The Third International Conference "Problems of Cybernetics and Informatics"
September 6-8, 2010, Baku, Azerbaijan. Section #5 "Control and Optimization"
www.pci2010.science.az/5/15.pdf

CONTROL OF INITIAL STATE OF DYNAMIC SYSTEMS BY MEANS OF SIGNAL INPUT

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Dynamics of controlled systems, including their free motion is investigated under non-zero initial conditions. It there is no access to inner structure of the object, to establish necessary initial conditions or "discharge", it may be performed by means of a signal input.

Equivalence of the initial state of dynamic system of Dirac's input δ(t) – function is the base of the given method.

Without loss of generality, consider a linear stationary object given by the state model:

$$dx/dt = Ax + Bu, \quad x(0) = x_0 = 0\,, \tag{1}$$

where $x = (x_1, x_2, ..., x_n)^T$ is a state vector; $u = (u_1, u_2, ..., u_m)^T$ is a control (signal input); A, B are constants of nxn and nxm matrices.

It is required to establish the initial condition of system (1) on $x(0) = \tilde{x}_0 \neq 0$. Let *m=n.* Then we can write

$$u = u_1 + B^{-1}\tilde{x}_0\delta(t) \quad , \tag{2}$$

where δ(t) is a scalar unit impulse.

In addition, the object equation:

$$dx/dt = Ax + B\left[u_1 + B^{-1}\tilde{x}_0\delta(t)\right], \quad x_0 = 0 \quad . \tag{3}$$

Taking into account the property of *L*[δ(t)]=1, under zero initial conditions the image of the state vector is in of the form:

$$X(s) = (sI - A)^{-1}\left[Bu_1(s) + \tilde{x}_0\right].$$

The appropriate pre-image:

$$x(t) = e^{At}\left[\tilde{x}_0 + \int_0^t Bu_1(\tau)d\tau\right] = e^{At}\tilde{x}_0 + \int_0^t e^{A(t-\tau)}Bu_1(\tau)d\tau\,. \tag{4}$$

Here $e^{At} = L^{-1}\left[(sI - A)^{-1}\right]$ is a transition matrix. Expression (4) is the known solution of linear system (1) under initial condition $\tilde{x}_0$.

For *m<n*, it is impossible to affect on all the initial states x_{i0}, $i = \overline{1,n}$ In this case, the problem has no complete solution. So, under scalar input *m*=1 the initial condition may be changed only of one variable $x_j(t)$, *j=1,2,...,n.*

The simulation scheme of equation (3) is shown in fig. 1.

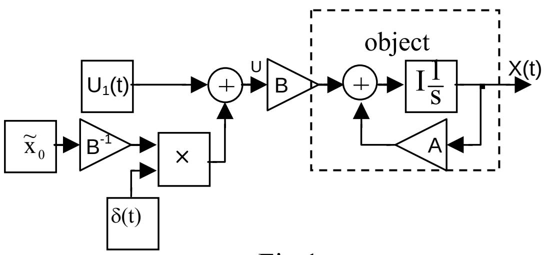


Fig.1

The unit impulse may be realized by the expression:

$$\delta(t) \approx h\left[1(t) - 1(t - \Delta)\right], \quad h = 1/\Delta, \quad \Delta = 10^{-(2\div5)}\,.$$

Here 1(t), 1(t-Δ) are the step functions shifted by the quantity *Δ*.

The case m=1 is oftenly met when reducing the model "input-input"

$$y^{(n)} = f\left(y, y', \ldots, y^{(n-1)}, u\right)$$

to the state model: In the linear case:

$$dx/dt = Ax + b\mathrm{u},$$

$$y = x_1.$$

Here,

$$A = \begin{bmatrix} 0 & 1 & 0 & \ldots \\ 0 & 0 & 1 & \ldots \\ \ldots & \ldots & \ldots & \ldots \\ -a_n & -a_{n-1} & \ldots & -a_1 \end{bmatrix}, \quad b = \begin{pmatrix} 0 \\ 0 \\ \ldots \\ \mathrm{b} \end{pmatrix}.$$

