Application of Viscoelastic Model for Investigating Nanocomposites

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***Abstract*— We consider a problem on application of viscoelasticity models. Casters continuum dynamics problems (micro polar medium) [1,2]. The medium of the nanocomposits is modeled as a periodic system of pairs of one-directed, parallel bars. Each pair contains two elastic or viscoelastic bars contacting between themselves. The materials of the viscoelastic bar is described by Boltzmann-Volterra model with arbitrary hereditary kernels. The relation between the parameters of the Cosser's medium and characteristics of the bars is defined.**

Keywords— viscoelasticy; structural elements;, hereditary kernels; nanocomposits; micropolar medium; dynamics problems

I. INTRODUCTION

Modeling problems were always actual at studying of mechanical properties of materials taking into account their structure as the decision of these problems leads to quantitative dependences of macroscopical characteristics of deformation, durabilities and resistance to destruction of a material from structure parameters.

Having the specified dependences, it is possible to carry out parametrical optimization at designing of new materials, designs, ways of tests. In variety of new materials the specialplace is occupied with the materials having nanoscale structure nanomateriales and the materials filled nanostructure with particles nano - and microscales (nanocomposites). High values of characteristics of deformation, durability and cracked durability, as a rule, are inherent in these materials that causes their perspectivity for various applications in the industry. Among extended nanomaterials are allocated carbon nanomaterials, such as carbon nano tubes. They can act both in the form of separate nano objects, and in the form of sets or a set of the particles shipped in a matrix of other material. From the moment of the first reception, carbon nano tubes remain object of constant scientific researches. From isotropic materials they are distinguished by special regular nuclear structure. The nano tubes rare combination of the linear sizes, relative density, deformation and durability characteristics is peculiar, therefore they find application in the technician and medicine. For real experiments on nanostructure nano - and microscale objects the difficult, high-precision and expensive equipment is required. Schemes of carrying out of experiments are often unique and innovative in each specific case. Therefore, though the quantity of the executed laboratory experiments also is great, they don't differ yet variety. There are no standardized schemes of mechanical tests nanostructure nano - and microscale objects. There is no metrological maintenance of such tests. In these conditions the special role is got by analitiko-numerical modeling of mechanical behavior, including mechanical tests, nanostructure nano - and microscale objects.

The use of carbon nanobodys as a filler in obtaining polymer- based nanocomposites essentially improves deformation-strength characteristics of these materials [1,2,3,4]. Essential difference between the technologies for obtaining new materials and possibilities of prediction of their physic- mechanical properties is observed. Creation and improvement of such models could allow obtaining new materials with regimen properties. This especially concerns the influence of local structure of the medium on its macro properties [1,3] . It is known that the classic phenomenological theory of continual medium does not take into account the materials microstructure and it is eligible for investigating such problems. Appearance of kasseris continuum model was marked by the beginning of transition in theory of continuum from Newton mechanics whose initial object is a material point, to Euler mechanics that has a solid body as an initial object. At present, insufficient practical application of Cosser's model mainly is stipulated by the absence of two factors : methods of reliable definition of material constants and concept of account of viscosity of such medium [3].

In the present paper, we attempt to apply viscoelastic models in Cosser's equation by setting the problem for a composite viscoelastic bar that is identical to Cosser's medium model by it dispersion properties.

II. PROBLEM STATEMENT

Strain state of Cosser's medium is described by asymmetric strain tensor and flexion-torsion tensor K_{ij} [1]:

$$\varepsilon_{ij} = \frac{\partial u_i}{\partial x_j} - f_{kij}, \quad k_{ij} = \frac{\partial \psi_i}{\partial x_j} \tag{1}$$

