ON SOME PROPERTIES STRONGLY AND WEAKLY SEPARABLE GAUSSIAN HOMOGENEOUS ISOTROPIC STATISTICAL STRUCTURES IN BANACH SPACE OF MEASURES

Lela Aleksidze[1], Zurabi Zerakidze[2]

Gori University, Gori, Georgia
[1]*lela_allk@bk.ru*, [2]*Z.Zerakidze@mail.ru*

On the interesting problems in the description of statistical structures. (See 1-3) Here we recall some definitions.

Definition 1. A statistical structures $\{E, S, \mu_i, i \in A\}$ is said to be weakly separable, it there exists a family $(X_i)_{i \in A}$ of measurable parts of E, such that the relations $(\forall i)(\forall j)(i \notin A \,\&\, j \in A \Rightarrow P_i(X_j)) = \begin{cases} 1, i = j \\ 0, i \neq j \end{cases}$ are fulfilled.

Definition 2. A statistical structure $\{E, S, \mu_i, i \in A\}$ is strongly separable, if there exists a disjunct family $(X_i)_{i \in A}$ of measurable parts of E, such that the relation $(\forall i)(i \in A \Rightarrow \mu_i(X_i) = 1)$ is fulfilled.

Let M^{σ} be linear real space of all finite measures with alternating signs on S.

Definition 3. A linear subset of measures $M_{\beta} \subset M^{\sigma}$ is said to be a Banach space of measures

1. one can introduce a norm $\|\mu\|$ on M_{β} is the Banach space, and for every mutually singular measures μ and ν, $\mu, \nu \in M_{\beta}$ and, for every real numbers $\lambda \neq 0$, we have $\|\mu + \lambda \nu\| \geq \|\mu\|$;
2. if $\mu \in M_{\beta}$ and $|f| \leq 1$, than $\nu_f(A) = \int\limits_A f(x)\nu(dx) \in M_{\beta}$, where $f(x)$ is an S-measurable real function and $\|\nu_f\| \leq \|\nu\|$.

In the sequel, it will be assumed that μ_i are orthogonal for all different *i* measures, i.e. the statistical structure $\{E, S, \mu_i, i \in A\}$ is the orthogonal Gaussian statistical structures.

We prove the following theorems

Theorem 1. Let $M_{\beta} = \bigoplus\limits_{i \in A} M_{\beta}(\mu_i)$ be Banach space of measures. For the Gaussian homogeneous isotropic orthogonal statistical structure $\{E, S, \mu_i, i \in A\}$ to be weakly separable, it is necessary and sufficient that the correspondence $f \to \ell_f$ given by the equality $\int f(x)\nu(dx) = \ell_f(\nu)$, $\forall \nu \in M_{\beta}$ be one-to-one, where $\ell_f(\nu)$ is a linear functional on M_{β}, $f \in F$ where F is the set of those f , for which $\int f(x)\nu(dx)$ is defined $\forall \nu \in M_{\beta}$.

Theorem 2. Let $M_{\beta} = \bigoplus\limits_{i \in A} M_{\beta}(\mu_i)$. Let E be a complete separable metric space, let S be the Borel σ-algebra, and $card A \leq 2^{\chi_0}$. Then, in the theory (ZFC) and (MA) for the Gaussian homogeneous isotropic orthogonal statistical structure $\{E, S, \mu_i, i \in A\}$ to be strongly separable, it is necessary and sufficient that the correspondence $f \to \ell_f$ given by the equality

$\int f(x)\nu(dx) = \ell_f(\nu)$, $\forall \nu \in M_\beta$ be one-to-one, where $\ell_f(\nu)$ is a linear functional on M_β, $f \in F$, where F is the set of those f , which $\int f(x)\nu(dx) = \ell_f(\nu)$ is defined $\forall \nu \in M_\beta$.

References

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