THE MODELS AND FORECASTING OF THE INCOMPLETE AFTER "DISORDER" FINANCIAL MARKETS

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The models of financial markets with disorder we have investigated in [1]-[3]. Here we represent one special model with disorder as the evolution of stock price process and find optimal in mean square sense forecasting estimation.

Let's consider a real valued stochastic process $S=(S_n)_{0\le n\le N}$ on a filtered probability space (Ω, F, F_n, P) and adopted to the filtration $(F_n)_{0\le n\le N}$ such that

$$S_n = S_0 e^{H_n},\ \ S_0>0, \tag{1}$$

where S_0 is deterministic and $H_n=\rho_1+\rho_2+...+\rho_n$ with

$$\rho_n=\rho_n^{(1)} I(n<\theta)+\rho_n^{(2)} I(n\ge\theta),\ \ n=1,2,...,N.$$

$I(A)$ is the indicator of an event $A\in F$;

$$\rho^{(i)}=(\rho_n^{(i)})_{1\le n\le N},\ i=1,2,$$

are adopted to the filtration F sequances of independent identically distributed random variables; $\rho^{(1)}$ takes the values a_1 and a_2 with probabilities $p_1^{(1)}=P(\rho_1^{(1)}=a_1),\ p_2^{(1)}=P(\rho_1^{(1)}=a_2)=1-p_1^{(1)}$ and $\rho^{(2)}$ takes the values $a_1,a_2,...,a_l$ with probabilities

$$p_k^{(2)}=P(\rho_1^{(2)}=a_k),\ k=1,2,...,l.$$

$\theta=\theta(\omega)$ is a random variable which takes the values from the set $\{1,2,...,N\}$ with probabilities

$$q_n=P(\theta=n),\ \ \Pi_n=P(\theta\le n),\ n=1,2,...,N.$$

We assume, that $\rho^{(1)}$ and $\rho^{(2)}$ are independent and they are jointly independent of θ, i.e. the vector $(\rho^{(1)},\rho^{(2)})$ is independent of θ.

From (1) it is clear, that

$$S_n=S_{n-1}e^{\rho_n},\ \ S_0>0. \tag{2}$$

We propose the process S given by (1) or (2) as stock price evolution model.

As we see before random moment θ there is complete (binomial) market and after disorder moment θ - incomplete (multinomial) market.

The problem we investigate is forecasting of stock price process S and estimation of disorder moment θ, i.e. we find $\hat{S}_n(m)=E[S_n/F_{n-m}^S]$, $m<n$ and $\hat{\theta}_n=E(\theta/F_n^S)$, where $F_n^S=\sigma\{S_0,S_1,...,S_n\}$.

Note, that from (2) for each r

$$F_r^S=F_r^\rho=\sigma\{\rho_1,...,\rho_r\}.$$

Lemma 1. The conditional expectation

$$P(\theta = n / F_r^S) = P(\theta = n / F_r^\rho) = \begin{cases} \dfrac{q_n u_n}{L_r}, \text{ if } r < n, \\ \dfrac{q_n u_{n-1}}{L_r}, \text{ if } r \geq n \end{cases} \quad (3)$$

where

$$u_k = U_k(\rho_1, ..., \rho_k) = \frac{P_1(\rho_1)P_1(\rho_2)\cdots P_1(\rho_k)}{P_2(\rho_1)P_2(\rho_2)\cdots P_2(\rho_k)}, u_0 = 1.$$

$$P_1(x) = P(\rho_1^{(1)} = x), i = 1,2, x \in \{a_1, a_2, ..., a_l\};$$

$$L_r = \sum_{k=1}^{r} q_k u_{k-1} + (1 - \Pi_r) u_r, \quad \Pi_r = \sum_{k=1}^{r} q_k .$$

The proof of Lemma1 is based on Bayes formula and straightforward calculations. The problem of finding this conditional expectation belongs to the general filtered probability-experiment framework, given in [4] and it is not difficult to obtain (3) from the results presented there.

Theorem 1. The optimal in mean square sense m-step forecasting estimation of $S = (S_n, F_n), n = 0,1,...,r$ described by (1) is

$$\hat{S}_n(m) = S_{n-m}[P(\theta \leq n / F_{n-m}^\rho)(Ee^{\rho_1^{(2)}})^m +$$
$$\sum_{k=2}^{m} P(\theta = n - m + k / F_{n-m}^\rho)(Ee^{\rho_1^{(1)}})^{k-1}(Ee^{\rho_1^{(2)}})^{m-k+1} + P(\theta > n / F_{n-m}^\rho)(Ee^{\rho_1^{(1)}})^m],$$

where the conditional expectations are defined from Lemma 1 formula (3).

Using Lemma 1 we obtain also optimal in mean square sense estimation of disorder moment θ

$$\hat{\theta}_r = E(\theta / F_r^\rho) = \frac{u_r E\theta - \sum_{k=1}^{r} k q_k (u_r - u_{k-1})}{L_r},$$

where $E\theta = \sum_{k=1}^{N} k q_k$.

References

1. O.Glonti, L.Jamburia and Z.Khechinashvili. The minimal entropy martingale measure in trinomial scheme with disorder. Bull. Georgian Acad. Sci. Tbilisi (2004), pp. 452-453.
2. O.Glonti, L.Jamburia and Z.Khechinashvili. Trinomial scheme with disorder. The minimal entropy martingale measure. Appl. Math. Inf. Mech. Tbilisi (2004), pp. 14-28.
3. O.Glonti. The optimal forecasting for the multinomial scheme with disorder. Georgian Math. J. Tbilisi (2010) (in press).
4. A.N.Shiryaev. On statistical models and optimal methods in problems of quickest detection. Teor. Verojatnost. i Primenen. Moscow (2008), pp. 417-436.