Prediction of Signal Characteristics Using Autoregressive Moving Average Method

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Abstract— **In this paper the autoregressive moving average (ARMA) method is used for forecasting of signal characteristics. The ARMA method is applied to seismoacoustic signals to predict estimates of signal characteristics. The analysis is verified through real signal characteristics.**

Keywords— Autoregressive Moving Average Method (ARMA); seismoacoustic signal charcteristics; noise variance; cross-correlation function; noise correlation

I. Introduction

The importance of time series analysis and forecasting in science, engineering, and business has, in the past, increased steadily and it is still of actual interest for engineers and scientists. Mathematical models used for time series analysis are generally regression models, time-domain models and frequency-domain models. [1,2]. The most popular regression models in engineering are the autoregressive model (AR) , moving-average model (MA), ARMA model, ARIMA model and CARIMA models.

Autoregressive models express the current value of a time series by a finite linear aggregate of previous values and by a shock μ_t [1]:

$$Z_t = a_1 Z_{t-1} + a_2 Z_{t-2} + \ldots + a_v Z_{t-v} + \mu_t \qquad (1)$$

where a_1 to a_v are the autoregressive parameters, μ_t is the white noise and v is the model order. The validity of an autoregressive model assumes that the time series to be modeled is stationary. Also, because of some possible internal cumulative effects, the autoregressive process will only be stable if the values of parameters a are within a certain range. It is common to write the autoregressive equation in terms of deviations $\tilde{Z}_t = Z_t - \mu_t$ generally using the variable Z and its deviation $\tilde{Z} = Z - \mu$. The individual terms of the time series now become $\tilde{Z}_t, \tilde{Z}_{t-1}, \tilde{Z}_{t-2}, \tilde{Z}_{t-3}, \ldots$, resulting in the autoregressive model [1].

$$\tilde{Z}_t = \phi_1 \tilde{Z}_{t-1} + \phi_2 \tilde{Z}_{t-2} + \phi_3 \tilde{Z}_{t-3} + \ldots + \phi_P \tilde{Z}_{t-p} + a_t \qquad (2)$$

where $\mu, \phi_1, \phi_2, \phi_3, \ldots, \phi_p, a_t$ are unknown parameters to be estimated from the observation data. Introducing the autoregressive operator [1]:

$$\phi(B) = 1 - \phi_1 B - \phi_2 B^2 - \phi_3 B^3 - \ldots - \phi_p B^p \qquad (3)$$

The autoregressive model can be written in the compact form

$$\phi(B)\tilde{Z}_t = a_t \qquad (4)$$

A crucial problem in modeling of autoregressive time series is the selection of the order of the model to be built. A useful approach in this case is the analysis of the related partial autocorrelation function and the inverse autocorrelation function, because using the autocorrelation function itself is computationally complicated in the case of building of higher order models. Alternatively, fitting the time series shape by models of progressively higher order can be used, along with the analysis of the residual sum of squares for each order.

Another approach frequently used in modeling of univariate time series is based on the moving-average model [1]:

$$\tilde{Z}_t = a_t - \theta_1 a_{t-1} - \theta_2 a_{t-2} - \theta_3 a_{t-3} - \ldots - \theta_q a_{t-q} \qquad (5)$$

Which expresses $\tilde{Z}_t$ in terms of an infinite weighted linear sum of $a_t, a_{t-1}, a_{t-2}, \ldots, a_{t-q}$. Introducing the moving-average operator of order q

$$\theta(B) = 1 - \theta_1 B - \theta_2 B^2 - \theta_3 B^3 - \ldots - \theta_q B^q \qquad (6)$$

The moving-average model can be written in the compact form as:

$$\tilde{Z}_t = \theta(B)\, a_t \qquad (7)$$

The model contains (q+2) unknown parameters $\mu, \theta_1, \theta_2, \theta_3, \ldots, \theta_q, a_t$ to be estimated from the observation data.

