COMBINED DECISION PRECISING FUZZY TECHNOLOGY FOR CREDIT RISK EVALUATION OF BANK INVESTMENTS

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The investment decision-making is influenced by the various uncertainty factors and, the need to formalize and process fuzzy, insufficient and, mainly, expert data. Therefore, ignoring the above mentioned factors results in inadequate and non-acceptable decisions. To examine the nature of risks related to financial decisions of banks, it is necessary to apply special mathematical methods. The methods could be based on fuzzy sets theory, for instance [1].

When a juridical person submits business-plan to the investment fund or the bank with the aim of receiving a credit, experts perform applicant's business analysis. In particular, they check up on certain factors that are essential to grant a credit. In fact, the experts commission selects a small set with minor credit risks from all submitted projects. Then, they perform additional evaluation to precise data within the selected set and take a final decision on granting a credit. Considering the procedure, the support decision making technology should involve two stages: a) choose the method of condensing the experts' knowledge, which is based on objective-expert data; b) choose the method, which works solely with expert data to support the decision.

The authors are well experienced in applying heuristic methods to the decision-making problems which are based on the objective and expert data [4],[5],[7]-[9]. By comparing various methods and evaluating their reliability, authors decided on two methods, which were subsequently applied to a problem of investment decision making.

To support the first stage, the Kaufmann - expertons method is used [2],[3]. At this stage the method selects projects with small or minor risks. The second stage makes more precise decisions with the method of possibilistic discrimination analysis. The latter was developed by the authors as a possibilistic generalization of well-known fuzzy discrimination analysis [6]. Possibilistic discrimination analysis is applied to a relatively small number of projects, selected at the first stage, to compare and sort out high-quality projects. As a result a new combined technology has been developed which makes it possible to identify investment projects with minimal risks and formulate levels of their crediting possibilities in the form of advice.

As the expertons method is well-known, here we will describe only a method of possibilistic discrimination analysis.

Let the set of all possible factors be $\Omega = \{w_1, w_2, ..., w_n\}$. The set of decisions $D = \{d_1, d_2, \ldots, d_m\}$ represents all competitors with minimum risks selected by the expertons method. It is possible to apply discrimination method provided that a tabular-numeric knowledge base $\{ f_{ij} \}$ could be build [7]. The classic version of the fuzzy discrimination analysis uses so called frequency tabular-numeric knowledge base, where f_{ij} designates the fraction of decisions d_i that were correct when w_j factor was present. Such knowledge base can be build, if databases of statistical information on successfully implemented investment projects exist. In decision making problems of investment projects, the values f_{ij} can only be received by the psychometric survey of the experts, since one can hardly hope for the availability of statistical information data bases. Then f_{ij} will designate the fraction of the experts who consider d_i being correct when w_j factor is present. So, if N experts participate in the psychometric survey, then $f_{ij} = N_{ij} / N$, where N_{ij} is the fraction of experts who supported decision d_i when for the d_i competitor w_j factor was present. Instead of the frequency tabular-numeric knowledge base it is advisable to build so called possibilistic tabular-numeric knowledge base $\{\pi_j^i\}$. For instance, by normalizing each row of $\{f_{ij}\}$ we receive

$$\pi_j^i = f_{ij} \Big/ \max_{j=\overline{1,n}} f_{ij} .$$

After creation of the possibilistic distribution table, the algorithm of a possibilistic variant of the discrimination analysis method can be described as follows:

1) Apply the well known transformation principle, the possibilistic distribution table is transformed to the probabilistic distribution table. For example, for $\forall d_i$ (i=1,2,…,m) let $\pi_{j1}^i \ge \pi_{j2}^i \ge \cdots \ge \pi_{jn}^i$, then the conditional probability f_j^i corresponding to the possibility π_j^i is expressed by the formula

$$f_{j_s}^i = \sum_{\ell=s}^{n} \left(\pi_{j_\ell}^i - \pi_{j_{\ell+1}}^i\right) \Big/ s \text{, where } s = 1, 2, \ldots, n \text{, } \pi_{j_{n+1}}^i \equiv 0 ; \tag{1}$$

2) Build positive and negative discriminations on $D \times \Omega$ and calculate the specific compatibility levels which determine to what extent the given factor influences (positive discrimination) and to what extent it does not influence (negative discrimination) the decision as compared to other factors:

$$p_{ij} = \frac{1}{n+1}\left\{1 + \sum_{k:\, f_k^i < f_j^i} \left(f_j^i - f_k^i\right)^{\alpha_1} + \frac{1}{1 + \sum_{k:\, f_k^i > f_j^i} \left(f_k^i - f_j^i\right)^{\alpha_2}}\right\}, n_{ij} = \frac{1}{n+1}\left\{1 + \sum_{k:\, f_k^i > f_j^i} \left(f_k^i - f_j^i\right)^{\alpha_1} + \frac{1}{1 + \sum_{k:\, f_k^i < f_j^i} \left(f_j^i - f_k^i\right)^{\alpha_2}}\right\},$$

$$\alpha_s > 0, \quad s = 1,2. \tag{2}$$

The positive discrimination p_{ij} is defined as the level of w_j factor influence on the decision d_i as compared with other factors. The negative discrimination n_{ij} shows the level to which the w_j factor does not influence the decision d_i as compared with other factors.

