ON ONE NUMERICAL ALGORITHM OF OIL FILTRATION PROCESS SIMULATION WITH CONSIDERATION FOR RELAXATION EFFECTS

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1. Introduction

During usage of depleted oil-and-gas-bearing fields and raising seam productivity methods of influencing a near-face zone are employed for the purpose of improving hydro dynamical conditions of ousting. When polymers and other reagents are added to injected water the behaviour of fluids under seam conditions changes and in case of presence of oil having increased content of high-molecular components non-Newtonian laws of filtration come into play, disbalance effects appear characterized by velocity inertia and its lag depending on pressure gradient, pressure relaxation etc. The paper [1] considers a problem of filtrating oil with abnormal properties characterized by displaying pressure and velocity relaxation. At sharp and significant changes in pressure the behaviour of porous medium will also be of relaxation nature manifesting through lag in reaching steady balanced state in micro pores [2, p.15]

2. Statement of the problem

A system of equations presenting oil filtration process with consideration for velocity, pressure and porosity relaxation is expressed in the form of motion equation

$$V + \tau_v V_t = -\frac{k}{\mu}\left(P_x + \tau_p P_{xt} + \tau_p \frac{V}{m} P_{xx} \right), \tag{2.1}$$

continuity equation

$$-(m\rho)_t = (\rho V)_X - \sum_{i=1}^{n} q_i(t)\delta_i(x), \tag{2.2}$$

and equations of oil and porous medium state

$$\rho = \rho_0\left[1 + \beta_ж (p - p_0)\right], \tag{2.3}$$

$$m + \tau_c \frac{\partial m}{\partial t} = m_0 + \beta_c (p - p_0) \tag{2.4}$$

The aim of the present research is to develop an efficient numerical algorithm for solving the system (2.1-2.4) in the area of $D\{0 < x \le L, 0 < t < T\}$ under the following initial and boundary conditions:

$$\begin{aligned} &P(x,0) = \varphi_1(x), \qquad 0 < x \le L, \\ &\chi_1 P + \chi_2 P_x\big|_{x=0} = \varphi_2(t), \quad 0 < t \le T, \\ &\chi_1 P + \chi_2 P_x\big|_{x=L} = \varphi_3(t), \\ &\frac{\partial P}{\partial n}\Big|\gamma_j = \frac{q_j \mu}{2\pi r_c kh}, \\ &V(x,0) = f_1(x), \qquad V(0,t) = f_2(t), \end{aligned} \tag{2.5}$$

Here k *is* rock permeability, μ is oil viscosity, m-rock porosity, ρ is oil density, P is pressure, t is time, $q_i -$ i-th well discharge, n is number of wells, $\tau_p, \tau_v, \tau_c -$ are times of pressure, velocity and porosity relaxation, respectively, $\beta_c, \beta_ж$ are coefficients of porous medium and liquid compressibility.

3. Solution method

By using the expression V from (2.1) the equation (2.2) is reduced to the following equation

$$a_1(P_{xt}, V_t, P_x)P_{xxx} + a_2(P_{xx})P_{xxt} + a_3(P_x, P_{xt}, V_t)P_{xx} + a_4(P_x, P_{xx})P_{xt} + \\ + a_5(P_x, V_t)P_x + a_6(Q) = a_7(P)P_t, \tag{3.1}$$

Where

$$a_1 = \tau_p \tau_V \rho \alpha V_t + \tau_p^2 \alpha \gamma(P) P_{xt} + \gamma(P)\tau_p \alpha P_x,$$
$$a_2 = \gamma(P)\tau_\rho - \tau_p^2 \alpha \gamma(P) P_{xx},$$
$$a_3 = -\gamma(P) - \tau_p \alpha \gamma'(P) P_x^2 - \tau_p \alpha \gamma(P) P_{xx} - \tau_\rho \tau_V \alpha \rho' P_x V_t -$$
$$- 0{,}5 \alpha \tau_p^2 \gamma'(P) P_x P_{xt} + \beta' P_t \left(\alpha^2(P)\tau_p^2 P_{xx} + 2\tau_p \alpha\right),$$
$$a_4 = -\tau^2{}_p \gamma'(P)\left(P_x + 0{,}5\tau_p \alpha P_x P_{xx}\right),$$
$$a_5 = -\gamma'(P) P_x,$$
$$a_6 = -\sum_{i=1}^{n} q_i \delta_i(x),$$
$$a_7 = -\beta'(P),$$
$$\alpha = \frac{k}{\mu},$$
$$\beta(P) = m(P)\rho(P),$$
$$\gamma(P) = \alpha\beta(P).$$

