MULTIDIMENSIONAL INVERSE BOUNDARY VALUE PROBLEM FOR THE SYSTEM OF HYPERBOLIC EQUATIONS IN THE BOUNDED DOMAIN

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In the paper we investigate the classical solution of a multi-dimensional inverse boundary value problem for the system of the linear hyperbolic equations in the bounded domain. It is offered that unknown coefficients and right hand side of the equation depends only on the argument t.

Let's consider:

$$\frac{\partial U(x,t)}{\partial t^2} - AU(x,t) = a_1(t)b(x,t)U(x,t) + c_1(t)d(x,t)\frac{\partial V(x,t)}{\partial t} + f(t)F(x,t) \tag{1}$$

$$\frac{\partial V(x,t)}{\partial t^2} - AV(x,t) = a_2(t)\tilde{b}(x,t)\frac{\partial U(x,t)}{\partial t} + c_2(t)\tilde{d}(x,t)V(x,t) + f_2(t)G(x,t)$$

$$(x,t) \in D_T = \overline{\Omega} \times [0,T] \tag{2}$$

$$U(x,0) = \varphi(x), \quad \left.\frac{\partial U(x,t)}{\partial t}\right|_{t=0} = \psi(x), \qquad x \in \overline{\Omega} \tag{3}$$

$$V(x,0) = \tilde{\varphi}(x), \quad \left.\frac{\partial V(x,t)}{\partial t}\right|_{t=0} = \tilde{\psi}(x), \qquad x \in \overline{\Omega} \tag{4}$$

$$U(x,t)\big|_{\Gamma_T} = 0, \quad V(x,t)\big|_{\Gamma_T} = 0, \qquad \Gamma = S \times [0,T], \tag{5}$$

$$U(x^i,t) = h_i(t) \quad (i = 1,2,3), \quad t \in [0,T], \tag{6}$$

$$V(x^i,t) = g_i(t) \quad (i = 1,2,3), \quad t \in [0,T]. \tag{7}$$

where $0 < T < \infty$, Ω is an arbitrary bounded n - dimensional domain, S is the boundary of the domain Ω, Γ_T is the lateral surface of the cylinder $\overline{D}_T$, $x^i (i = \overline{1,3})$ are the different fixed points in Ω, and the operator A has the form:

$$AU(x,t) = \sum_{i,j=1}^{n} \frac{\partial}{\partial x_i}\left(a_{ij}(x)\frac{\partial U(x,t)}{\partial x_i}\right) - K(x)U(x,t) \tag{8}$$

and $a_{ij}(x) = a_{ji}(x)$, $K(x) \ge 0$ are measurable functions everywhere on $\overline{\Omega}$, bounded in Ω and $\sum_{i,j=1}^{n} a_{ij}\xi_i\xi_j \ge \mu \sum_{i=1}^{n} \xi_i^2$, $\mu = const > 0$, $\xi_i (i = \overline{1,n})$ are any real numbers.

Functions
$b(x,t), \tilde{b}(x,t), d(x,t), \tilde{d}(x,t), F(x,t), G(x,t), \varphi(x,t), \tilde{\varphi}(x,t), \psi(x,t), \tilde{\psi}(x,t), h_i(t)$,
$g_i(t)$ $(i = 1,2,3)$ are given functions, and $U(x,t), V(x,t), a_1(t), a_2(t), c_1(t), c_2(t), f_1(t), f_2(t)$ are unknown functions.

Definition of the functions $\{U(x,t), V(x,t), a_1(t), a_2(t), c_1(t), c_2(t), f_1(t), f_2(t)\}$ is called a classical solution of the problem (1) - (7) if the following conditions are satisfied:
1. Functions $U(x,t), V(x,t)$ are twice continuously differentiable on $\overline{D}_T$.

2. Functions $a_1(t), a_2(t), c_1(t), c_2(t)$, $f_1(t)$ and $f_2(t)$ continuous on $[0,T]$..
3. Conditions (1) - (7) are satisfied in usual sense.

Let's assume that, the functions $a_{ij}(x)$ $(i,j=\overline{1,n}), k(x), b(x,t), \tilde{b}(x,t), d(x,t), \tilde{d}(x,t)$, $F(x,t), G(x,t), \varphi(x,t), \tilde{\varphi}(x,t), \psi(x,t), \tilde{\psi}(x,t), h_i(t)$ and $g_i(t)$ $(i=\overline{1,3})$ satisfy the following conditions:

1. Functions a_{ij} $(i,j=\overline{1,n})$ $\left[\frac{n}{2}\right]+2$ times, and the function $k(x)\ge 0$ $\left[\frac{n}{2}\right]+1$ times continuously differentiable on $\overline{\Omega}$.
2. $S\in C^{\left[\frac{n}{2}\right]+2}$
3. Eigenfunctions $\mu_k(x)$ of the operator A under the boundary conditions $\mu_k(x)\big|_S=0$, $(k=1,2,...)$ $\left[\frac{n}{2}\right]+3$ times continuously differentiable on $\overline{\Omega}$
4. Functions $\varphi(x)\in W_2^{\left[\frac{n}{2}\right]+3}(\Omega), \varphi(x)\big|_S = A\varphi(x)\big|_S = ... = A^{\left[\frac{n}{4}\right]+1}\varphi(x)\big|_S = 0$,

