Method of Numeral Modeling of Elastic Water Drive Mode of Development of Layer

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***Abstract*— We propose a method of numerical modeling elastic water drive mode in the of reservoir development when setting the law of motion of fluids. Applying the methods of rectifying the fronts and the difference approximation, the problem reduces to the problem of the difference. A computational algorithm for solving the resulting difference problem is given.**

Keyword*— oil reservoir; elastic water drive mode; boundary inverse problem; method of straightening the front; difference method

I. INTRODUCTION

It is known that in the oil fields with an initial reservoir pressure above the saturation pressure of oil gas in the initial stage of development an elastic regime. [1]. If the oil reservoir is connected with the water surrounding the reservoir system, the development process evolves elastic water drive mode. During manifestation of this mode the movement of oil wells are not only due to the potential energy of elastic deformation of the reservoir and oil, and also because of the pressure of the edge waters.

Experimental studies of various processes occurring in the development of oil fields during elastic water drive mode show that in most cases, the displacement of oil by edge water holds totally and a distinct boundary between two liquids, moving to an unknown pre-law is formed in the ledge.

It should be noted that the effective development of oil fields in the elastic water drive mode is largely determined by the modes of oil production, which affects the dynamics of changing of fluids boundary. And the rate of advance of the interface depends on timing of exploitation, water flood recovery factor of fields, etc. In this regard, the practice of oil fields with elastic water drive mode regime is very important to the question of regulating the movement of the interface of two fluids in the ledge.

II. PROBLEM FORMULATION

Suppose that we are considering the rigid, uniform horizontal oil ledge with the ledge constant thickness and width, bounded top and with bottom by impermeable planes. There is a gallery of development wells in the section of the ledge and the external boundary of reservoir is enclosed by the edge water, under pressure $P_s(t)$. Suppose that at the moment t=0 to the gallery operation of production is admitted wells elastic water drive mode in the reservoir occur. A straight line-parallel flow of oil to the wells occur. Due to the potential energy of elastic deformation due to oil and water pressure boundary as the selection of oil through the gallery of marginal water flows into the reservoir, completely replacing the long engaged in oil, and forms a clear boundary between the water-oil. It is assumed that oil is weakly fluid and its motion in the reservoir obeys Darcy's law. Then the equation describing the unsteady rectilinear-parallel flow of oil in the reservoir using the active promotion of the boundary of water can be represented as follows

$$\frac{\partial P}{\partial t} = \chi \frac{\partial^2 P}{\partial x^2}, \qquad (x,t) \in \Omega_S = \{0 < x < s(t),\, 0 < t \le T\}, \quad (1)$$

where $\chi = \dfrac{k}{\mu\beta m}$, $P(x,t)$ is the pressure in the reservoir; k is the absolute permeability of the reservoir; μ is the viscosity of oil ; β is the coefficient of volume compression of oil; m is porosity reservoir; $s(t)$ is the position of the interface between the water-oil.

Let at the initial time $t = 0$ the distribution of pressure in the oil reservoir and the position of the interface of fluids are known, i.e. we have the following initial conditions for (1):

$$s(0) = L\,, \quad (2)$$

$$P\big|_{t=0} = \varphi(x)\,, \qquad 0 \le x \le s(0)\,. \quad (3)$$

We assume that the change of the pressure over time in a gallery of production wells is described by a function $f(t)$. Then on the boundary of the ledge $x = 0$ we will have the following condition

$$P\big|_{x=0} = f(t) \quad (4)$$

At the water-oil boundary oil pressure must be equal to the edge waters

$$P\big|_{x=s(t)} = p_s(t) \quad (5)$$

and the condition of the material balance must be satisfied

$$m\frac{ds}{dt} = -\frac{k}{\mu}\frac{\partial P}{\partial x}\Big|_{x=s(t)}\,. \quad (6)$$

It should be noted that the direct problem of formation of elastic water drive mode consists of finding functions satisfying the equations (1), (6) with given coefficients k, μ, β, m, and in additional given criteria (2) - (5). An important feature of the direct problem is the presence of interface fluid displacement is determined by the law in the course of solving the problem. Direct problem elastic water drive mode of the development is the formation problem of Stefan type [2].

However, for oil-bearing strata developed in elastic water drive mode of practical importance is the problem, in which a predetermined law of motion of the interface between liquids are studied the operating modes of the gallery in which such motions are possible. Considering the pressure boundary of water given, put the following inverse problem: find an operating mode of the gallery, which would provide the interface of fluids moving in a given law.

