

# The Eigensensitivity–Based Finite Element Model Updating for Structural Parameter Identification

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**Abstract**—Shown the eigensensitivity–based finite element model updating and given its application to four storey space steel frame structure. The monitoring, experimental details and data-processing techniques, determination of dynamic characteristics are described. Finite element model of the structure was evaluated as a primer numerical model. The output-only modal identification results were used to update a finite element model of the building. Parameters of the starting finite element model were modified using an automated procedure to improve the correlation between measured and calculated modal parameters. Mention structure was build for the testing with aim comparing several identification techniques (including ambient vibration) and other various structural engineering research studies. Copyright © 2003 OMU MF-046

**Keywords**— system identificaitn; finite element model updating; ambient vibration

## I. INTRODUCTION

Output -only modal identification techniques efficiently is used with model updating tools to develop reliable finite element models of structures. For the modal updating of the structure it is necessary to estimate sensitivity of reaction of examined system to change the parameters of a building [9, 10]. System identification is the process of developing or improving a mathematical representation of a physical system using experimental data [6, 7, 2, 14, 15 ...]. In engineering structures there are three types of identification: modal parameter identification; structural-modal parameter identification; control-model identification methods are used.

## II. DESCRIPTION OF THE STRUCTURE

The four storey space steel frame structure was build by the Earthquake Engineering Research Laboratory at the University of Ondokuz Mayıs (in scope of the research project MF-046 is supported by the University research fond) for the testing with aim comparing several identification techniques(including ambient vibration) and other various structural engineering research studies (Figure 1). It is two-by-two bay, 3.0 m x 5.0 m in plan and 4.86 m in height. Details of the structure are given in [16, 17]. All of devices with appropriate software and necessary instruments for structural monitoring are placed in mobile vehicle designed in scope of the research project MF-046 and used as mobile structural monitoring system. (Figure 1 and website: [www2.omu.edu.tr/docs/bcalismalar/1528.pdf](http://www2.omu.edu.tr/docs/bcalismalar/1528.pdf)).



Figure 1. Mobile structural monitoring system and steel frame benchmark structure.

### III. TEST DESCRIPTION

Ambient excitation was provided mainly by traffic and partly human activity. For the effort of excitation during monitoring, vehicle (fire engine) with 176.58 kN weight and 20-30 km/h speed is turned around of the laboratory building. Six accelerometers were used for the ambient vibration measurements, three of which were allocated as reference sensors always located on the 4<sup>th</sup> floor [16, 17].

### IV. FEM UPDATING STUDY

This study involved the comparison of the natural frequencies and mode shapes of the experimental model analysis and FE models until an acceptable correlation was achieved. Details of the FE model used for this study and the parameters selected for the model updating [17] are given in the following sections.

#### 4.1 Finite Element Model Calibration of the Building

A finite element model was generated in FEMtools, SAP2000 and dynamic analysis program DAP, ver.2, [11, 12]. Beams and columns were modeled as 3D beam-column elements. To better the model of the beam-column connections, a small element of 50 mm length was added at the ends of each beam to allow for variance of the stiffness of the connection without changing the properties of the entire beam. At the base of the structure in the model, the ends of every element were fixed against translation and rotation for the 6-DOF. In modeling of the steel space frame young's module  $E=2.0 \times 10^5$  MPa, the material mass density  $\rho = 77.0085$  kN/m<sup>3</sup>, the Poisson ratio  $\mu = 0.3$ . In total model consisted of 432 beam-column elements (it includes 196 beam-column connection elements with 50 mm length), 16 shell elements (for modeling of the lead plate loads) and it contained 301 nodes. Dynamic analysis result of the finite element structure model is shown in Table 1.