The combination of the AR and MA models makes up the ARMA model [1]:

$$\begin{aligned}\widetilde{Z}_t &= \phi_1\widetilde{Z}_{t-1} + \phi_2\widetilde{Z}_{t-2} + \phi_3\widetilde{Z}_{t-3} + \ldots + \phi_p\widetilde{Z}_{t-p} + \\ &+ a_t - \theta_1 a_{t-1} - \theta_2 a_{t-2} - \theta_3 a_{t-3} - \ldots - \theta_q a_{t-q}\end{aligned} \tag{8}$$

Rewriting the model as:

$$\begin{aligned}&\widetilde{Z}_t - \phi_1\widetilde{Z}_{t-1} - \phi_2\widetilde{Z}_{t-2} - \phi_3\widetilde{Z}_{t-3} - \ldots - \phi_p\widetilde{Z}_{t-p} = \\ &= a_t - \theta_1 a_{t-1} - \theta_2 a_{t-2} - \theta_3 a_{t-3} - \ldots - \theta_q a_{t-q}\end{aligned} \tag{9}$$

and rearranging it as:

$$\begin{aligned}&(1 - \phi_1 B - \phi_2 B^2 - \ldots - \phi_p B^p)\widetilde{Z}_t = \\ &= (1 - \theta_1 B - \theta_2 B^2 - \ldots - \theta_q B^q)\, a_t\end{aligned} \tag{10}$$

The model can finally be written in compact form as [1]:

$$\phi(B)\widetilde{Z}_t = \theta(B)\, a_t \tag{11}$$

Where B is a delay operator. The derived compact model contains $(p+q+2)$ unknown parameters $\mu, \phi_1, \phi_2, \phi_3, \ldots, \phi_p$ and $\theta_1, \theta_2, \theta_3, \ldots, \theta_q, a_t$ that are to be estimated from the given time series data..

When $\phi(B) = 1$, we have ARMA(p,q) = MA(q), and, when $\theta(B) = 1$, we have ARMA(p,q) = AR(p). Such processes are often denoted as ARMA(0,q) and ARMA(p,0) to stress the fact that the moving average model and the autoregressive model are members of the family of ARMA models.

Let

$$Z_t - \phi_1 Z_{t-1} - \ldots - \phi_p Z_{t-p} = a_t \tag{12}$$

we assume that the roots of $\phi(z)$ are outside the unit circle. When $\tau > p$, the linear combination minimizing the mean square linear prediction error is

$$f(p) = \sum_{j=1}^{p} \phi_j Z_{\tau-j} \tag{13}$$

Now the partial autocorrelation function (PACF) for $\tau > p$ namely:

$$\begin{aligned}\phi_{\tau\tau} &= corr\{Z_\tau - f(p), Z_0 - f(p)\} = \\ &= corr\{a_\tau, Z_0 - f(p)\} = 0\end{aligned} \tag{14}$$

Since, by causality, $Z_{\tau-j}$ does not depend on the future noise value a_τ. When $\tau \le p, \phi_{pp} \ne 0$ and $\phi_{11}, \ldots, \phi_{p-1,\, p-1}$ are not necessarily zero.

II. USING ARMA METHOD FOR PREDICTION OF CHARACTERISTICS

In Gum Island RNM ASP station in Azerbaijan Republic the experiment results in a time period from 15/01/2012 to 18/01/2012 is shown in Fig. 1. In 16/01/2012 an earthquake is registered in northeastern of Iran and is shown in Fig. 1.

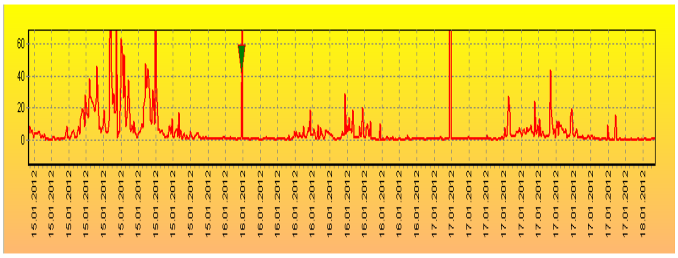

Figure 1. The experiment results in a time period from 15/01/2012 to 18/01/2012 in Qum Island RNM ASP station.

Estimates of noise variance of noise, noise correlation and cross-correlation function between the useful signal and noise, meanwhile, increase sharply in the same period, over 5-10 hours before the earthquake allowed the system to register the beginning of the origin of all abnormal seismic processes in that period within a radius over 150-200 km [3-6]. It is clear that ASP lasted more than 10 hours in 15/01/2012 (see Fig. 1.). During that period ARMA method can predict future estimations and it will be useful for earthquake prediction.