And we can affect only on the initial condition of the variable $x_n(t) = y^{(n-1)}(t)$. Expression (2) takes the form:

$$u = u_1 + \mathrm{b}^{-1}\tilde{x}_{n0}\delta(t).$$

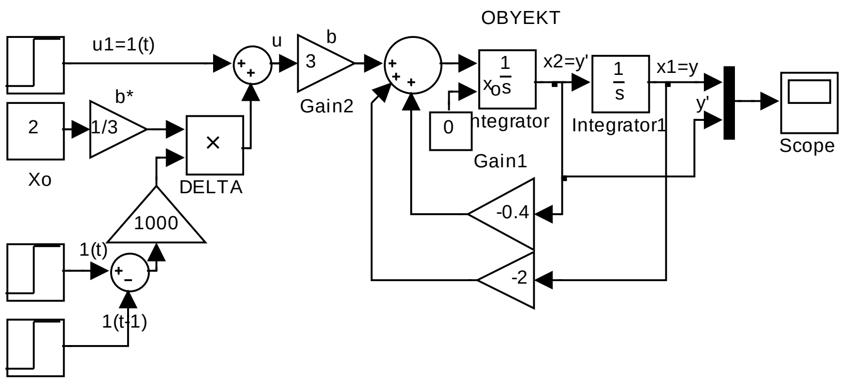
Example 1. The scalar case $m = 1$. The object equation is given in the form "input-input"

$$\ddot{y} + 0{,}6\dot{y} + 2y = 3u.$$

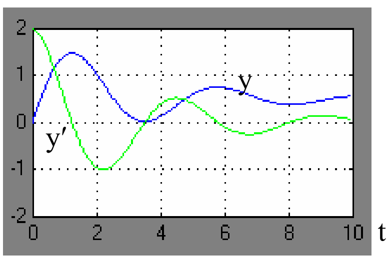
Introducing the new variables $x_1 = y,\ x_2 = \dot{y}$ we can write:

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -2 & -0{,}4 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} 0 \\ 3 \end{pmatrix} u.$$

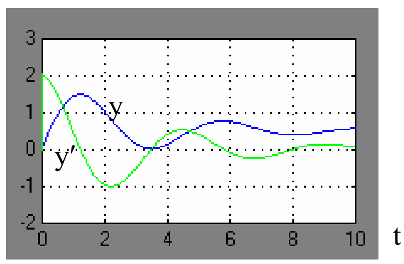
Expression (2): $u = u_1 + (1/3)\tilde{x}_{n0}\delta(t)$. Let the control signal u_1=1(t), the required initial condition $x_{20} = 2$. The simulation scheme in the pack SIMULINK is shown in figure 2.



a)



b)

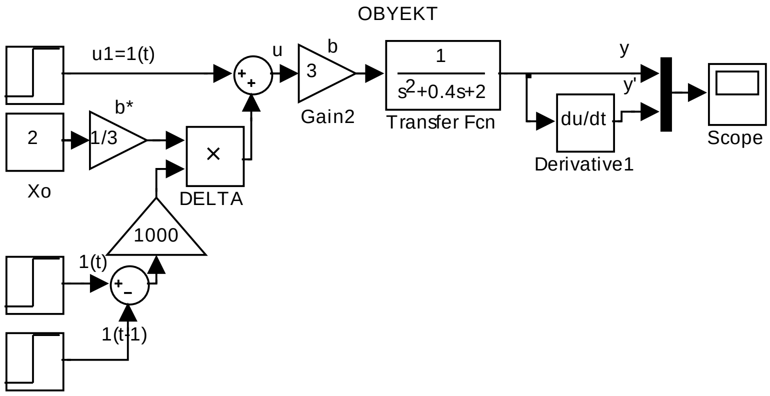


v)

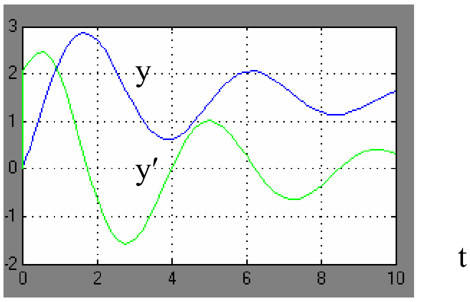
Fig.2

As it is seen from fig. 2.b, the trajectory $x_2(t) = \dot{y}(t)$ begins from the required initial state $y'(0) = 2$. In fig. 2.v, the value $x_2(0)=2$ is realized by changing the initial condition of the first integrator for $\delta(t) = 0$. As it is seen, in the both cases the same transients $x_1(t) = y(t)$ and $x_2(t) = \dot{y}(t)$ are obtained.

