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APPLICATION OF THE DIFFERENCE METHOD FOR SOLUTION OF A NONLINEAR PROBLEM ON FILTRATION OF COMPRESSIBLE LIQUID IN HEREDITARY MEDIUM

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The processes in saturated porous media are on the base of many natural phenoma. Their study is related with many practical problems of undergroud and surface constraction. The results of these investigations are also widely claimed in mining, development and operation of oil and gas deposits.

In practice, in great majority of cases we have to do with complicated problems related with fluid filtration in complicated rheology porous medium. The experiments on geomaterials show that in different levels of stresses and temperatures after attaining the limit of elasticity, there arises irreversible internal microdestruction in rocks. Growth of creep flow is obseved. This makes necessary to take into account hereditary factors in hydrodynamical calculations. Today, there are works on account of history and prehistory of loading on deformation process, filtration process and porousity growth process. A number of simpliest problems on studying the simpliest non-stationary one-dimensional filtration flows were considered, and hydrodynamic analysis of peculiarities of development of oil pools possessing viscoelastic properties are carried out. Various methodts of not in equilibrium filtration – relaxational filtration taking into account both not in equilibrium character of the filtration law and relaxational properties of porosity are suggested. In the present paper, mathematical problems on one-dimensional filtrational flow for nonlinear-relaxational porosity model and hereditary character of Darcy filtration law are formulated. Fulfilment of the rest conditions are assumed: i.e. pressure, porosity and density are constant and equal their initial values.

Write the known law of conservation of mass in the form used in the papers [4, 5]:

$$\frac{\partial(\rho m)}{\partial t}+\frac{\partial(\rho w)}{\partial x}=f(x,t)\,. \tag{1}$$

Here, m is a porosity factor, ρ is fluid's density, w is a filtration flow rate. Relation (1) holds under availability of sources or flow interior to the formation. Furthermore, the product ρm defines the mass of fluid in a unit volume, $f(x,t)$ is mass flow rate in a time unit in the availability of flows and sources.

In the place of determining relation connecting the filtration rate w with pressure gradient, we accept the hereditary model used in [5]:

$$w=-\frac{k_0}{\mu_0}\int_0^t H(t-\tau)\frac{\partial P(\tau,x)}{\partial x}d\tau\,. \tag{2}$$

Here we use the denotation $P(t,x)=\mathrm{P}\,(t,x)-p_0$ that determines pressure increment with respect to its initial value. The permeability factor is assumed to be constant.

In the place of a porosity model we take a hereditary model that takes into account dependence of porosity on the pressure change history [3], but with correction for nonlinearity

$$m=m_0+\alpha_m\int_0^t E(t-\tau)P^n(\tau,x)d\tau\,. \tag{3}$$

Here m_0 is initial porosity, n is a nonlinearity index.

Assume that the functional dependence of density on pressure is known

$$\rho = \rho(P). \tag{4}$$

For weakly compressible fluid we expand this dependence in Taylor's series in the vicinity of the initial value of pressure, retain only a linear term in the expansion and get the following formula:

$$\rho = \rho_0(1+\alpha_\rho P). \tag{5}$$

Here and in the last formulae we accepted the following denotation: k_0 is a constant coefficient of permeability velocity, μ_0 is viscosity factor, α_m is compressibility rate of a porous medium, α_ρ is fluid's compressibility rate coefficient; $H(t)$ is filtration rate relaxation kernel; $E(t)$ is a porosity relaxation kernel.

Represent equation (1) in the form:

$$\frac{\partial\rho}{\partial t}m + \frac{\partial\rho}{\partial x}w + \rho\left(\frac{\partial m}{\partial t} + \frac{\partial w}{\partial x}\right) = f(x,t). \tag{6}$$

Allowing for dependence (4), we find

$$\partial\rho/\partial t = \rho'(P)\cdot P_t'; \qquad \partial\rho/\partial x = \rho'(P)\cdot P_x', \tag{7}$$

where $P_t' = \partial P/\partial t$; $P_x' = \partial P/\partial x$.