3) To decrease the informational entropy, build the following average positive and negative possibilistic discriminations on the set of solutions D:

$$\pi_i = \sum_{j=1}^{n} p_{ij} \Big/ n, \qquad v_i = \sum_{j=1}^{n} n_{ij} \Big/ n ; \tag{3}$$

4) Build possibilistic distribution $\forall i = 1,2,...,m$ on D:

$$\delta_i = \left(\pi_i^\beta + (1 - v_i)^\beta\right)/2, \quad \beta > 0 . \tag{4}$$

5) Regard the decision δ_{i_0} which has a maximum value on the possibilistic distribution $\{\delta_i\}$ as the most believable (convincing) decision:

$$\delta_{i_0} = \max_i \delta_i . \tag{5}$$

Let us consider an example of the application of the combined decision-making technology for the evaluation of investment projects.

Assume there are ten members ($i = \overline{1,10}$) of the investments fund commission, and the number of possible risk estimates for a given competitor, i.e. the number of possible decisions (crediting risks) equals to four (P_j, $j = \overline{1,4}$): P_1- crediting with an insignificant risk; P_2- crediting with a low risk; P_3- crediting with an average risk; P_4- crediting with a high risk. Experts provide confidence intervals which are included in the interval $[0,1]$: $[a_1, a_2] \subset [0,1]$, where a_1 is the pessimistic level of given risk and a_2 is the optimistic level of the risk. The aggregate table of experts' estimates may have the following form:

Experts i	Possible decisions P_i P_1	P_2	P_3	P_4
1	[0.2,0.3]	[0.3,0.4]	[0.6,0.7]	0.3
2	[0.5,0.6]	0	[0.5,0.6]	[0.2,0.4]
3	[0.1,0.7]	[0.1,0.2]	[0.8,0.9]	[0.1,0.2]
4	[0.3,0.4]	[0.1,0.3]	1	0
5	0.6	[0.1,0.2]	[0.6,0.8]	[0.5,0.6]
6	[0.8, 1]	0.4	[0.2,0.3]	0.3
7	[0.4,0.8]	[0.3,0.7]	[0,0.1]	[0.6,0.7]
8	[0.4,0.5]	[0.8,1]	1	0.4
9	[0,0.1]	0	[0.9,1]	[0.3,0.4]
10	[0.2,0.4]	0.5	[0.4,0.6]	[0.2,0.5]

Let us consider 11 α-cuts from 0 to 1, and for each of the possible decisions $P_i, i=\overline{1,4}$ calculate two statistics for each level: one for the lower boundary of an interval and, the other, for the upper boundary. By extending these statistics to the set of levels $\{0, 0.1, 0.2, \ldots, 0.9, 1\}$, we receive a table which is an experton.

An experton is then transformed as follows: a) an averaged experton is calculated by taking a mean arithmetic value of each interval boundaries; b) the averaged experton is reduced to a fuzzy set by calculating mean values; c) if necessary, a nonfuzzy set, the closest to the fuzzy one, is found.

As a result, on $\{P_1, P_2, P_3, P_4\}$ we obtain a possibilistic distribution of risk identification for a certain applicant: each P_i will be associated with the definite number established with the experts' common opinion taken into account. In a given example the possibilistic distribution of identified risks of the considered competitor look like:

P_1	P_2	P_3	P_4
0.495	0.377	0.682	0.395

To obtain a unique solution we use the principle of the maximum: $\delta(P_{i\,0}) = \max_i(P_i)$. In our case $\delta(P_{i\,0}) = P_3$. This means that in accordance to the common opinion of the experts the experton gives preference to the decision P_3, i.e. considered competitor has a low crediting risk.

Processing the information with the expertons method, allows for selecting only those competitors whose profile provides either insignificant- or, possibly, low - risk credit.

The next stage chooses from the number of the selected candidates by evaluating certain factors characteristic to these candidates.

Let us determine main $w_k, k=\overline{1,9}$ factors, by which all of the commission experts will score the candidate seeking the credit: w_1 - business profitability; w_2 - purpose of the credit; w_3 - pledge guaranteeing repayment of the credit; w_4 - credit amount; w_5 - payment of interest; w_6 - credit granting date; w_7 - credit repayment date; w_8 - monthly payment of a portion of the principal and accrued interest (repayment scheme); w_9 - per cent ratio of the pledge to the credit monetary amount.

Suppose the table of π_j^i possibilistic distribution is

D	Ω w_1	w_2	w_3	w_4	w_5	w_6	w_7	w_8	w_9
d_1	0.25	0.75	1.0	0.75	0.5	0.25	0.5	0.5	0.25
d_2	0.75	1.0	1.0	0.75	0.75	0.5	0.25	1.0	0.75
d_3	0.4	0.4	0.2	0.2	0.4	0.6	1.0	0.6	0.4
d_4	1.0	0.25	0.25	0.75	0.75	1.0	0.5	0.25	1.0

Firstly, we convert it to the table of conditional probabilistic distribution f_j^i (see formula (1)).

Further, to calculate the tables of positive and negative discriminations, we take the values $\alpha_1 = 0.3$, $\alpha_2 = 0.95$ (chosen empirically for the "spectral decomposition" of the values p_{ij} and n_{ij}) as the coefficients of α_s, $s = 1,2$. As a result, we receive the table of positive and negative discriminations (see formula (2))

We proceed with calculating π_i and ν_i representing the values of positive and negative discriminations for the i-th competitor (see formula (3)).Taking the coefficient value equal to $\beta = 0.85$ (chosen empirically for the "spectral decomposition" of the values δ_i), we determine the possibilistic distribution on $D = \{d_1, d_2, d_3, d_4\}$ (see formula (4)):

D	π_i	ν_i	δ_i
d_1	0.390223	0.27055	0.60709
d_2	0.404293	0.274846	0.612043
d_3	0.381682	0.262803	0.606352
d_4	0.404923	0.283352	0.608552

The final decision is $\delta_2 = \max_j \delta_j$ (see formula (5)), i.e. the investment project of the competitor d_2 receives the credit.

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