CONTROL PROBLEM FOR THE HYPERBOLIC EQUATIONS WITH PHASE RESTRICTION

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Let controlled process in $Q=(0,l)\times(0,T)$ be described by the hyperbolic equation

$$\frac{\partial^2 u(x,t)}{\partial t^2}-\frac{\partial^2 u(x,t)}{\partial x^2}= f\left(x,t,u(x,t),v(x,t)\right) \tag{1}$$

with initial and boundary conditions

$$u(x,0)=u_0(x),\ \frac{\partial u(x,0)}{\partial t}=u_1(x),\ x\in(0,l), \tag{2}$$

$$\frac{\partial u(0,t)}{\partial x}=0,\ \frac{\partial u(l,t)}{\partial x}=0,\ \ t\in(0,T), \tag{3}$$

where $u_0\in W_2^1(0,l),\ u_1\in L_2(0,l)$ - given functions, $f(x,t,u,v)$ - given Karateodori function, i.e., it is measurable on $(x,t)\in Q$ for all $(u,v)\in R\times[m_1,m_2]$, continuous on $(u,v)\in R\times[m_1,m_2]$ for almost all $(x,t)\in Q$, has the bounded derivative with respect to u for almost all $(x,t)\in Q$ and for all $(u,v)\in R\times[m_1,m_2]$, m_1,m_2 - given numbers.

As a class of admissible controls U_d is taken the set of measurable, bounded on Q functions $v(x,t)$ with values from the interval $[m_1,m_2]$.

For each admissible control $v(x,t)$ under the solution of the problem (1) - (3) is understood a function $u(x,t)\in W_2^1(Q)$ (the generalized solution). Note, that such solution is bounded on Q.

On the set U_d it is required to minimize the functional

$$\begin{aligned} J_\alpha(v)=&\int_0^T\left\{\beta_0[u(0,t)-f_0(t)]^2+\beta_1[u(l,t)-f_1(t)]^2\right\}dt+\\ &+\alpha\int_0^l\int_0^T[v(x,t)-w(x,t)]^2dxdt, \end{aligned} \tag{4}$$

with additional phase restriction

$$r_1\le u(x,t)\le r_2, \tag{5}$$

where $f_0(t),\ f_1(t)\in L_2(0,T), w(x,t)\in L_2(Q)$ - given functions, $\alpha\ge 0$, $\beta_0\ge 0$, $\beta_1\ge 0$, $\beta_0+\beta_1>0, r_1, r_2$ - given numbers.

By the help of penalty function the problem (1) - (5) is reduced to the following problem: to find a minimum functional

$$\tilde{J}(v)=J_\alpha(v)+P_k(v)$$

by restrictions (1) - (3), where

$$P_k(v) = A_k \int_0^l \int_0^T \left[\varphi^1(u) + \varphi^2(u)\right] dxdt,$$

$A_k > 0$ are such numbers, that $\lim_{k\to\infty} A_k = \infty$, $\varphi^1(u) \equiv [\max(r_1 - u(x,t);0)]^2$, $\varphi^2(u) \equiv [\max(u(x,t) - r_2;0)]^2$.

In the work the conjugate problem is introduced:

$$\frac{\partial^2 \psi(x,t)}{\partial t^2} - \frac{\partial^2 \psi(x,t)}{\partial x^2} - \frac{\partial f(x,t,u,v)}{\partial u}\psi(x,t) = A_k \left[\varphi_u^1(u) + \varphi_u^2(u)\right], \ (x,t) \in Q, \quad (6)$$

$$\psi(x,T) = 0, \frac{\partial \psi(x,T)}{\partial t} = 0, \ x \in (0,l), \quad (7)$$

$$\frac{\partial \psi(0,t)}{\partial x} = 2\beta_0 [u(0,t) - f_0(t)], \ \frac{\partial \psi(l,t)}{\partial x} = -2\beta_1 [u(l,t) - f_1(t)], \ t \in (0,T). \quad (8)$$

Using the problem (6)-(8) and assuming that $f(x,t,u,v)$ has a derivative with respect to v, that belongs to $L_\infty(Q)$, is proved, that the functional $\tilde{J}(v)$ is differentiable on v and

$$\frac{\partial \tilde{J}(v)}{\partial v} = -\frac{\partial H(x,t,u,v,\psi)}{\partial v},$$

where

$$H(x,t,u,\psi,v) = \psi f(x,t,u,v) - \alpha \|v - w\|_{L_2(Q)}^2 .$$