Allowing for (7), equation (6) will be

$$\rho'(P)(mP_t' + wP_x') + \rho(m_t' + w_x') = f(x,t). \tag{8}$$

Substitute formulae (2) and (3) into (8) and find:

$$\rho'(P)\left\{P_t'(t,x)\left(m_0 + \alpha_m\int_0^t E(t-\tau)P^n(\tau,x)d\tau\right) + P_x'(t,x)\left(-\frac{k_0}{\mu_0}\int_0^t H(t-\tau)P_x'(\tau,x)d\tau\right)\right\} +$$

$$+\rho(P)\left\{\alpha_m\cdot\frac{\partial}{\partial t}\int_0^t E(t-\tau)P^n(\tau,x)d\tau - \frac{k_0}{\mu_0}\int_0^t H(t-\tau)P_{xx}'(\tau,x)d\tau\right\} = f(x,t). \tag{9}$$

Assume that the porosity relaxation kernel $E(t)$ is regular, then we have:

$$\frac{\partial}{\partial t}\int_0^t E(t-\tau)P^n(\tau,x)d\tau = E(0)P^n(t,x) + \int_0^t E'(t-\tau)P^n(\tau,x)d\tau. \tag{10}$$

Allowing for this equation, (9) will take the form

$$\rho'(P)\left\{P_t'(t,x)\left(m_0 + \alpha_m\int_0^t E(t-\tau)P^n(\tau,x)d\tau\right) - \frac{k_0}{\mu_0}P_x'(t,\tau)\int_0^t E'(t-\tau)P_x'(\tau,x)d\tau\right\} +$$

$$+\rho(P)\left\{\alpha_m E(0)P^n(t,x) + \alpha_m\int_0^t E'(t-\tau)P^n(\tau,x)d\tau - \frac{k_0}{\mu_0}\int_0^t H(t-\tau)P_{xx}''(\tau,x)d\tau\right\} = f(x,t). \tag{11}$$

Carry out regrouping in (11):

$$\rho'(P)\left\{m_0P_t'(t,x) + \int_0^t\left[\alpha_m E(t-\tau)P^n(\tau,x)P_t'(t,x) - \frac{k_0}{\mu_0}H(t-\tau)P_x'(\tau,x)P_x'(t,x)\right]d\tau\right\} +$$

$$+\rho(P)\left\{\alpha_m E(0)P^n(t,x) + \int_0^t\left[\alpha_m E'(t-\tau)P^n(\tau,x) - \frac{k_0}{\mu_0}H(t-\tau)P_{xx}''(\tau,x)\right]d\tau\right\} = f(x,t). \tag{12}$$

Further, for the dependence (4) we accept the form (5). We will assume that the coefficient α_ρ and α_m are small. Substituting (5) into (12) and rejecting the addends from the coefficients α_ρ and α_m, we get

$$\alpha_\rho m_0 P_t'(t,x) + \alpha_m E(0) P^n(t,x) + $$
$$+\alpha_m \int_0^t \left[E'(t-\tau)P^n(\tau,x) - \frac{k_0}{\alpha_m \mu_0} H(t-\tau) P_{xx}''(\tau,x) \right] d\tau = \frac{f(x,t)}{\rho_0}. \quad (13)$$

Accept the denotation:

$$N(t,\tau,x) = E'(t-\tau)P^n(\tau,x) - \frac{k_0}{\alpha_m \mu_0} H(t-\tau) P_{xx}''(\tau,x). \quad (14)$$

Then (13) is written in the form:

$$\alpha_\rho m_0 P_t'(t,x) + \alpha_m E(0) P^n(t,x) + \alpha_m \int_0^t N(t,\tau,x) d\tau = \frac{f(x,t)}{\rho_0}. \quad (15)$$

Solve this equation with respect to the pressure increment function $P(t,x)$:

$$P(t,x) = \frac{1}{\sqrt[n]{\alpha_m E(0)}} \left\{ \frac{f(x,t)}{\rho_0} - \left[\alpha_\rho m_0 P_t'(t,x) + \alpha_m \int_0^t N(t,\tau,x) d\tau \right] \right\}^{1/n}. \quad (16)$$

Write it in the form:

$$P(t,x) = \sqrt[n]{\frac{f(x,t)}{\rho_0 \alpha_m E(0)}} \left\{ 1 - \frac{\rho_0}{f(x,t)} \left[\alpha_\rho m_0 P_t'(t,x) + \alpha_m \int_0^t N(t,\tau,x) d\tau \right] \right\}^{1/n}. \quad (17)$$

