

# Computer Experiment for the Spray Drying Process Considering Concomitant Coalescence

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**Abstract**— System of integro-differential equations to describe the processes occurring in the drying columns with spray is suggested. Differential equations, composed of the functions of distribution of substances in size and the longitudinal coordinate system, described as a process of drying the droplets, and coalescence. The properties of the model are obtained by computer simulation.

**Keywords**— process of spray drying; integro-differential equations; coalescence of the distribution function of particle size

## I. STATEMENT OF THE PROBLEM AND BASIC DEFINITIONS

The study of spray drying involves many physical processes of heat - and mass transfer in a disperse medium aerated. A very effective method is the use of mathematical modeling, through which we can formulate the basic laws and the interaction of these processes is followed by a parametric identification of the real data of the existing kilns. This is an important area of engineering, manufacturing processes to date is hardly elaborated.

The mathematical description of the dynamics of formation of the distribution function and its evolution over time with changes in the parameters of the perturbation and control can be performed based on the laws of the drying droplets of coagulation and coalescence, and gravitational settling. To this end, proposed to use two-dimensional distribution function. As one of the independent variable used in the longitudinal coordinate of the drying apparatus, measured from the point of entering the dried solution  $-\infty < \ell < \infty$  (Fig. 1). In this paper we consider one-dimensional problem in space, ie the problem of zero gradient along the radial coordinate. Model spray dryer is built on the following idealized situation:

- The granules and drops make a chaotic motion with laminar and turbulent components [1];
- The length of free path of particles is much greater than their diameter;
- The collision of two grains is not sticking (coagulation);
- Collision of drops with a drop of granule birth only if its mass exceeds the mass of granules;

- Drop into a solid particle, if the moisture content by weight greater than a given value  $\varepsilon$ ;
- As a result of a collision with a solid particle droplet is formed particle, aggregate state, which is determined by the size of the resulting moisture content.

The simultaneous collision of three particles has zero credibility.

Building such a model could be implemented with the introduction of a number of two-dimensional distribution functions.

### A. The distribution function of droplet size and the longitudinal coordinate system:

The rate drops in the jet spray is incomparably greater than the maximum speed of random motion, and free of precipitation. Therefore, the state of relatively rapid processes fill tube space drops occurring after leaving the nozzle, made quasi-stationary. Consequently, the differential distribution function of the introduced droplet size, and the longitudinal coordinate staff in the following form:

$$\varphi_m = \varphi_m(x, \ell, \mathbf{k}_v) \quad (1)$$

where  $x$  – the mass of the drop;  $\mathbf{k}_v$  - vector of parameters to be omitted in further notation.

The physical meaning of this function is that it expresses the distribution of droplets of different sizes along the length of the drying unit after input portion (single pulse) of the liquid solution is dried.

### B. The concept of the lifetime of the droplets.

The time required for drying the droplets can be expressed as a function of droplet size:

$$\theta = \theta(x) \quad (2)$$

which makes it possible to move on to the particle distribution function for the lifetime:

$$\varphi_b = \varphi_b(\theta) \quad (3)$$

C. The function of the velocity distribution of droplets on the death of space and size of droplets due to drying:

$$w_d(x, l, t) = g[t - \theta(x)] \cdot \varphi_m(x, l) \quad (4)$$

The peculiarity of this function is that it is not uniquely defined at a given point  $x$ , and requires taking into account the domain of its argument  $\tau = t - \theta(x)$  in the plane  $(t, x)$ .

D. Function of deposition rate, the same for droplets and granules (dried droplet) on the size:

$$\mu = \mu(x) \quad (5)$$

E. Two-dimensional distribution function of the solid (dry) particles.

$$\psi = \psi(x, l, t) \quad (6)$$

F. Functions expressing the density of both dispersed phases, at the time:

$$\alpha = \alpha(l, t) \quad \beta = \beta(l, t) \quad (7)$$

In connection with these parameters the following relations:

$$\alpha = \frac{1}{S_a} \int_0^{\infty} \varphi(x, l, t) dx, \quad \beta = \frac{1}{S_a} \int_0^{\infty} \psi(x, l, t) dx, \quad (8)$$

where  $S_a$  – cross sectional area of the unit.

## II. PROBABILITY OF COLLISION AND PARTICLE VELOCITY COALESCENCE

The decisive role for the formation of state functions  $\varphi(x, l, t), \psi(x, l, t)$  accounts for coalescence. The meeting of two droplets in chaotic motion, as well as a drop on a solid particle is kolalestsentsiyu (adhesion). We assume that the mass growth rate, or decreasing substances, grouped by state and by size, proportional to the probability of collision, and their masses. The probability of collision particles (droplets or granules) within the unit of time with any other particle depends on the concentration, ie of the distribution function, density of the aerosol cloud and the effective rate statistic.