When the object is given by the transfer function, the simulation scheme is shown in fig. 3. In this case, the initial conditions are zero $y(0) = \dot{y}(0) = 0$ and it is impossible to change them by meddling in internal structure of the object.



a)



b)

Fig.3

As it is seen from fig. 3,b, the process in $\dot{y}(t)$ begins from the required initial state $\dot{y}(0) = 2$.

Example 2. Now, let's consider the vector case $m = n = 2$. The object equation is given in the form of the state model:

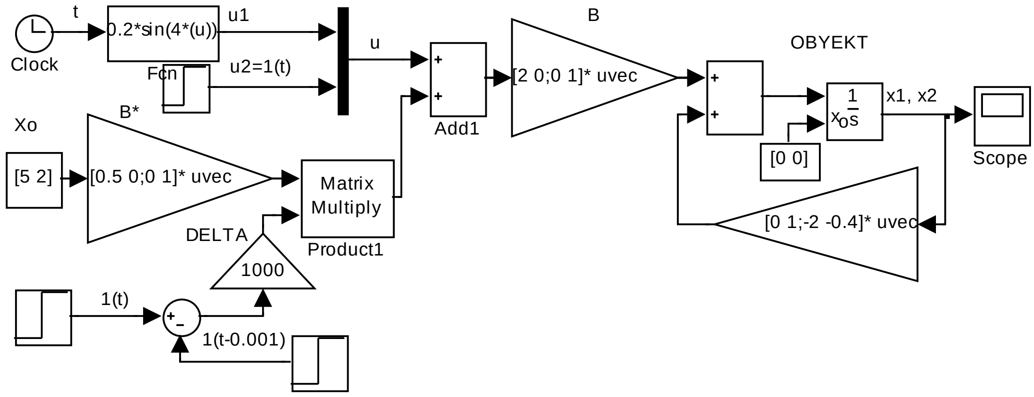
$$\dot{x}_1 = x_2 + 2u_1,$$
$$\dot{x}_2 = -2x_1 - 0{,}4x_2 + u_2.$$

Here,

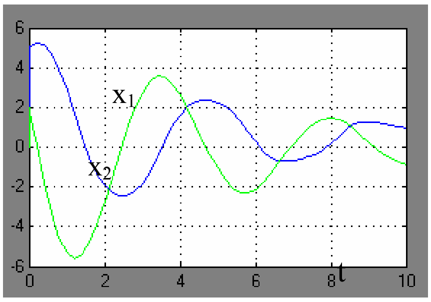
$$A = \begin{bmatrix} 0 & 1 \\ -2 & -0{,}4 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}, \quad B^{-1} = \begin{bmatrix} 0{,}5 & 0 \\ 0 & 1 \end{bmatrix}$$

Let $u_1 = 1 + 0{,}2sin(4t)$, $u_1 = 1(t)$. The required initial state $\tilde{x}_0 = (5{,}2)^T$.

The scheme of vector realization of the object's model is represented in fig. 4.



a)



b)
Fig.4

As it is seen from fig.4,b the processes x_1(t) and x_2(t) begin from the required initial state $x_1(0)=5$ and $x_2(0)=2$. The similar results are obtained for $\delta(t)=0$ and initial condition of the integrator $x_0=[5\ 2]$.

The considered case may be used for pre-excitation of internal state of the elements, computing techniques and control systems. And also to realize "discharge" of the current state.

The solution of model problems in MATLAB/SIMULINK showed high reliability of theoretical aspects and allowed to make some positive conclusions of high applied value.