$\tilde{\varphi}(x)\in W_2^{\left[\frac{n}{2}\right]+3}(\Omega), \tilde{\varphi}(x)\big|_S = A\tilde{\varphi}(x)\big|_S = ... = A^{\left[\frac{n}{4}\right]+1}\tilde{\varphi}(x)\big|_S = 0$,

$\psi(x)\in W_2^{\left[\frac{n}{2}\right]+3}(\Omega), \psi(x)\big|_S = A\psi(x)\big|_S = ... = A^{\left[\frac{n}{4}\right]+1}\psi(x)\big|_S = 0$,

$\tilde{\psi}(x)\in W_2^{\left[\frac{n}{2}\right]+3}(\Omega), \tilde{\psi}(x)\big|_S = A\tilde{\psi}(x)\big|_S = ... = A^{\left[\frac{n}{4}\right]+1}\tilde{\psi}(x)\big|_S = 0$.

5. Functions

$$\frac{\partial^i b(x,t)}{\partial x_1^{\alpha_1} x_2^{\alpha_2}...x_n^{\alpha_n}}, \frac{\partial^i \tilde{b}(x,t)}{\partial x_1^{\alpha_1} x_2^{\alpha_2}...x_n^{\alpha_n}}, \frac{\partial^i d(x,t)}{\partial x_1^{\alpha_1} x_2^{\alpha_2}...x_n^{\alpha_n}}, \frac{\partial^i \tilde{d}(x,t)}{\partial x_1^{\alpha_1} x_2^{\alpha_2}...x_n^{\alpha_n}}, \ \left(i=1,\left[\frac{n}{2}\right]+2\right)$$

belongs to the space $C(\overline{D})$ and $\frac{\partial^j b(x,t)}{\partial x_1^{\alpha_1} x_2^{\alpha_2}...x_n^{\alpha_n}}=0$, $\frac{\partial^j \tilde{b}(x,t)}{\partial x_1^{\alpha_1} x_2^{\alpha_2}...x_n^{\alpha_n}}=0$,

$$\frac{\partial^j d(x,t)}{\partial x_1^{\alpha_1} x_2^{\alpha_2}...x_n^{\alpha_n}}=0,\ \frac{\partial^j \tilde{d}(x,t)}{\partial x_1^{\alpha_1} x_2^{\alpha_2}...x_n^{\alpha_n}}=0,\ \left(t\in[0,T], x\in S;\ j=\overline{0,2\left[\frac{n+2}{2}\right]}\right)$$

6. Functions $F(x,t), G(x,t)$ belongs to the space $W_{x,t,2}^{\left[\frac{n}{2}\right]+2,0}(D_T)$, and

$F(x,t)\big|_{\Gamma_T} = AF(x)\big|_{\Gamma_T} = ... = A^{\left[\frac{n+2}{4}\right]}F(x)\big|_{\Gamma_T} = 0$,

$G(x,t)\big|_{\Gamma_T} = AG(x)\big|_{\Gamma_T} = ... = A^{\left[\frac{n+2}{4}\right]}G(x)\big|_{\Gamma_T} = 0$.

7. Functions $h_i(t)\ne 0$, $g_i(t)\ne 0$ $(i=\overline{1,3})$ are twice continuously differentiable on $[0,T]$, and $h_i(0)=\varphi(x^i), h'_i(0)=\psi(x^i), g_i(0)=\tilde{\varphi}(x^i), g'_i(0)=\tilde{\psi}(x^i)$ $(i=\overline{1,3})$.

8. $$\Delta(t)=\begin{vmatrix} b(x^1,t)h_1(t) & d(x^1,t)g'_1(t) & F(x^1,t) \\ b(x^2,t)h_2(t) & d(x^2,t)g'_2(t) & F(x^2,t) \\ b(x^3,t)h_3(t) & d(x^3,t)g'_3(t) & F(x^3,t) \end{vmatrix} \neq 0 \quad \forall t\in[0,T],$$

$$\Delta(t)=\begin{vmatrix} b(x^1,t)h'_1(t) & d(x^1,t)g_1(t) & G(x^1,t) \\ b(x^2,t)h'_2(t) & d(x^2,t)g_2(t) & G(x^2,t) \\ b(x^3,t)h'_3(t) & d(x^3,t)g_3(t) & G(x^3,t) \end{vmatrix} \neq 0 \quad \forall t\in[0,T].$$

under the conditions 1-8, by applying the Fourier method and considering conditions (6) and (7), we will reduce the solution of the problem (1) - (7) to the solution of the following system of integral equations:

$$U(x,t)=\sum_{k=1}^{\infty}\varphi_k \cos\lambda_k t\ \mu_k(x)+\sum_{k=1}^{\infty}\frac{\psi_k}{\lambda_k}\sin\lambda_k t\ \mu_k(x)+\sum_{k=1}^{\infty}\frac{1}{\lambda_k}\int_0^t\int_\Omega\Big[a_1(\tau)b(\zeta\tau)U(\zeta,\tau)$$
$$+c_2 d(\zeta,\tau)\frac{\partial V(\zeta,\tau)}{\partial\tau}+f(\tau)F(\zeta,\tau)\Big]\sin\lambda_k(t-\tau)\,\mu_k(\zeta)d\zeta d\tau\bullet\mu_k(x) \tag{9}$$

$$V(x,t)=\sum_{k=1}^{\infty}\tilde{\varphi}_k \cos\lambda_k t\,\mu_k(x)+\sum_{k=1}^{\infty}\frac{\tilde{\psi}_k}{\lambda_k}\sin\lambda_k t\,\mu_k(x)+\sum_{k=1}^{\infty}\frac{1}{\lambda_k}\int_0^t\int_\Omega\Big[a_2(\tau)\tilde{b}(\zeta\tau)\frac{\partial U(\zeta,\tau)}{\partial\tau}+$$
$$+c_2(\tau)\tilde{d}(\zeta,\tau)V(\zeta,\tau)+f_2(\tau)G(\zeta,\tau)\Big]\sin\lambda_k(t-\tau)\,\mu_k(\zeta)d\zeta d\tau\bullet\mu_k(x) \tag{10}$$

$$\begin{cases} a_1(t)=\dfrac{1}{\Delta(t)}\displaystyle\sum_{i=1}^{3}A_{i1}(t)\varphi_i(U,V,a_1,c_1,f_1;t) \\ c_1(t)=\dfrac{1}{\Delta(t)}\displaystyle\sum_{i=1}^{3}A_{i2}(t)\varphi_i(U,V,a_1,c_1,f_1;t) \\ f_1(t)=\dfrac{1}{\Delta(t)}\displaystyle\sum_{i=1}^{3}A_{i3}(t)\varphi_i(U,V,a_1,c_1,f_1;t) \end{cases} \tag{11}$$

$$\begin{cases} a_2(t)=\dfrac{1}{\tilde{\Delta}(t)}\displaystyle\sum_{i=1}^{3}\tilde{A}_{i1}(t)\tilde{\varphi}_i(U,V,a_1,c_1,f_1;t) \\ c_2(t)=\dfrac{1}{\tilde{\Delta}(t)}\displaystyle\sum_{i=1}^{3}\tilde{A}_{i2}(t)\tilde{\varphi}_i(U,V,a_1,c_1,f_1;t) \\ f_2(t)=\dfrac{1}{\tilde{\Delta}(t)}\displaystyle\sum_{i=1}^{3}\tilde{A}_{i3}(t)\tilde{\varphi}_i(U,V,a_1,c_1,f_1;t) \end{cases} \tag{12}$$

where $\varphi_i(U,V,a_1,c_1,f_1;t)=h_i''(t)+\sum_{k=1}^{\infty}\lambda_k^2\varphi_k\cos\lambda_k t\ \mu_k(x^i)+\sum_{k=1}^{\infty}\lambda_k\psi_k\sin\lambda_k t\ \mu_k(x^i)+$

$$+\sum_{k=1}^{\infty}\int_0^t\int_\Omega\lambda_k\left[a_1(\tau)b(\zeta,\tau)U(\zeta,\tau)+c_1(\tau)d(\zeta,\tau)\frac{\partial V(\zeta,\tau)}{\partial\tau}+f_1(\tau)F(\zeta,\tau)\right]\sin\lambda_k(t-\tau)\ \mu_k(\zeta)d\zeta d\tau\ \ \mu_k(x^i)$$

$$(i=\overline{1,3}),$$

$$\widetilde{\varphi}_i(U,V,a_2,c_2,f_2;t) = g_i''(t) + \sum_{k=1}^{\infty} \lambda_k{}^2 \widetilde{\varphi}_k \cos\lambda_k t \;\; \mu_k(x^i) + \sum_{k=1}^{\infty} \lambda_k \widetilde{\psi}_k \sin\lambda_k t \;\; \mu_k(x^i) +$$

$$\sum_{k=1}^{\infty} \int_0^t \int_\Omega \lambda_k \left[a_2(\tau)\widetilde{b}(\zeta\tau)\frac{\partial U(\zeta,\tau)}{\partial\tau} + c_2(\tau)\widetilde{d}(\zeta,\tau)V(\zeta,\tau) + f_2(\tau)G(\zeta,\tau)\right] \sin\lambda_k(t-\tau)\,\mu_k(\zeta)\,d\zeta d\tau \bullet \mu_k(x)$$

$$(i=\overline{1,3}),$$

$A_{ij}(t)$ is a cofactor of the element b_{ij} of the determinant $\Delta(t)$, and $\widetilde{A}_{ij}(t)$ is a cofactor of the element $\widetilde{b}_{ij}$ of the determinant $\widetilde{\Delta}(t)$. They have the following:

Theorem: *Let conditions 1-8 be satisfied. Then, for sufficiently small values of T problem (1) - (7) has a unique classical solution.*

References

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