Here u_i are the displacement tensor component; x_i i=1,2,3 is Cartesian coordinate; ψ_i is the component of rotation vector that is cinematically independent of displacements; f_{kij} is Levy-Chivita pseudo tensor (i=1,2,3; j=1,2,3; k= 1,2,3). While considering adiabatic processes of elastic strain it is necessary to take into account change and dependence of internal energy on the invariants of strain measures. It is accepted that for *t=0 the* medium is at rest, $\varepsilon_{ij}=0,\ K_{ij}=0$. We expand the internal energy E in Taylor series and take only the linear part. For a separate isotropic, homogeneous cylindrical shell we get an expansion in the form similar to [1]:

$$E=0.5((\mu+\alpha)\varepsilon_{ij}\varepsilon_{ji}+(\mu-\alpha)\varepsilon_{ji}\varepsilon_{ij}+(\eta+\nu)k_{ij}k_{ji}+ (\eta-\nu)k_{ji}k_{ij}+\beta k_{ii}k_{jj} \quad , \qquad (2)$$

where λ,μ are Lame constants, α,β,η,ν are new elastic constants of the micro polar material that should satisfy the following constraints [2]:

$$\begin{gathered}\alpha\geq 0,\ \ \eta+\nu\geq 0,\ \ 3\beta+2\eta\geq 0,\\ -(\eta+\nu)\leq\eta-\nu\leq\eta+\nu.\end{gathered} \qquad (3)$$

Some dependence between these parameters are suggested in technical references.

On their base, we can suggest a partially generalized variant in the form

$$\eta(\mu-\alpha)=\mu(\beta+\nu) \qquad (4)$$

We can represent the volume density of the kinetic energy in the form of two summands

$$E_k=0.5(\rho u_i u_i+c_0\psi_i\psi_i), \qquad (5)$$

where ρ is mediums density, c_0 *=const* is characterizes inertia properties of macro volume and equals to the product of moment of inertia of the substance of particle with respect to the axis passing through its gravity center by the number of particles in the volume unit. In such representation, the stress state is defined by asymmetric stress tensor σ_{ij} and moment stresses tensor m_{ij} expressed by the internal energy

$$\sigma_{ij}=\frac{\partial U}{\partial\eta_{ij}},\qquad m_{ij}=\frac{\partial U}{\partial k_{ij}}\ . \qquad (6)$$

According to [3], Cosser's continuum dynamics equation is of the form

$$\begin{gathered}\Delta\vec{u}-c_0 rotrot\,\vec{u}+c_1 rot\,\vec{\psi}=\rho\vec{u}_{tt},\\ \Delta\vec{\psi}-d_0 rotrot\,\vec{\psi}+d_1 rot\,\vec{u}-d_2\vec{\psi}=\rho_1\vec{\psi}_{tt},\end{gathered} \qquad (7)$$

where

$$c_0=\frac{\mu+\alpha}{\lambda+2\mu},\ c_1=\frac{2\alpha}{\lambda+2\mu},\ \rho_0=\frac{\rho}{\lambda+2\mu},\ d_0=\frac{\eta+\nu}{\beta+2\eta},$$

$$d_1=\frac{2\alpha}{\beta+2\eta},\ \rho_1=\frac{\rho}{\beta+2\eta}.$$

Due to the work [3], system (7) allows to describe longitudinal elastic waves (dilatation waves), shear waves and rotation waves subdivided into longitudinal (dilatation) rotation waves and lateral rotation waves. Linear dependence of shear waves and lateral rotation waves are taken into account. By investigating the one-dimensional problem ($x_1=x$) system (7) is transformed to the form:

$$\begin{gathered}\varphi_{xx}-n_0\varphi_x=\rho_2\varphi_{tt},\\ \psi_{xx}-n_1\psi_x+n_2\psi=c_1\psi_{tt}.\end{gathered} \qquad (8)$$

Here $\varphi(x,t),\psi(x,t)$ are displacement and rotation vector components,

$$n_0=2\alpha/(\mu+\alpha),\ \rho_1=\rho/(\mu+\alpha),\ n_1=2\alpha/(\eta+\nu),$$
$$n_2=2n_1,\ c_1=c_0/(\eta+\nu).$$

By means of mathematical transformations, we reduce system (8) to an equation with respect to shear displacements:

$$\varphi_{tt}-\frac{\mu}{\rho}\varphi_{xx}+\frac{c_0}{4\alpha}\varphi_{tttt}-c_2\varphi_{xxtt}+c_3\varphi_{xxxx}=0, \qquad (9)$$

where

$$c_2=((\mu+\alpha)c_0+\rho(\eta+\nu))/(4\alpha\rho),$$
$$c_3=(\mu+\alpha)(\eta+\nu)/(4\alpha\rho).$$

