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MULTIPAGE PHASE STEREO A PORTRAIT OF THREE-DIMENSIONAL DYNAMIC SYSTEM AND VISUALISATION OF PROCESS OF THE CONTROL BY MOVEMENT

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Modern achievements in the field of the nonlinear dynamics, based on a method of computing experiment, and modern computer technologies of creation of animation effects, transform into a reality strategy predicative control strategy by physical objects in real time. On a way of effective introduction of this strategy to practice except a construction problem 3D phase portraits of dynamic systems there is variety of unresolved questions in which number there is a problem of creation of visual representations about all features of three-dimensional families of trajectories of operated objects on the "two-dimensional" screen of visual supervision. Creation of effects of animation, for example, realization of periodic purposeful turns of the image, serial switching 2D projections of images, etc. can serve success in the decision of the given problem.

Working out of strategy of management with use of phase portraits is complicated as well by that there is a quantitative and qualitative influence of a vector of management on them. So the problem of dimension which at all high possibilities of productivity of modern computers demands special algorithms for the decision is born.

Let's assume the dynamic system is set

$$\frac{d\boldsymbol{x}}{dt} = \boldsymbol{f}(\boldsymbol{x},\boldsymbol{u}); \quad \boldsymbol{x} = \{x_1, x_2, x_3\}; \quad \boldsymbol{u} = \{u_1, u_2\}; \quad \boldsymbol{f} = \{f_1, f_2, f_3\}; t > 0 , \tag{1}$$

Nonlinear, having in the considered limited area of phase conditions G three points of balance $\boldsymbol{x}_j^* = \{x_{j1}^*, x_{j2}^*, x_{j3}^*\}$; $j = \overline{1,3}$ defined by conditions $f_i(\boldsymbol{x}_j^*, \boldsymbol{u}) = 0; i = \overline{1,3}$. Among points of balance two points $\boldsymbol{x}_1^*, \boldsymbol{x}_2^*$ are steady. We assume, that a vector of control $u_k(t); k = \overline{1,2}$, the partially constant functions limited to intervals $u_{k\min} \le u_k \le u_{k\max}$, have influences not only on co-ordinates of points of balance, but also can qualitatively change structure of families of phase trajectories. In particular, there can be a merge of two equilibrium positions in one and be born бифуркация decisions of dynamic system. It is considered, that area G can be divided only on two separate sub areas G_1 and G_2, including accordingly x_1^*, x_2^* as the centers of gravity of the trajectories, all time remaining in them while management is invariable, i.e. $u_k^n(t) = const; k = \overline{1,2}$. Under the same condition unstable position of balance x_3^* remains a limiting point for both subsets.

Definition 1. A phase stereo a portrait of dynamic system we will name flat display to the panel supervision (monitor) of family of the three-dimensional phase trajectories constructed in additional system of co-ordinates with the beginning which located in a point of balance of system (1) and which makes turn round one of the axes on one of below-mentioned laws:

$$\varphi(t) = \alpha_0 t + \alpha_{\max} \sin\frac{t}{T}; \; \varphi(t) = \alpha_0 t + \alpha_{\max} \dim\left(\frac{t}{T}\right), \tag{2}$$

Where $\alpha_0, \alpha_{\max}$ – regular and harmonious components of angular speeds of rotation of co-ordinate axes, and designation dim shows function of allocation of a fractional part of the private.

Definition 2. A stereo the image of area of kept conditions (AKC) we will name one of closed surfaces S_1, or S_2, allocating according to area G_1 or G_2-vector fields with the image of current condition $\boldsymbol{x}(t)$ (a red point) which is presented also in rotating on one of laws (2) system of co-ordinates.

Definition 3. *A multisheet phase stereo a portrait*, (*a multisheet stereo image AKC)* we will name sets of phase portraits of a stereo (set of images of stereo AKC) which each element corresponds to one vector of control $\boldsymbol{u} \in U$.

Biunique conformity of each phase portrait to some vector of management is fair only within certain topological quality of area of decisions of system (1). Therefore, it is necessary to demand reparability, general set U on convex subsets $U^{\lambda} \subset U; \lambda = 1,2,...,\Lambda$ of different topological qualities, i.e. on subsets in which limits the system is characterized qualitatively one type by phase portraits.

Let rectangles are set:

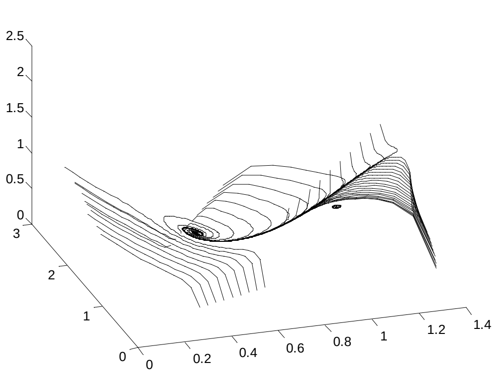
$$u_{k\min}^{\lambda} < u_k^{\lambda} < u_{k\max}^{\lambda}; k = 1,2; \lambda = \overline{1,\Lambda}, \tag{3}$$