TABLE I. DYNAMIC ANALYSIS RESULT AT THE FE MODEL

Mode No	1	2	3	4	5	6	7	8	9	10	11
Period (sec)	0.313	0.281	0.270	0.267	0.247	0.204	0.137	0.036	0.024	0.022	0.221
Mode Type	Y	X	...	Y	T	...	...	...	...	Y	T

#### 4.2 Selection of Parameters for Model Updating

A sensitivity analysis of the dynamic response of the finite element model of the structure to a change in element properties was first conducted on a large number of parameters. A parameter refers to a selected property of a given element. The selected parameters for the sensitivity analysis were the second moment of inertia (I) of the beam-column connection elements (by 50 mm length) in both principal directions (I<sub>2</sub>, I<sub>3</sub>).

#### 4.3 The eigensensitivity-based finite element model updating

In mention method, the relationship between the perturbation in the updating parameters ( $\delta\{P\} = \{P\} - \{P_{cur}\}$ ) and the difference ( $\delta\{D\} = \{D_{mea}\} - \{D_{cal}\}$ ) between the measured ( $\{D_{mea}\}$ ) and calculation results ( $\{D_{cal}\}$ ) from the

finite element model can be represented by a sensitivity matrix ( $[S]$ ) as [5]:

$$\delta\{D\} = [S]\delta\{P\} \quad (1)$$

in which  $\{P\}$  and  $\{P_{cur}\}$  are updated and current vectors of the updating parameters, respectively; Elements of the sensitivity matrix are determined as:

$$S_{ij} = \frac{\partial\{D_i\}}{\partial P_j} \quad (2)$$

Where  $\{D_i\}$  the  $i$ -th component of the modal is vector, and  $\{P_j\}$  is the  $j$ -th component of the updating parameter vector. Through differentiating the eigen equation ( $[k]\{\phi\} = \lambda[m]\{\phi\}$ ) of a structural system with respect to updating parameters ( $\{P_j\}$ ), the derived formula for natural frequencies can be obtained as follows [8]:

$$\frac{\partial\lambda_k}{\partial P_i} = \{\phi_k\}^T \frac{\partial[k]}{\partial P_i} \{\phi_k\} - \lambda_k \{\phi_k\}^T \frac{\partial[m]}{\partial P_i} \{\phi_k\} \quad (3)$$

Where  $\lambda_k$  is the current  $k$ -th eigen values;  $\frac{\partial\lambda_k}{\partial P_i}$  is the notation for the sensitivity of the  $k$ -th eigen values ( $\lambda_k$ ) with respect to updating parameter ( $P_i$ );  $\{\phi_k\}$  is the current  $k$ -th mode shape which is normalized to the mass matrix  $[m]$ ;  $[k]$  is the current stiffness matrix. In ambient tests, higher natural frequencies are often obtained with less accuracy than the lower order ones. Therefore, a weighting matrix  $[W_p]$ , whose entries are often obtained from the reciprocals of the variance of the corresponding modal data, is introduced in the FE model updating algorithm. If only the weighting matrix of the updating parameters  $[W_p]$  is considered, the best estimation for the updating parameters can be obtained through the weighted least squares method. In this way, the solution for simultaneous equation (1) can be obtained by considering a constrained optimization problem as follows:

$$\text{Minimize } \delta\{P\}^T [W_p] \delta\{P\} \quad \text{subject to}$$

$$\delta\{D\} = [S]\delta\{P\} \quad (4)$$

Its corresponding solution is

$$\delta\{P\} = [W_p]^{-1} [S]^T ([S][W_p][S]^T)^{-1} \delta\{D\} \quad (5)$$

If both the weighting matrices  $[W_p]$ ,  $[W_D]$  are included, the best estimation of the updating parameters can be obtained by the Bayesian estimation technique. The associated FE model updating procedure can be regarded as seeking the solution of the following constrained optimization problem:

Minimize

$$(\delta\{D\} - [S]\delta\{P\})^T [W_D] (\delta\{D\} - [S]\delta\{P\}) + \delta\{P\}^T [W_p] \delta\{P\}$$

Subject to

$$\delta\{D\} = [S]\delta\{P\} \quad (6)$$

The corresponding solution can be obtained as [3]:

$$\delta\{P\} = [W_P]^{-1}[S]^T \left( [W_D]^{-1} + [S][W_P]^{-1}[S]^T \right)^{-1} \delta\{D\} \quad (7)$$

In order to avoid the updated results being physically meaningless, the lower and upper limits for the updating parameters are necessarily set in the FE model updating procedure, these are listed in Table 2.