The effective rate is a statistical indicator of regular and chaotic components. The latter depends on the gas-dynamic conditions in this area. Without touching the laws of

formation of this situation, we introduce a function of the effective random velocity component

$$K_{ST} = K_{ST}(x, \eta, \varepsilon) \quad (9)$$

where  $\eta$  – viscosity of the gas,  $\varepsilon$  – turbulence is introduced by a jet of spray. This rate will be determined as the ratio of sputtering power on the discharge capacity of drying gas. Regular component is determined by the rate of deposition of particles.

Thus, the probability of collision of two drops of the masses belonging to the interval  $(x, x + dx)$ ,  $(\tilde{x}, \tilde{x} + dx)$  expressed as:

$$P(x, \tilde{x}, \alpha, K_{ST}, dx) = K_{ST} \alpha^2 x \varphi(x) \tilde{x} \varphi(\tilde{x}) d^2 x \quad (10)$$

The likelihood of encountering a drop with a solid particle we write, following a similar concept:

$$P(x, \tilde{x}, \alpha, \beta, K_{ST}, dx) = K_{ST} \alpha \beta x \varphi(x) \tilde{x} \psi(\tilde{x}) d^2 x \quad (11)$$

## III. BASIC EQUATIONS OF DYNAMICS

We write the material balance for the differential of  $(x, x + dx) \times (l, l + dl)$  with respect to the liquid dispersed phase. The share of the total mass of droplets contained within this differential area is:

$$d\Phi = \varphi(x, l) dx dl \quad (12)$$

The rate of change in birth rates balanced by spraying droplets of injected fluid, drying droplets, formation of new particles in the layer as a result of the differential adhesion, as well as the loss of particles (going beyond the limits of the differential area) due to the same drying effect, trapping ash layer from the differential  $dl$  due to gravitational settling. The rate includes all these components, awarded as follows:

$$W = \frac{d}{dt} d\Phi = \frac{d\varphi(x, l, t)}{dt} dx dl \quad (13)$$

Balance velocities for the liquid aerosol phase we write, taking into account the eight major components  $W$

$$W = W_1 + W_2 + W_3 + W_4 + W_5 + W_6 + W_7 + W_8 \quad (14)$$

who commented below:

- The rate increase imposed by a spray of droplets:

$$w_1 = g(t) \cdot \varphi_m(x, l) dx dl \quad (15)$$

- The rate of decrease due to loss of droplets due to drying:

$$w_2 = g[t - \theta(x)] \cdot \varphi_m(x, l) dx dl \quad (16)$$

- The rate of increase made by gravity settling of new drops:

$$W_3 = (V - \mu(x))\varphi(x, l, t)dx \quad (17)$$

- The rate of decrease due to carried away by gravitational flow of drops from elementary layer:

$$W_4 = -(V - \mu(x))\varphi(x, l + dl, t)dx \quad (18)$$

- Birth rate drops due to the trapping of two randomly drops associated with the probability of collisions (11) and the width of the elementary layer  $dxdl$ .

- In this case, we consider the collision between the particles whose total mass is given,  $x$ , ie  $x = \tilde{x} + \xi$ . Using  $\xi$  in the integrand as a variable, write the following formula for the velocity:

$$W_5 = 2K_{ST}\alpha \cdot dxdl \cdot \int_0^{0.5x} (x - \xi)\xi \varphi(x - \xi)\varphi(\xi)d\xi \quad (19)$$

Note that for brevity in (17) and later in the records of the functions  $\varphi(x, l, t)$ ,  $\xi(x, l, t)$  will be passed arguments  $l, t$ .

- Birth rate drops from sticking to the grains smaller than their size, expressed as a similar integral:

$$W_6 = K_{ST}\sqrt{\alpha\beta} dxdl \cdot \int_0^x (x - \xi)\xi \psi(x - \xi)\varphi(\xi)d\xi \quad (20)$$

The lower limit of the integral in this expression takes into account that according to the fourth assumption underlying the proposed model, the birth of droplets leads to only those accessions that satisfy the condition  $x > \xi$ .

- The rate drops due to loss of adhesion to the beads in excess of the mass, ieadhesion resulting in the granules are produced:

$$W_7 = -K\sqrt{\alpha\beta} dxdl \cdot \int_0^{0.5x} (x - \xi)\xi \varphi(x - \xi)\psi(\xi)d\xi \quad (21)$$

- The rate of decrease due to loss of droplets from any attachment is determined by the formula:

$$W_8 = -K_{ST}\alpha \cdot dxdl \cdot \int_0^x x\xi \varphi(x)\varphi(\xi, l, t)d\xi \quad (22)$$

Taking into account (3), (8), (15) - (19), we write:

$$\begin{aligned} \frac{\partial \varphi(x, l, t)}{\partial t} = & -(V - \mu(x))\frac{1}{dl} [\varphi(x, l + dl, t) - \varphi(x, l, t)] + \\ & + \{g(t) - g[t - \theta(x)]\}\varphi_m(x, l) + \\ & + 2K_{ST}\alpha \int_0^{0.5x} (x - \xi)\xi \varphi(x - \xi, l, t)\varphi(\xi, l, t)d\xi + \\ & + K_{ST}\sqrt{\alpha\beta} \int_0^x (x - \xi)\xi \psi(x - \xi, l, t)\varphi(\xi)d\xi - \end{aligned} \quad (23)$$

$$\begin{aligned} & - K_{ST}\sqrt{\alpha\beta} \int_{0.5x}^{\infty} (x - \xi)\xi \varphi(x - \xi, l, t)\psi(\xi, l, t)d\xi - \\ & - K_{ST}\alpha \int_0^{\infty} x\xi \varphi(x, l, t)\varphi(\xi, l, t)d\xi \cdot \end{aligned}$$

