STOCHASTIC SIMULATION OF QUEUES DESCRIBED BY BEHAVIOR OF MOVING PARTICLES

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Mathematical model. We consider the motion (without overtaking) of S transport units on a circle divided into N equal parts. The mathematical models of these systems have been considered in [1] and later on [2,3,4] and it was derived that separately considered particle makes binomial random walk. In [5] that fact was used for describing of traffic jam, wich can be happened in transport systems and for calculation the road capacity.

In this paper the following mathematical model is considered. There are n stations and S transport units, which sequentially pass stations. At each station stationary Poisson flow of customers arrives to service. It is supposed transport unit has infinite capacity, i.e. if transport unit passes some station then all customers, which are waiting for service immediately will get service. It is assumed that the distance between the stations and the velocities of the transport units are the same. In the capacity of efficiency index we take customer average waiting time before service.

The aim of this paper to calculate different efficiency indexes for various systems. Such mathematical models have complicated structure and their analytical research faces with some troubles. Hence, we'll use simulation of such systems, which allows to calculate (numerically) different characteristics and later on to make statistical analysis of these data.

Simulation. We consider the motion of S transport units without overtaking on a circle divided into N equal parts. We'll fix some point A on the circle and calculate customer waiting time before service at this point. The customer waiting time before service is defined as a time interval from customer arrival into the system until the instant when any particle will pass the point. For calculation of customer waiting time before service we'll calculate total waiting time before service of all customers and divide to the number of the customers.

For simulation, first of all, we have to generate customer arriving instant into the system. We assume that Poisson flow of customers with intensity λ arrives to service and denote $t_1, t_2, ..., t_i$ sequential customer arriving instants into the system. Then $t_1 = -\frac{1}{\lambda}\ln(1-\alpha_1)$, $t_2 = t_1 - \frac{1}{\lambda}\ln(1-\alpha_2)$, where ($\alpha_i$) is a pseudo-random number, having standard normal distribution.

Denote the total waiting time by *W*, the number of requests by B and determine the expression of the mean waiting time as $w = \frac{W}{B}$

The results of simulation. As a result of simulation for customer average waiting time before service we get the following graph.

As it is seen from Figure 1, customer average waiting time before service tends to some constant when the number of particles moving on a circle increases.

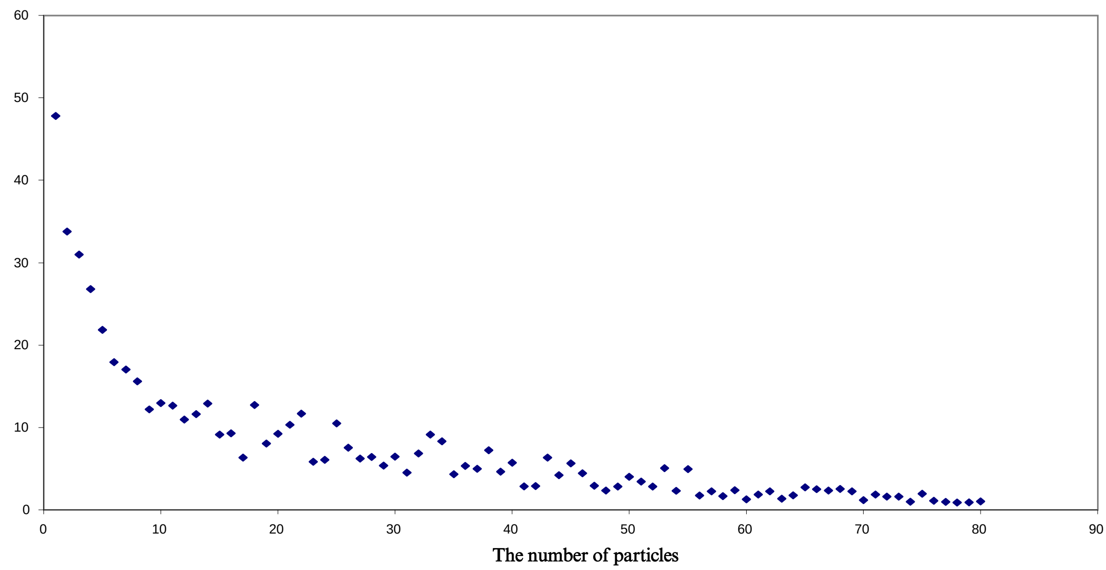

Fig.1

We have also investigated the changing of average waiting time depending on the λ parameter. The graphs are constructed for variable values of λ parameter $\lambda = 0.5$, $\lambda = 1$, $\lambda = 1.5$ (Figure 2)