TABLE II. THE LOWER AND UPPER LIMITS OF THE UPDATED PARAMETERS

FE model updating parameters	Lower limits (m <sup>4</sup> , zero %)	Upper limits (m <sup>4</sup> , 100%)
I <sub>2</sub> (second moment of inertia local 2 axis)	7.319e-8	1.4638e-7
I <sub>3</sub> (second moment of inertia local 3 axis)	7.848e-7	1.5696e-6

(Second moment of inertia of the beam elements by the length 0.05m added in the beam-column connection points)

The convergence criteria were also set in each iteration loop as follows:

$$|f_k - f_{\bullet k}| \leq \text{Specified limit of natural frequency difference} \quad (8)$$

$$\text{MAC}(d_k, d_{\bullet k})_{k=1,n} \geq \alpha \quad (9)$$

$$\{P_{lower}\} \leq \{P_k\} \leq \{P_{upper}\} \quad (10)$$

where  $f_k, f_{\bullet k}$  are the current analytical and corresponding experimental values of the natural frequency, respectively;  $\{P_{lower}\}, \{P_{upper}\}$  are the lower and upper limits of the updating parameters, respectively;  $\alpha$  is the lower limits of the MAC matrix;  $n$  is the compared appropriate mode's number, another word it is the considered number of compared degree of freedom of the structural system;  $\text{MAC}(d_k, d_{\bullet k})_{k=1,n}$  is the modal assurance criterion indices for between the FE computational ( $d_k$ ) and experimental ( $d_{\bullet k}$ ) mode shapes, which indicate how well the FE mode shapes fit to the corresponding measured ones and calculated as:

$$\text{MAC}(d_k, d_{\bullet k})_{k=1,n} = \frac{\left( \sum_{j=1}^n \phi_{jk} \phi_{\bullet jk} \right)^2}{\sum_{j=1}^n (\phi_{jk})^2 \sum_{j=1}^n (\phi_{\bullet jk})^2} \quad (11)$$

in which  $\phi_{jk}, \phi_{\bullet jk}$  are the  $j$ -th coordinates of the  $k$ -th analytical and measured mode shapes, respectively. Once all the conditions listed in equations (8-11) are satisfied, the iteration

process ends, and the final FE model updated results are obtained.

## V. STRUCTURAL PARAMETER IDENTIFICATION

Based on the eigensensitivity-based FE model updating procedure described in the previous sections FE model updating methodology is developed and applied to the steel frame structure for structural parameter identification. As a result of this application the parameters-mode shape responses sensitivity relationship are given in Figure 3. (The sensitivity change table is four pages, because not include to the paper). The analysis showed that the dynamic response of the FE model was more sensitive to a change first seventeen parameters. These parameters are I<sub>2</sub> of the beam-column connection elements all floors.

## VI. MODAL UPDATING RESULTS

The results are summarised in table 3 and 7. It can be seen that the 2, 3 experimental modes and the appropriate 1, 2 analytical mode shapes are well correlated (interval of correlation approximately is between 89%-97%). In table 3 and 6 it can be obtained that the 2, 3 experimental modes and the appropriate 1, 2 analytical mode shapes were well correlated before updating.