Thus, the law of movement of the interface between fluids $s(t)$ is known and is required to determine the function $f(t)$, $P(x,t)$ of equation (1) and the additional conditions (2) - (6).

III. Method of Solution

Using the method of straightening the front, we transform the problem (1) - (6). By replacing the variables

$$y=\frac{x}{s(t)}, \qquad t=t\,, \quad P(x,t)=P(y,t)\,,$$

domain of definition of (1) Ω_S display the per area $\Omega=\{0<y<1,\, 0<t\le T\}$.

Then equation (1) and additional conditions (2) - (6) take the form

$$\frac{\partial P}{\partial t}=r(t)\frac{\partial^2 P}{\partial y^2}+u(y,t)\frac{\partial P}{\partial y},\ \ (y,t)\in\Omega=\{0<y<1,\,0<t\le T\}, \tag{7}$$

$$P\big|_{t=0}=\varphi(y)\,, \qquad 0\le y\le 1 \quad , \tag{8}$$

$$P\big|_{y=1}=p_s(t)\,, \tag{9}$$

$$\frac{\partial P}{\partial y}\Big|_{y=1}=w(t)\,, \tag{10}$$

$$P\big|_{y=0}=f(t)\,, \tag{11}$$

where

$$r(t)=\frac{\chi}{s^2(t)}; \quad u(y,t)=\frac{y}{s(t)}\frac{ds}{dt}; \quad w(t)=-\frac{\mu s(t)m}{k}\frac{ds}{dt}.$$

The advantage of this transformation is that the resulting problem (7) - (11) is considered in a rectangular domain Ω with fixed boundaries. Since the unknowns are the functions $P(y,t)$ and $f(t)$ consequently, the problem (7) - (11) belongs to a class of boundary inverse problems [3] - [6].

For the numerical solution of (7) - (11), we use the approach proposed in [6]. To move to the difference problem we introduce the uniform difference grid in the area $\overline{\Omega}$

$$\varpi_{h\tau}=\{(y_i,t_j):\, y_i=ih,\ t_j=j\tau,\ i=0,1,2,...,N,\ \ j=0,1,2,...,M\}$$

with the steps: $h=\dfrac{1}{N}$ and $\tau=\dfrac{T}{M}$.

Difference analogue of equation (7) on the grid $\varpi_{h\tau}$ can be written as the following implicit scheme

$$\frac{P_i^{j+1}-P_i^j}{\tau}=r^{j+1}\frac{P_{i+1}^{j+1}-2P_i^{j+1}+P_{i-1}^{j+1}}{h^2}+u_i^{j+1}\frac{P_i^{j+1}-P_{i-1}^{j+1}}{h}$$

$$i=\overline{1,N-1}\,,\ j=\overline{0,M-1}\,.$$

Difference analogy of the initial condition (8) and boundary conditions (9) - (11) can be written as

$$P_i^0=\varphi_i\,, \quad i=\overline{0,N}\,,$$

$$\frac{P_N^{j+1}-P_{N-1}^{j+1}}{h}=w^{j+1}\,,$$

$$P_N^{j+1}=p_s^{j+1}\,,$$

$$P_0^{j+1}=f^{j+1}\,.$$

The resulting system of difference equations to the form

$$a_iP_{i-1}^{j+1}-c_iP_i^{j+1}+b_iP_{i+1}^{j+1}=-d_i\,, \tag{12}$$

$$i=\overline{1,N-1}\,,\ j=\overline{0,M-1}\,,$$

$$P_i^0=\varphi_i \qquad i=\overline{0,N} \tag{13}$$

$$P_N^{j+1}=P_{N-1}^{j+1}+hw^{j+1} \tag{14}$$

$$P_N^{j+1}=p_s^{j+1} \tag{15}$$

$$P_0^{j+1}=f^{j+1} \tag{16}$$

where

$$a_i=r^{j+1};\quad b_i=r^{j+1}-hu_i^{j+1};\quad c_i=a_i+b_i+h^2/\tau;\quad d_i=P_i^jh^2/\tau\ .$$

