, T_1 = 1{,}9528,$ $T_{02} = 0{,}4882, T_2 = 0{,}9729$.

References

[1] R., Haber and H. Unbehauen. "Structure identification of nonlinear dynamic systems – a survey on input/output approaches". Automatica, Vol. 26, No. 4. 1990, pp. 651-667.

[2] B.G. Shanshiashvili. "Frequency method for identification of a model structure of nonlinear continuous – time systems." Preprints of the 9 th IFAC/IFORS Symposium on Identification and System Parameter Estimation. Budapest.: Pergamon Press, vol. 1, pp. 640 – 643, 1990.

[3] A. A., Rurua and B.G. Shanshiashvili. "Structure identification of nonlinear continuous – time stationary systems." Proceedings of the II Iinternational Conference "System Identification and Control Problems" SICPRO '03. Moscow. .A. Trapeznikov Institute Of Control Sciences, pp. 572-582. 2003 (in Russian).

[4] G.A. Parker and E.L Moore. "Practical nonlinear system identification using a modified Volterra series approach." Automatica, vol. 18, no. 1, pp. 85-9 1982.

[5] P. Alper. "A consideration of the discrete Volterra series." IEEE Trans. Automat Control. AC-10, pp. 322-327, 1965.

[6] A. N. Kolmogorov. "Interpolation and extrapolation of stationary random series." Bulletin of the Academy Sciences of USSR. Mathematical series,. vol. 5, no. 1, pp. 3-14, 1941

[7] D. Gabor, P.L. Wilby and R. Woodcook "A universal nonlinear filter predictor and simulator which optimizes itself by a learning process." IEE Proc., vol.108, part B. pp. 422-433, 1961.

[8] Z. Hasiewisz, and G. Mzyk. "Combined parametric-nonparametric identification of Hammerstein systems." IEEE Transactions on Automatic Control, vol. 49, no. 8. pp. 1370-1375, 2004.

[9] W.-X. Zhao. "Parametric Identification of Hammerstein systems with consistency results using stochastic Inputs." IEEE Transactions on Automatic Control, vol. 55, no. 2, pp.474 – 480, 2010.

[10] E.-W. Bai. "Frequency domain identification of Wiener models," Automatic, vol. 39, no. 9, pp. 1521–1530. 2003.

[11] L.G. Kruglikova and R.I. Krulov. Reference Book on Numerical Harmonic Analysis. Nauka i Tekhnica, Minsk 1968 (in Russian).

[12] R.W. Hamming. Numerical methods for scientists and engineers. New York, MC Graw-hill book company, 1962.

[13] A.A. Vavilov, E.V. Sergeev and A.I. Solodovnikov. Computers for an experimental research of continuous and discrete systems of automatic control. In: Analog and analog - digital computer engineering, V.G. Ushakov (ed.), Mockow, Sovetskoe Radio, publication 2, pp. 39-55. 1968, (in Russian).