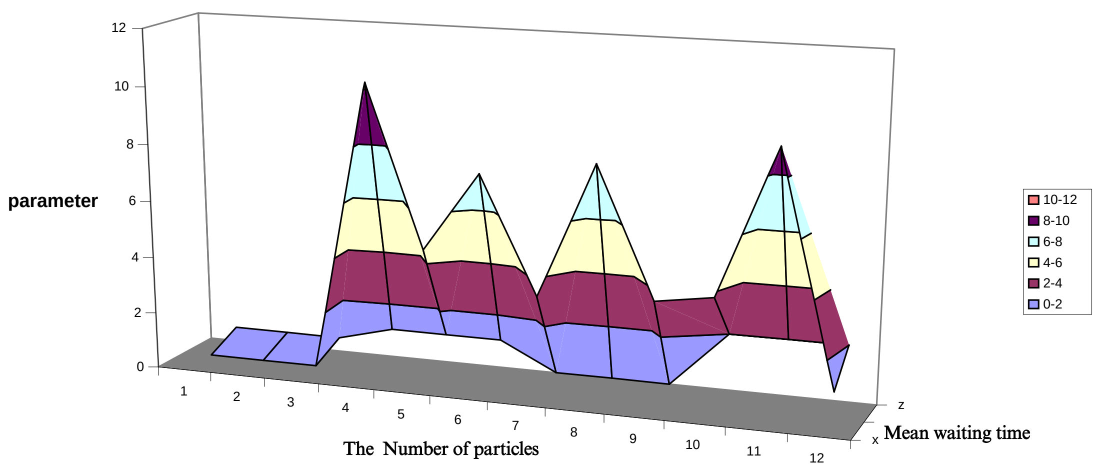

Fig.2

The changing of average waiting time for various parameters are shown here.

Customer average waiting time for various values of S is shown in Table 1.

Table 1

Number of the particles	1	2	3
Parametr (λ)	0,5	0,5	0,5
Mean waiting time (W)	20,59644	11,38839	7,68931
Parametr (λ)	1	1	1
Mean waiting time (W)	20,37312	11,27111	7,697607
Parametr (λ)	1,5	1,5	1,5
Mean waiting time (W)	20,268025	10,76061	7,590578
Parametr (λ)	2	2	2
Mean waiting time (W)	20,56960	11,05717	7,700163

The program for simulation has been prepared in BORLAND PASCAL programming language.

Remark. When the number of particles increases, to some level, the mean waiting time decreases.

We should also notice that when the number of the particles will pass that level, the mean waiting time will increase and after some time there may happen a plug in the system.

References

1. Belyayev Yu. K . On simplified model of motion without overtaking. Izv. AN SSSR, Tekhnicheskaya kibernetika, №3, 1969, pp. 17-21.
2. Zahle U. Generalization of motion of model without overtaking. Izv. AN SSSR, Tekhnicheskaya kibernetika, №5, 1972, pp. 100-103.
3. Hajiyev A. H. On a random walk of particles on a ring. Matem. zametki, №6, Vol. 47, 1990, pp. 140-143.
4. A.Hajiyev,H.Jafarova, T,Mammadov. Mathematical models of moving particles without overtaking. RUSSION acad.Sci.,Doklady,Mathem, 2010, v. 81, №3, pp. 1-4.
5. Hasratova M.H. Modeling of moving particles on a closed contour without overtaking// Proceedings of IMM of NAS of Azerbaijan, v. XXVIII (XXXVI), 2008, pp. 189-194.