We represent the solution of (12) - (16) as

$$P_{i+1}^{j+1}=\alpha_{i+1}P_i^{j+1}+\beta_{i+1}\,, i=0,1,2,\ldots,N-1\,, \tag{17}$$

where α_{i+1},β_{i+1} is yet unknown factors. We write a similar expression for P_i^{j+1}

$$P_i^{j+1}=\alpha_iP_{i-1}^{j+1}+\beta_i\,.$$

Substituting P_i^{j+1},P_{i-1}^{j+1} into equation (12), we obtain the following formulas for the coefficients α_i,β_i

$$\alpha_i=a_i/(c_i-\alpha_{i+1}b_i),\ \ \beta_i=(b_i\beta_{i+1}+d_i^{j+1})/(c_i-\alpha_{i+1}b_i),$$

$$i=N-1,\ N-2,\ ...,1$$

$$\alpha_N=1, \qquad \beta_N=hw^{j+1}.$$

Once the coefficients are α_i,β_i found for all $i=\overline{1,N}$, it is possible to determine the relationship between P_N^{j+1} and P_0^{j+1} in explicit form. To do this, (17) can be written in $i=N-1$

$$P_N^{j+1}=\alpha_NP_{N-1}^{j+1}+\beta_N\,.$$

Substituting the expression fo

$$P_{N-1}^{j+1},\ P_{N-1}^{j+1} = \alpha_{N-1} P_{N-2}^{j+1} + \beta_{N-1},$$

we have

$$P_N^{j+1} = \alpha_N \alpha_{N-1} P_{N-2}^{j+1} + \alpha_N \beta_{N-1} + \beta_N .$$

Then, substituting in this equation the expression for

$$P_{N-2}^{j+1}, P_{N-3}^{j+1}, \ldots, P_1^{j+1},$$

we obtain a formula which P_N^{j+1} is expressed through P_0^{j+1}

$$P_N^{j+1} = P_0^{j+1} \prod_{i=1}^{N} \alpha_i + \sum_{i=1}^{N-1} \beta_i \prod_{n=i+1}^{N} \alpha_n + \beta_N .$$

Hence, using (15) (16), we obtain

$$f^{j+1} = \frac{p_s^{j+1} - \sum_{i=1}^{N} \beta_i \prod_{n=i+1}^{N} \alpha_n - \beta_N}{\prod_{i=1}^{N} \alpha_i} . \qquad (18)$$

Having determined f^{j+1} by the formula (18), we can consistently find $P_1^{j+1}, P_2^{j+1}, \ldots, P_{N-1}^{j+1}$ the recurrence formula (17). In the transition to the next layer is the time this procedure is repeated calculations again.

Thus, the proposed numerical method allows for each time step to determine the operational mode of the gallery and the pressure distribution in the oil reservoir.

IV. The Results of Numerical Experiments

To determine the effectiveness of the practical application of the proposed computational algorithm were carried out numerical simulations to model problems. The scheme of the numerical experiment is as follows. For the given functions $f(t)$, $s(t)$ of the direct problem was solved (7), (8), (10) (11). This dependence $P(1,t)$ was taken as accurate for the numerical solution of the inverse problem of reconstruction $f(t)$. The first series of calculations performed with the use of the unperturbed data. The second series of calculations were performed when applied to a function $P(1,t)$ that models the error of experimental data

$$\tilde{P}(1,t) = P(1,t) + \delta(\sigma(t) - 0.5),$$

where $\sigma(t)$ is a stochastic process, modeled using a random numbers, δ is the level of error.

The calculations were performed on the spatial-temporal difference grid with the steps $h = 0.04$, $\tau = 5$ a day. The results of numerical experiments conducted for the case of $\beta = 10^{-8}$ Pa^{-1}, $k = 2 \cdot 10^{-12}$ m^2, $\mu = 0.003$ Pa.s, L =200 m, $s(t) = L - 0.5t$, $m = 0.35$, $f(t) = 150 - 50\sin(\pi t/12)$ atm., at the level of error, the value determined by the $\delta = 5$ atm. presented in the table; in it t is the time; f^t is the exact values of $f(t)$, $\overline{f}$ is calculated values for the unperturbed data, $\tilde{f}$ is calculated values for the perturbed data.

TABLE I.

t, day	**10**	**20**	**30**	**40**	**50**	**60**	**70**	**80**	**90**	**100**
f^t, Atm.	225	293	200	293	225	250	275	206	300	206
$\overline{f}$, Atm..	225	293	200	293	225	250	275	206	300	206
$\tilde{f}$, Atm.	222	291	201	291	224	250	277	205	298	205

V. Conclusion

As the results of numerical experiments, using the unperturbed input the desired function is restored exactly. When using the perturbed input data, in which the error is the fluctuation nature of the desired function is restored with a relative error of about 2%. With a decrease in the level of decision error is restored more accurately.

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