NECESSARY OPTIMALITY CONDITION IN THE CONTROL PROBLEM DESCRIBED BY THE SYSTEM OF VOLTERRA TYPE TWO-DIMENSIONAL INTEGRAL EQUATIONS

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Let the motion of the controlled object be described by the following system of Volterra type non-linear two- dimensional integral equations:

$$z(t,x)=\int_{t_0}^{t}\int_{x_0}^{x} f(t,x,\tau,s,z(\tau,s),u(\tau,s))dsd\tau,\ (t,x)\in D=T\times X=[t_0,t_1]\times[x_0,x_1] \tag{1}$$

Here $z(t,x)$ is an n- dimensional vector of phase variables, $u(t,x)$ is an r-dimensional piece-wise continuons vector of control actions with values from the given non-empty bounded set $U\subset R^r$, $f(t,x,\tau,s,z,u)$ is a given n- dimensional vector- function continuons in $D\times D\times R^n\times R^r$ together with $f_z(t,x,\tau,s,z,u)$.

Consider a problem on minimum of the terminal functional

$$S_0(u)=\varphi_0(z(t_1,x_1)) \tag{2}$$

involving functional restrictions of inequality type on the right end of trajectory,

$$S_i(u)=\varphi_i(z(t_1,x_1))\le 0,\qquad i=\overline{1,p} \tag{3}$$

It is assumed that $\varphi_i(z), i=\overline{0,p}$ satisfy the Lipschitz condition and have derivatives in any direction.

If the solution $z(t,x),(t,x)\in D$ of system (1) corresponding to the control $u(t,x)$ satisfies restrictions (3), such a control is said to be an admissible control.

In what follows, the problem on minimum of functional (2) under restrictions (1), (3) is called problem (1)-(3), the admissible control $u(t,x)$ being a solution of this problem an optimal control.

The present paper is devoted to obtaining the necessary optimality condition in the considered problem.

Let $(u(t,x),z(t,x))$ be a fixed admissible process.

Introduce the following denotation:

$$f_z[t,x,\tau,s]\equiv f_z(t,x,\tau,s,z(\tau,s),u(\tau,s)),$$

$$\Delta_v f[t,x,\tau,s]\equiv f(t,x,\tau,s,z(\tau,s),v)-f(t,x,\tau,s,z(\tau,s),u(\tau,s)),$$

$$l(m,\theta_j,\mu_j,v_j,l_j)=\sum_{j=1}^{m} l_j\left(\Delta_{v_j} f[t_1,x_1,\theta_j,\mu_j]+\int_{\theta_j}^{t_1}\int_{\mu_j}^{x_1} R(t_1,x_1,\tau,s)\Delta_{v_j} f[\tau,s,\theta_j,\mu_j]dsd\tau\right),$$

where m is an arbitrary natural number, $v_j\in U$, $l_j\ge 0, j=\overline{1,m}$ are arbitrary real numbers, $(\theta_j,\mu_j)\in[t_0,t_1)\times[x_0,x_1), j=\overline{1,m}$ are arbitrary continuity points of the control $u(t,x),(t,x)\in T\times X$, and $R(t,x,\tau,s)$ is an $(n\times n)$-matrix being a solution of the following Volterra type integral equation

$$R(t,x,\tau,s)=\int_{\tau}^{t}\int_{s}^{x} R(t,x,\xi,\eta)f_z[\xi,\eta,\tau,s]d\xi d\eta+f_z[t,x,\tau,s]$$

Assume

$$I(u) = \{i : \varphi_i(z(t_1, x_1)) = 0,\ i = \overline{1,p}\},$$
$$J(u) = \{0\} \cup I(u).$$

The following theorem is proved.

Theorem. For optimality of the admissible control $u(t,x)$ in problem (1)-(3) it is necessary that for any natural number m the inequality

$$\max_{i \in J(u)} \frac{\partial \varphi_i(z(t_1, x_1))}{\partial l(m, \theta_j, \mu_j, v_j, l_j)} \geq 0$$

be fulfilled for all $v_j \in U$, $l_j \geq 0$, , $(\theta_j, \mu_j) \in [t_0, t_1) \times [x_0, x_1)$, $j = \overline{1,m}$.

References

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