It is easy to see that the first term in (20) represents the partial derivative of  $l$  of the function  $\varphi = \varphi(x, l, t)$ . Therefore:

$$\begin{aligned} \frac{\partial}{\partial t} \varphi(x, l, t) = & -[V - \mu(x)]\frac{\partial}{\partial l} \varphi(x, l, t) + \\ & + \{g(t) - g[t - \theta(x)]\} \cdot \varphi_m(x, l, t) + \\ & + 2K_{ST}\alpha \int_0^{0.5x} (x - \xi)\xi \varphi(x - \xi, l, t)\varphi(\xi, l, t)d\xi - \\ & K_{ST}\alpha \int_0^{\infty} x\xi \varphi(x, l, t)\varphi(\xi, l, t)d\xi + \\ & + K_{ST}\sqrt{\alpha\beta} \int_{0.5x}^x (x - \xi)\xi \psi(x - \xi, l, t)\varphi(\xi, l, t)d\xi - \\ & - K_{ST}\sqrt{\alpha\beta} \int_{0.5x}^{\infty} (x - \xi)\xi \varphi(x - \xi, l, t)\psi(\xi, l, t)d\xi \cdot \end{aligned} \quad (24)$$

Thus, we obtain an integro-differential equation describing the evolution of the main regularities of the distribution function of droplets in an aerosol cloud of the size and space. A similar argument leads to the following equation for granules dried ingredients:

$$\begin{aligned} \frac{\partial}{\partial t} \psi(x, l, t) = & -(V - \mu(x))\frac{d}{dl} \psi(x, l, t) + g[t - \theta(x)]\varphi_m(x, l) - \\ & - K_{ST}\sqrt{\alpha\beta} \int_{0.5x}^x (x - \xi)\xi \psi(x - \xi, l, t)\varphi(\xi, l, t)d\xi + \\ & + K_{ST}\sqrt{\alpha\beta} \int_{0.5x}^{\infty} (x - \xi)\xi \varphi(x - \xi, l, t)\psi(\xi, l, t)d\xi \cdot \end{aligned} \quad (25)$$

The validity of the preconditions imposed on the basis of the construction of structures above equations efficiently check through computational experiments on the model (24) and (25). In this case the input data for the solution of this system are a function of  $\varphi_m = \varphi_m(x, l)$ ,  $g(t)$ ,  $\mu(x)$  and the parameters  $V, K_{ST}, S_a$ . system was integrated numerically on a sufficiently large interval change  $x$  and  $l$ . Figure 1 shows a cross section in a time sequence of the obtained solutions.

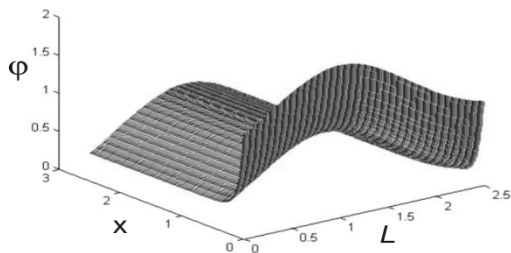


Figure 1. The stationary solution of (24), (25).

The resulting solution corresponds to the input function

$$\varphi_{in}(x, l) = \frac{1.6l - 0.3l^2}{2\pi\sqrt{\sigma_1}} \exp\left(-\frac{(x-\bar{x})^2}{2\sigma^2}\right); \mu(x) = 151.3x - 253.6x^2$$

and parameters

$$S_a = 1M^2; L_1 = 3M; L_2 = 8M;$$

$$g(t) = const = 0.2kg/c; \quad V = 2M^3/c.$$

The resulting evolution equation for two-dimensional distribution functions  $\varphi(x, l, t)$  and  $\psi(x, l, t)$  formed the basis of a mathematical model of spray drying of the composite in the production of surfactant-active substances for which solved the problem of parametric identification according to the existing installation.

#### REFERENCES

- [1] Nigmatulin R.I. Fundamentals of mechanics of heterogeneous media. Pergamon Press, 1978, P.336.
- [2] Reineccius G.A. The spray drying of food flavors. Drying Technology, 2004. 22(6), 1289-1324.
- [3] Guignon B., Duquenoy A. & Dumoulin E. Fluid bed encapsulation of particles and practice. Drying Technology, 2002, 20(2), 419-447.
- [4] Buffo R.A., Probst K., Zehentbauer G., Luo Z. & Reineccius G.A.. Effects of agglomeration on the properties of spray-dried encapsulated flavours. Flavour and Fragrance Journal, 2002, P.-17, 292-299.