On Identification of Nonlinear Binary Dynamic Systems

Yakup Haci
Canakkale Onsekiz Mart University, Canakkale, Turkey
yhaciyev@comu.edu.tr

***Abstract*— In this paper, the identification problem of nonlinear dynamic systems is transformed into the discrete optimization problem of functional defined by a set of $[\mathrm{GF}(2)]^{q}$ and it is solved by recursive algorithms of adaptive approximation.**

Keywords*— *identification problem; recursive algorith; adaptive approximation

In this paper, the adaptive algorithms are given for non-linear binary dynamic systems (BDS). The general form of such dynamic systems is represented by ℓ^{th} order logical difference equations as

$$y[n] = f(y[n-1], \dots, y[n-\ell], x[n-1], \dots, x[n-\ell_1]) \quad , (1)$$

where $y[n]$, $x[n]$ is input and output variable respectively. $f(.)$ is a non-linear Boole function given on the arbitrary complete system.

Nonlinear BDS' can be given by means of first order logical difference equations .

$$\vec{y}[n] = f(\vec{y}[n-1], \vec{x}[n-1]), \qquad (2)$$

where

$$\vec{y}[n] = (y_1[n], \dots, y_{\ell}[n]) \,,$$
$$\vec{x}[n] = (x_1[n], \dots, x_{\ell_1}[n]) \,. \qquad (3)$$

Are input and output boole vectors respectively.

Identification of binary dynamic systems consists of constructing the equation of dynamic systems corresponding to input and output variables.

For different representation of nonlinear binary dynamic systems, let us apply adaptive approximation to identification problem.

To solve this identification problem, according to ℓ^{th} order, logical difference equations of the dynamic systems need to be determined in the best way. So we consider $\ell + \ell_1$ dimensional state vector,

$$\vec{u}[n] = (y[n-1], \dots, y[n-\ell], x[n-1], \dots, x[n-\ell_1]) \; .(4)$$

By virtue of (4), (1) can be rewritten as

$$y[n] = f(\vec{u}[n]). \qquad (5)$$

Suppose that we consider the logical system {Λ,V,–}. We can rewrite right hand-side of (5) with accuracy of undetermined coefficient in the following form.

$$f(\vec{u}[n]) = a_1 u_1[n] \vee a_{\bar{1}}\overline{u_1}[n] \vee a_2 u_2[n] \vee a_{\bar{2}}\overline{u_2}[n] \vee \dots, \vee a_{\ell+\ell_1} u_{\ell+\ell_1}[n] \vee a_{\overline{\ell+\ell_1}}\overline{u_{\ell+\ell_1}}[n] \vee a_{12} u_1[n] \vee a_{\bar{1}2}\overline{u_1}[n]\overline{u_2}[n] \vee \dots, \vee a_{\overline{\ell+\ell_1-1}}\overline{u_{\ell+\ell_1-1}}[n] u_{\ell+\ell_1}[n] \vee a_{123} u_1[n] u_2[n] u_3[n] \vee a_{\bar{1}23}\overline{u_1}[n] u_2[n] u_3[n] \vee a_{1\bar{2}3} u_1[n]\overline{u_2}[n] u_3[n] \vee \dots, \quad \vee a_{123,\dots,\ell+\ell_1-1\;\ell+\ell_1} u_1[n]\overline{u_2}[n] \dots \overline{u_{\ell+\ell_1-1}}[n] u_{\ell+\ell_1}[n] \vee a_{\bar{1}\bar{2}\,\bar{3}\dots\overline{\ell+\ell_1}}\overline{u_1}[n]\overline{u_2}[n] \dots \overline{u_{\ell+\ell_1}}[n]. \qquad (6)$$

Instead of the right hand side of (6), we may write arithmetic function as

$$\hat{f}(\vec{u}\,, \vec{a}) = \vec{a}^T \varphi(\vec{u}), \qquad (7)$$

where

$$\vec{a} = (a_1, a_{\bar{1}}, a_2, a_{\bar{2}}, \dots, a_{\bar{1}\bar{2}\,\bar{3}\dots\overline{\ell+\ell_1-1}\,\overline{\ell+\ell_1}}) \;, \qquad (8)$$

$$\varphi(\vec{u}[n]) = (u_1[n], (1-u_1[n]), u_2[n], (1-u_2[n]), \dots, (1-u_1[n])(1-u_2[n]) \dots \big(1-u_{\ell+\ell_1-1}[n]\big), \dots, (1-u_1[n])(1-u_2[n]) \dots (1-\; u_{\ell+\ell_1}[n])$$

and

$$a_1, a_{\bar{1}}, a_2, a_{\bar{2}}, \dots, a_{\bar{1}\bar{2}\,\bar{3}\dots\overline{\ell+\ell_1-1}\,\overline{\ell+\ell_1}} \qquad (9)$$

are arbitrary binary numbers.