In which inequalities $b_{\lambda} < \psi(\boldsymbol{u}) < b_{\lambda+1}$; $\lambda = \overline{1,\Lambda}$ allocating Λ numbers convex areas of a vector of management at which the system (1) gives rise to various topological qualities of space of decisions are defined:

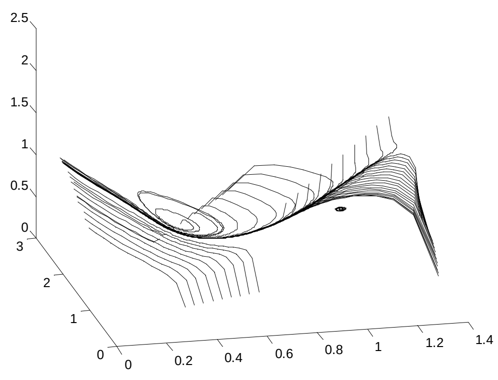
We name character of movement of system conditionally free if motive forces are initial deviation $\boldsymbol{x}(t_0)$ of a representing point from centre of gravity $\boldsymbol{x}_1^*$, and casual fluctuations from external impulses, times which actions are scornfully small in comparison with characteristic time of system (1), and the average period of repetition makes considerable size. Performance of these conditions provides scornfully small dependence of phase portraits on influences of external impulses. Drift of a representing point from the centre of gravity can lead $\varepsilon -$ to affinity to surface S_1. This situation we will consider as occurrence in a zone of dangerous border of loss of keeping area G_1.

On fig. 1. Topological qualities for the decision of following differential equations are presented:

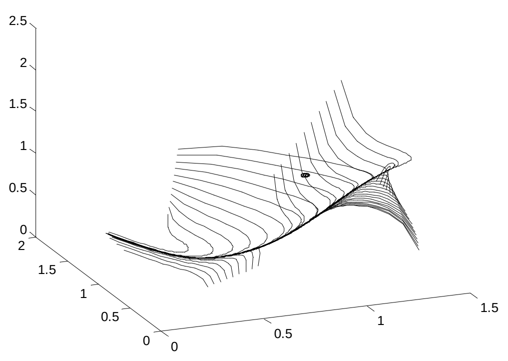
$$\begin{aligned}
&\frac{dx_1}{dt} = 0.6(x_3 - x_1) - 2.1w;\\
&\frac{dx_2}{dt} = c_1 - c_2; \quad c_1 = 0.11w + 0.16u_1; \quad c_2 = (0.23 + u_2u_1)x_2 + 0.37\\
&\frac{dx_3}{dt} = 10u_1(x_1 - x_3) + 100u_1^2(th(2.4x_2 - 2.4) + 2); \quad w = x_1\exp\left(-0.3\frac{1-x_2}{x_2}\right);
\end{aligned} \tag{4}$$



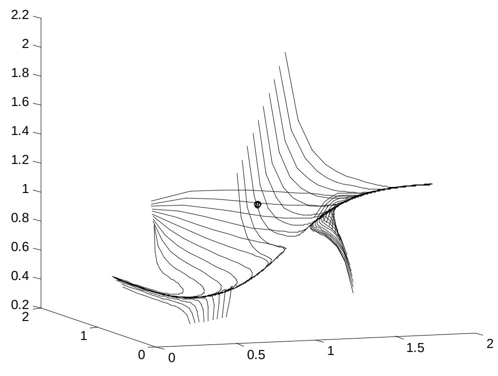
a)



b)



c)



d)

Fig. 1. Phase portraits with various topological qualities:
a)With a stationary point of type unstable "focus"; b)With a stationary point of type steady "focus"; c)Loss of one stationary point; d) the Phase portrait with three stationary conditions.

The closed surfaces allocating AKC, it is convenient to approximate in spherical system of co-ordinates.

The spherical system of co-ordinates with the beginning coinciding with one of steady stationary conditions is entered:

$$x_1 = x^*_{j1} + \rho\sin\theta\cos\varphi;$$
$$x_2 = x^*_{j2} + \rho\sin\theta\sin\varphi; \qquad (5)$$
$$x_3 = x^*_{j3} + \rho\cos\theta; \quad j = 1,\text{or}\,2$$

Where $x^*_{ji}; i = \overline{1,3} -$ co-ordinates of the Cartesian system of one of two steady points of balance; $\theta, \varphi, \rho -$ сферические co-ordinates of any points of space concerning the chosen point of balance; $x_1, x_2, x_3 -$ co-ordinates of system of Descartes in conformity with system (1).