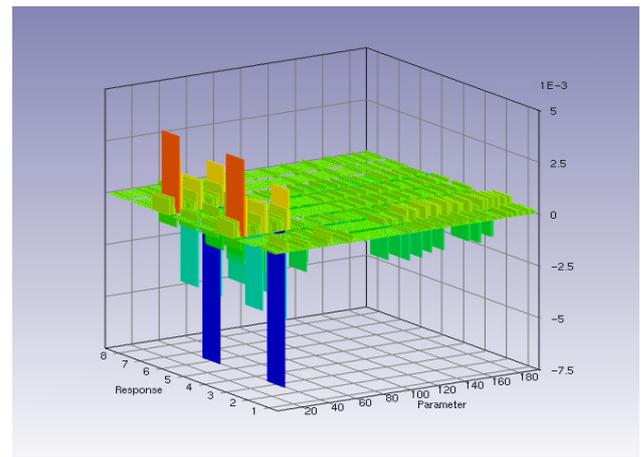


Figure 3. 3D view of the parameters-shape modes response.

3D plots of MAC matrices to first eleven mode shapes of structure before and after updating are given in Figure 4.

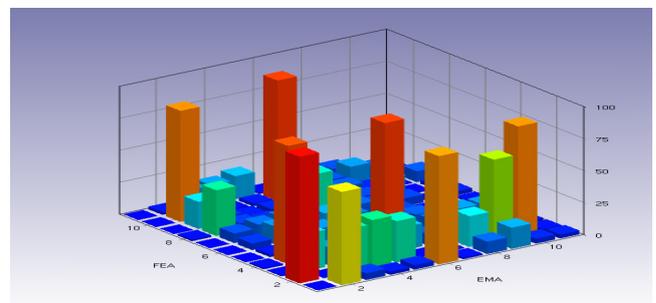


Figure 4. Comparison of 3D plots of MAC matrices to first eleven mode shapes of structure.

Mode shape pairs to first eleven mode shapes of building before and after updating are given in Table 3, 4 appropriately.

TABLE III. MODE SHAPE PAIRS BEFORE UPDATING FOR 2 MODES

FEA	Hz	EMA	Hz	Diff.	MAC
1	3.20	2	3.11	2.91	72.9
2	3.57	3	4.00	-10.77	34.2

TABLE IV. MODE SHAPE PAIRS AFTER UPDATING FOR 2 MODES

FEA	Hz	EMA	Hz	Diff.	MAC
1	3.19	2	3.11	2.67	73.1
2	3.60	3	4.00	-10.08	34.3

A summary of changes of the FEM update result to the EMA results showed that the parameter  $I_2$ ,  $I_3$  for the all of beam-column connection elements in Y direction approximately is increased two times (107%). But same parameter ( $I_2$ ,  $I_3$ ) for beam-column connection elements (joints) in the global X direction of the structure approximately is changed 11%. As seen from the modal updating result the actual system beam-column connection elements (joints) rigidity in global Y direction approximately two times more than that in the X direction. This could be explained by analyzing the configuration of the connection. In the global Y (strong) direction connection the beam is attached to the flange (15 mm thick) of the columns. In the global X (weak) direction connection the beam is attached to the web (9 mm thick) of the columns. Therefore naturally first mode shape of the actual system is obtained as the vibration in the global X direction. (Figure 5) Main difference between mode shapes of the FEM and EMA may explained with beginning incorrect acceptance about equal rigidity of the beam-column connection elements (joints) in the global X and Y direction of the structure’s finite element model.

## VII. CONCLUSION

Shown the eigensensitivity-based finite element model updating and given its application to four storey space steel frame structure. The fundamental periods and corresponding mode shapes for the 4-storey space steel structure were determined experimentally using ambient vibration measurements. The modal parameters obtained experimentally were used to calibrate a finite element model of the building. Based on the eigensensitivity-based FE model updating procedure a summary of the changes the FEM results to the EMA results is presented graphically and numerically in percent to the initial state of the structure. As seen from the modal updating result the actual system beam-column connection elements (joints) rigidity in global Y direction approximately two times more than that in the X direction. This was explained by analyzing the configuration of the connection. MAC values were generated between analytical and experimental mode shapes. Main difference between mode shapes of the FEM and EMA was explained. For more details and last applications see references [18,19].

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