As indicated above for an arbitrary coefficient $\vec{a}$, we have

$$f(\vec{u}) \neq \hat{f}(\vec{u}\,, \vec{a}). \qquad (10)$$

Thus, the determination problem of the coefficient arises. So any function

$$F(\varepsilon(\vec{a}, \vec{u})),$$

which is dependent of the neighborhood,

$$\varepsilon(\vec{a}, \vec{u}) = y - \vec{a}^T \varphi(\vec{u}),$$

has to be minimal.

To apply adaptive approach on solution of the problem it is assumed that $\{\vec{u}[n]\}$ is a stationary random sequence. Then it is transformed to

$$J(\vec{a}) = M_u[F(\varepsilon(\vec{a},\vec{u}))] \to \min_{\vec{a}\ \in [GF(2)]^q} \quad (11)$$

where

$$[\mathrm{GF}(2)]^{\mathrm{q}} = \underbrace{\mathrm{GF}(2) \times \mathrm{GF}(2) \times \ldots \times \mathrm{GF}(2)}_{\mathrm{q}}, \quad \mathrm{q} = \ell + \ell_1$$

The considered problem is transformed into the problem of minimal discrete functional defined on a set of $[\mathrm{GF}(2)]^{\mathrm{q}}$.

In this representation, distribution of the stationary process is unknown but realization of function is mesurable. We can solve the problem by applying adaptive approximation with the aid of recursive algorithms providing that problem has a solution with pure and mix strategies. Finally, the problem is transformed to stochastic programming problem and the solution of this problem is shown at [4]. For instance, when we apply recursive algorithms to pure strategies, we get the following equation.

$$\vec{a}\,[n] = \vec{a}\,[n-1] - \gamma[n]\widetilde{\nabla}_a F(\varepsilon(\vec{a}[n-1],\vec{u}[n])). \quad (12)$$

If the following conditions are satisfied,

1) $\gamma[n] > 0$ and when $n \to \infty$, $\gamma[n] \to 0$,
2) $F(\varepsilon(a,u))$ is bounded for each $a[n], u[n]$,
3) $\sum_{n=1}^{\infty} \gamma[n] = +\infty$, $\sum_{n=1}^{\infty} \gamma^2[n] < \infty$,

then the functional $J(\vec{a})$ is minimal when $a = a^*$.

It is much more convenient to use (2) in some cases where output variables are given as boole vector. Thus, the previous identification method changes. Let us take $\{\wedge, \vee, -\}$ system as a complete system and substitute the function

$$f_\mu(\vec{y},\vec{x},\vec{a}) = \sum_{v=1}^{\hat{q}} a_v \varphi_{\mu v}(\vec{y},\vec{x}) \qquad \mu = 1,2,\ldots,\ell,$$

$$\hat{q} = 2^{\ell+\ell_1}, \quad (13)$$

or the vector

$$\hat{f}(\vec{y},\vec{x},\vec{a}) = \phi(\vec{y},\vec{x})\vec{a}$$

with the components of boole vector function $\widehat{f}(\vec{y},\vec{x})$ where

$$\phi(\vec{y},\vec{x}) = \|\varphi_{\mu v}(\vec{y},\vec{x})\|$$

is a $\ell \times \hat{q}$ matrice involving known arithmetic function.

Moreover, identification problem of BDS' is transformmed to the following minimal problem of expected value .

$$J(\vec{a}) = M\{F(\vec{y}[n] - \phi(\vec{y}[n-1],\vec{x}[n-1]\vec{a}))\}, \quad (14)$$

where F is an arbitrary pseudoboole function which has a pseudogradient.

By applying recursive algorithms, we have the following form.

$$\vec{a}[n] = \vec{a}[n-1] - \gamma[n]\widetilde{\nabla}_a F(\vec{y}[n-1],\vec{x}[n-1],\vec{a}[n-1]).$$

If the necessary conditions are satisfied then the functional $J(\vec{a})$ is minimal when $a = a^*$.

REFERENCES

[1] A. J. Anderson, Automata Theory with Modern Applications, Cambridge University Press, 37-99, 2006.

[2] A. J. Anderson, Discrete Mathematics with Combinatorics, New Jersey, second edition, 9-38, 2004.

[3] H. K. Rosen, Discrete Mathematics and Its Applications, (3 rd. Ed.), Mc Graw-Hill, 611-622 p. , 1995.

[4] Y. M. Yermolyev, Stochastic Programming Methods, Nauka, 3-240,1976. (in Russian)

[5] Y. Z. Sıpkin, Adaptation and Teaching of Automatic Systems, Nauka,61-75, 149-162, 1968. (in Russian)

[6] A. Gill, Introduction the Theory Finite -State Machines, Mc Graw-Hill Company , 6-17, 19-44, London, 1962.