ARMA modeling is based on a unique decomposition of a strictly proper order of the numerator is less than the order of the denominator, discrete transfer function. Once the time series model has been developed and tested it can be used for forecasting the future time series values at various time distances *d*. Of course, the forecasting does not deliver the exact future values of data that the given time series will really have, but rather their estimates. Using the autoregressive model:

$$Z_t = \phi_1 Z_{t-1} + \phi_2 Z_{t-2} + \varepsilon_t \tag{15}$$

Based on a one-step movement along the time series:

$$Z_{t+1} = \phi_1 Z_t + \phi_2 Z_{t-1} + \varepsilon_{t+1} \tag{16}$$

We can formally write the predicted value to be:

$$\widetilde{Z}_{t+1} = \phi_1 Z_t + \phi_2 Z_{t-1} \tag{17}$$

For the two-steps ahead prediction, based on a two-steps movement along the time series, we can also formally write

$$Z_{t+2} = \phi_1(\phi_1 Z_t + \phi_2 Z_{t-1} + \varepsilon_{t+1}) + \phi_2 Z_t + \varepsilon_{t+2} \tag{18}$$

And the predicted value to be

$$\widetilde{Z}_{t+2} = \phi_1\widetilde{Z}_{t+1} + \phi_2 Z_t \tag{19}$$

Or we have:

$$\widetilde{Z}_{t+2} = \phi_1(\phi_1 Z_t + \phi_2 Z_{t-1}) + \phi_2 Z_t \tag{20}$$

The scaled estimates of variance of noise and its prediction result using ARMA method is shown in Fig. 2. The 100 samples of recorded of variance of noise estimates are applied to forecast the next 10 samples of variance of noise estimates using ARMA method.

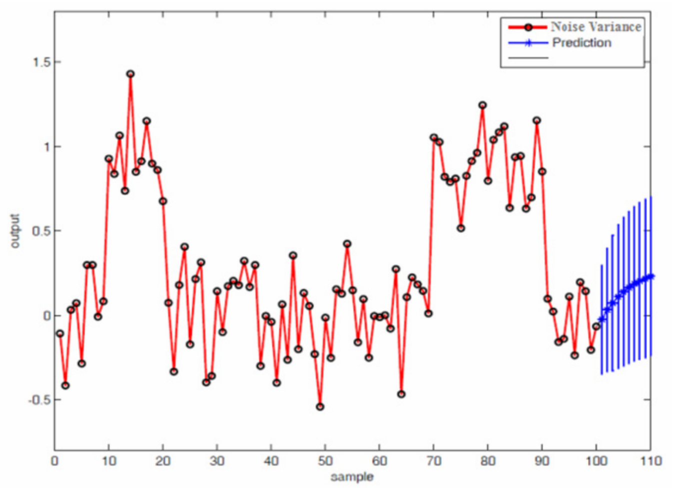

Figure 2. The noise variance estimations and its prediction result using ARMA method.

REFERENCES

[1] Ajoy K. Palit and Dobrivoje Popovic. *Computational Intelligence in time series forecasting*, Springer, 2005, 393 p.

[2] Karanasos, M. (2001). *Prediction in ARMA models with GARCH in mean effects.* //Journal of Time Series Analysis, 22, pp 555– 576, 2001.

[3] Telman Aliev. *"Digital noise monitoring of defect origin"*. Springer, London, 2007, 223 p.

[4] T.A. Aliev, G.A. Guluyev, F.H. Pashayev, A.B. Sadygov. *Noise monitoring technology for objects in transition to the emergency state*. Mechanical Systems and Signal Processing, Volume 27, February 2012, pp. 755–762.

[5] A. Abbasov, T. Aliyev, A. Ali-zada, G. Etirmishli, G. Guluyev, F. Pashayev. *Intellectual seismoacoustic telemetric station* // The third international conference "Problems of cybernetics and informatics" Volume II, September 6-8, Baku, Azerbaijan, pp. 83-85, 2010.

[6] T.A. Aliev, A.A. Alizadeh, G.D. Etirmishli, G.A. Guluev, F.G. Pashaev, and A.G. Rzaev. *Intelligent Seismoacoustic System for Monitoring the Beginning of Anomalous Seismic Process* // Seismic Instruments, 2011, Vol. 47, No.1, pp. 1–9. © Allerton Press, Inc., 2011.