The waves described by system (8) or equation (9) have dispersion. Dispersive dependence of wave number k to from frequency ω looks like. The law of dispersion is determined by the solution of the equation

$$4\alpha c_3k^4+(d_3-d_4)k^2-d_5=0 \qquad (10)$$

Where

$$d_3=4\alpha\mu/(c_0\rho),\ d_4=4\alpha\rho c_0c_2\omega^2,\ d_5=\frac{\omega^2}{c_0}(4\alpha-\omega^2).$$

Applying the suggested method, we investigate the union of viscoelastic bars that are in lateral contact with each other. The contact interaction force between them is assumed to be linear viscoelastic. Mechanical properties of the bars are described by the relations of Boltzmann -Volterra relaxation function. We consider the variants of the pairs of bars when one of them is elastic, another one is viscoelastic and both of them are viscoelastic. In this statement, the solutions of the problems are reduced to the solution, of the system of motion equations:

$$\begin{gathered}u_{1xx}-R(0)(u_1-u_2)-R(t)(u_{1t}-u_{2t})=\rho_1u_{1tt}\\ u_{2xx}-R(0)(u_2-u_1)-R(t)(u_{2t}-u_{1t})=\rho_2u_{2tt}\end{gathered} \qquad (11)$$

$$R(t)=\int_0^t\Gamma(t-\tau)d\tau,\ R(0)=R(t)_{t=0}$$

In this expression relaxation and creep function in advance analytically aren't set, i.e. can have any kind. It gives the chance to the decision of a return problem, for reception of a material with the set properties, optimization of mechanical characteristics and reinforcing principle nanocomposites.

This system of the integer-differential equations allows considering viscoelastic effects at deformation nanocomposites. on a polymeric basis. The problem is solved by the method represented in [1,2] for the function of any form. In the [4,5] the method is developed for solving the problems on free vibrations of viscoelastic structural elements made of linear viscoelastic material under arbitrary hereditary kernels was elaborated. The further decision the approximately-analytical way, resulted in [4,5] doesn't present difficulty.

III. CONCLUSIONS

The problem can be generalized for case n cores. Instead of entering an equivalent homogeneous environment, as physic mechanical characteristics of a polymeric basis its modules of elasticity, and under elastic and geometrical characteristics of cylindrical cores «effective rigidity» are used. With use of assumptions of classical mechanics concerning character of deformation of environments expressions for kinetic and potential energy of deformations are deduced. Such approach to a solution of a problem is based on the assumption that the characteristic cross-section sizes of supporting elements (nano cores) and distances between them are small in comparison with the sizes of viscously elastic file and characteristic sizes of moving of viscously elastic environment at deformation.

Cores are accepted as cylindrical elements of final length.

Length of this element is accepted as unit, and the area of cross-section S_0 .All cylinders have identical directions on axes, and an arrangement of cylinders in section of environment the casual. If the semi space A surface contains n cores in the area and it is possible to enter concept about the density of cores defined as follows: $\eta = n \cdot S_0 / A$.

According to the theory of the effective module, "the large-scale" behavior of the reinforced semi space can be studied by its research taking into account uniformity but anisotropies of a material. This properties of materials considers parities for physically nonlinear continuous environments. The effective law of Huka for such environment by definition registers in a kind the following form:

$$\sigma_{ij} = C^*_{ijk\ell} \cdot e_{k\ell}.$$

Constant effective modules of the elasticity, which number it is established from symmetry reasons. For a case of an irregular, chaotic arrangement of chinks it is possible to accept a case transversally isotropy and to express parities between pressure and deformations by means of five modules of elasticity which then are replaced with hereditary operators of linear viscoelastic environments, at any kind of functions of creep and a relaxation.

REFERENCES

[1] Novatsky V. The theory of elasticity. – M. "MIR". 1975.-872c.(in Russian)

[2] Yerofeev V. I. Wave processes in firm bodies with a microstructure. - M.: The Moscow State University, 1999, 328 p. .(in Russian)

[3] Yerofeev V. I, Kazhaev V. V, Semerikova N.P.//the Computing mechanics of continuous environments. 2009. T.2., №2. .(in Russian)

[4] Hasanov A.B. Reaction of mechanical systems to non-stationary external influences. Baku, 2004, 250 p. .(in Russian)

[5] Ilyasov M. H. Non-stationary viscoelastic waves. Baku, 2011. 352 p.,(in Russian)