Fixing in (4) variables θ, φ, i.e. building a grid on central values $\theta = \{0, h_1, ..., m \cdot h_1, ..., 2\pi\}, h_1 = \dfrac{2\pi}{M}; \varphi = \{0, h_2, ..., n \cdot h_2, ..., 2\pi\}, h_2 = \dfrac{2\pi}{N}$, in areas, it is possible to carry out approximation of sets of every possible trajectories or every possible dividing surfaces on the remained two other variables, x^*_{ji}, ρ. Considering, that the last are control functions, it is possible to write:

$$x_1(\boldsymbol{u}, m, n) = x^*_{j1}(\boldsymbol{u}) + \rho(\boldsymbol{u}, m, n);$$
$$x_2(\boldsymbol{u}, m, n) = x^*_{j2}(\boldsymbol{u}) + \rho(\boldsymbol{u}, m, n);$$
$$x_3(\boldsymbol{u}, m, n) = x^*_{j3}(\boldsymbol{u}) + \rho(\boldsymbol{u}, m, n); \qquad (6)$$
$$j = 1,\text{or}\,2;\ m = \overline{1, M};\ n = \overline{1, N}.$$

Functions $x^*_{ji}(\boldsymbol{u})$ are easy for receiving on the equations of stationary decisions which represent two-parametrical diagram's bifurcation. Discrete representation of area of admissible managements

$$u_i \mapsto \left\{ u_{i\min} + k_i \cdot \delta_i ; \delta_i = \frac{u_{i\max} - u_{i\min}}{K_i} ; k_i = 0,1,...,K_i - 1; i = 1,2 \right\}; \quad (7)$$

Creates $K = K_1 \times K_2$ numbers the fixed vectors $\{u_k ; k = \overline{1,K}\}$, i.e. K number of sheets in a multisided phase stereo portrait of system.

Definition $\rho(\boldsymbol{u},m,n)$ should be carried out by results of computing experiment on model (1) on construction of dividing surfaces. And, it agree K to number of the sheets approximating area $\boldsymbol{u} \in U$, vectors $\rho_{\boldsymbol{k}} ; k = \overline{1,K}; \dim \rho_{\boldsymbol{k}} = n \times m$ should be defined.

Digitization of numerical set $(\theta,\varphi) \in \mathbf{R}^2$ in the form of a uniform grid with $n \times m$ numbers central points, i.e. creation of a template of vectors of the fixed directions, requires working out of algorithms of their generation.

The algorithm of generation of set of vectors, secants a sphere surface in set W number equidistant points, is formulated in a following kind:

$$m = \overline{1,L};$$

$$\theta(m,n) = \frac{2\pi}{L \cdot \sin\frac{\pi}{L}} \frac{n}{m} ; n = \overline{1, \ floor\left(L \sin\frac{\pi}{L} m \right)}; \quad (8)$$

$$\varphi(m) = \frac{\pi}{L} m;$$

Where $\theta(m,n), \varphi(m) -$ accordingly discrete spherical co-ordinates; $m,n -$ numbers of steps of increments of angular co-ordinates; $L -$целое the number, defining "subtlety" of a grid, i.e. - size$W$; $floor(.) -$ the function returning a smaller nearest integer. In particular, have conformity places: - at $L = 20, W = 120;$ $L = 30, W = 268;$ $L = 40, W = 486;$ $L = 50, W = 766,$, etc.

On fig. 2 the net covering (template) on a sphere surface is resulted at $L = 50, W = 766$.

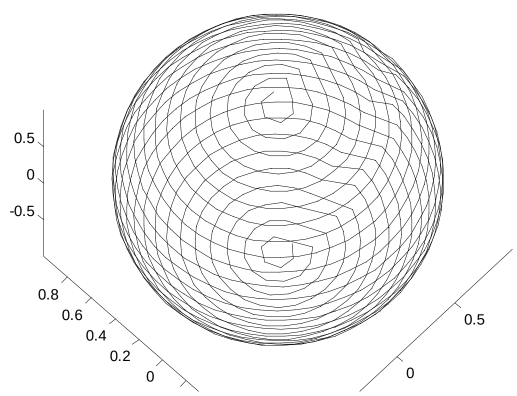


Fig. 2. A template for approximation of areas of an attraction in spherical system of co-ordinates.

In software products on visualization of calculations and the computer drawing of leading firms effective algorithms of construction of coverings of objects of not geometrical forms in which basis the method of a triangulation Delon lays are used [and]. In this case the approximation, using a template, considerably simplifies as well a triangulation problem. It is simple to build in regular intervals distributed triangular grid with the set quantity of knots on a surface of sphere of individual radius, i.e. to define corresponding discrete spherical co-ordinates $\theta(m), \varphi(m,n)$ in knots with numbers m,n. Further considering the measured length of polar radius, are easily recalculated $x_i, i = \overline{1,3}$ image co-ordinates.

Thus, the given strategy of "storing" 3D phase portraits and-or surfaces of division ОУС differs high profitability and in the speed of reproduction. In the course of the control in real time there is only a sheet disclosing in infinitely page FP which corresponds to vector $\boldsymbol{u}$. This conformity can be set not only in the form of the table, and also can be defined by functional dependence:

$$\begin{aligned} &\Psi_{ji}(x_{ji}, \boldsymbol{u}) = 0;\ j = \overline{1,3};\ i = \overline{1,3}; \\ &\Xi_k(\rho_k, \boldsymbol{u}) = 0; k = \overline{1,K}\ , \end{aligned} \tag{9}$$

Approximating tabular data. We will note, in this case FP for dynamic system (1) to name not much page, and infinitely page phase portrait more precisely.