Taking into account smallness of the coefficients α_ρ and α_m and assuming that their smallness order are same, expand (17) in series in the vicinity of a unit and retain only the linear terms with respect to these coefficients and get

$$P(t,x) = \sqrt[n]{\frac{f(x,t)}{\rho_0 \alpha_m E(0)}} \left\{ 1 - \frac{\rho_0}{n f(x,t)} \left[\alpha_\rho m_0 P_t'(t,x) + \alpha_m \int_0^t N(t,\tau,x) d\tau \right] \right\}. \quad (18)$$

Accept the following denotation:

$$A(t,x,P_t') = \sqrt[n]{\frac{f(x,t)}{\rho_0 \alpha_m E(0)}} \left[1 - \frac{\rho_0 \alpha_\rho m_0}{n \cdot f(x,t)} P_t'(t,x) \right], \quad (19)$$

$$K(t,\tau,x,P,P_{xx}'') = \sqrt[n]{\frac{f(x,t)}{\rho_0 \alpha_m E(0)}} \cdot \frac{\rho_0 \alpha_m}{n \cdot f(x,t)} \int_0^t N(t,\tau,x) d\tau. \quad (20)$$

Within these denotation we can write (18) as the following nonlinear Volterra equation with respect to the pressure increment function:

$$P(t,x) = A(t,x,P_t') - \int_0^t K(t,\tau,x,P,P_{xx}'') d\tau. \quad (21)$$

Let's consider the numerical solution of equation (21). To that end, we partition the domain of definition of the function $P(t,x)$ on the squares by means of the following points:

$$t_i = ih \quad (i = 0,1,\dots,N), \qquad x_i = a + jh \quad (j = 0,1,\dots,M),$$

here, $0 < h$ is an integration step. For calculating the derivatives we use the following schemes:

$$P_t'(t_n,x) = \frac{P(t_n+h,x) - P(t_n-h,x)}{2h}, \quad P_{x^2}'(t,x_n) = \frac{P(t,x_n+h) - 2P(t,x_n) + P(t,x_n-h)}{h^2}.$$

Then, from (21) we get

$$P(t_n, x_m) \approx A(t_m, x_m, P(t_{n+1}, x_m), P(t_{n-1}, x_m)) -$$

$$-\int_0^{t_n} K(t_n, \tau, x_m, P(\tau, x_m), P(\tau, x_{m+1}), P(\tau, x_{m-1}))dt \quad (n = 0,1,\ldots,N;\ m = 0,1,\ldots,M). \quad (22)$$

Using the results from [6] and [7], we can write:

$$P(t_{n+1}, x_m) - P(t_n, x_m) = A(t_{n+1}, x_m, P(t_{n+2}, x_m), P(t_n, x_m)) - A(t_n, x_m, P(t_{n+1}, x_m), P(t_{n-1}, x_m)) -$$

$$-hK(\xi_{n+1}, \xi_{n+1}, x_m, P(\xi_{n+1}, x_m), P(\xi_{n+1}, x_{m+1}), P(\xi_{n+1}, x_{m-1})) -$$

$$-\int_{t_n}^{\xi_{n+1}} K(t_n, \tau, x_m, P(\tau, x_m), P(\tau, x_{m+1}), P(\tau, x_{m-1}))d\tau -$$

$$-\int_{\xi_{n+1}}^{t_{n+1}} K(t_{n+1}, \tau, x_m, P(\tau, x_m), P(\tau, x_{m+1}), P(\tau, x_{m-1}))d\tau,$$

where $t_n < \xi_{n+1} < t_{n+1}$. To the calculation of the integrals we apply the Euler implicit and explicit methods, respectively. As a result, we get a system of difference equations. After solving this system we found approximate values of the solution of equation (1). In one variant, the mentioned system of difference equations of the form:

$$P(t_{n+1}, x_m) - P(t_n, x_m) = A(t_{n+1}, x_m, P(t_{n+2}, x_n), P(t_n, x_m)) - A(t_n, x_m, P(t_{n+1}, x_m), P(t_{n-1}, x_m)) -$$

$$-h(K(t_{n+1}, t_{n+1}, x_m, P(t_{n+1}, x_m), P(t_{n+1}, x_{m+1}), P(t_{n+1}, x_{m-1})) +$$

$$+K(t_{n+1}, t_n, x_m, P(t_n, x_m), P(t_n, x_{m+1}), P(t_n, x_{m-1}))/2 \quad (n = 0,1,2,\ldots,N).$$

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