For solving the equation (3.1) under the conditions (2.5) finite difference method is used. We'll introduce a uniform three-dimensional network in the (O, L) area:

$$\overline{\omega}_h = \{x_i = ih,\ i = 0,1,\ldots.., N; h = L/N\},$$

and a uniform time network in the (O,T) area:

$$\varpi_\tau = \{t^j = j\tau,\ j = 0,1,\ldots.., M; \tau = T/M\},$$

where h, τ are three-dimensional and temporal steps, N is number of break-down points(O,L), M is number of temporal layers.

For numerical solution of the problem is proposed an iterative implicit scheme on a three-dimensional node network. By approximating P_x and P_{xx} on a five-point picture $\{x_{i-2}, x_{i-1}, x_i, x_{i+1}, x_{i+2}\}$, and by approximating P_{xxx} on *a* seven-point picture $\{x_{i-3}, x_{i-2}, x_{i-1}, x_i, x_{i+1}, x_{i+2}, x_{i+3}\}$ [3, p.571-574] we shall have

$$P_x \approx (P_{i-2} - 8P_{i-1} + 8P_{i+1} - P_{i+2})/(12h),$$
$$P_{xx} \approx -(P_{i-2} - 16P_{i-1} + 30P_i - 16P_{i+1} + P_{i+2})/(12h^2), \tag{3.2}$$
$$P_{xxx} \approx (P_{i-3} - 8P_{i-2} + 13P_{i-1} - 13P_{i+1} + 8P_{i+2} - P_{i+3})/(8h^3),$$
$$P_t \approx \left(P_i^{j+1} - P_i^j\right)/\tau, \quad V_t \approx \left(V_i^{j+1} - V_i^j\right)/\tau.$$

Using the approximations (3.2) we shall write a two-layer implicit scheme for the equation (3.1). The following system of algebraic equations is obtained, for determining values of the desired function on a new layer *t=t_j+1:*

$$A_i^s P_{i+3}^{s+1} + B_i^s P_{i+2}^{s+1} + C_i^s P_{I+1}^{s+1} + D_i^s P_i^{s+1} + E_i^s P_{i-1}^{s+1} + G_i^s P_{i-2}^{s+1} + H_i^s P_{i-1}^{s+1} = F_i^s, \left(i = \overline{4, N-3}\right) \quad (3.3)$$

The coefficients $A_i, B_i, C_i, D_i, E_i, G_i, H_i, F_i$ are functions of the sought-for solution, that's why they are computed on the preceding iteration *s* for linearization of the system (3.3). For solving the system (3.3) run method is proposed and the solution is sought in the form

$$P_i = \alpha_i P_{i-1} + \beta_i P_{i-2} + \gamma_i P_{i-3} + \theta_i, \quad \left(i = \overline{N-3,4}\right) \quad (3.4)$$

For determination of unknown running coefficients $\alpha_i, \beta_i, \gamma_i, \theta_i$ the following relations are derived by elimination method:

$$\alpha_i = \left(-A_i\left(\alpha_{i+2}\alpha_{i+3}\beta_{i+1} + \beta_{i+1}\beta_{i+3} + \alpha_{i+3}\gamma_{i+2}\right) + \\ + B_i\left(\alpha_{i+2}\beta_{i+1} + \gamma_{i+1}\right) + C_i\beta_{i+1} + E_i\right)\Big) / ZN,$$

$$\beta_i = -\left(A_i\left(\alpha_{i+2}\alpha_{i+3}\gamma_{i+1} + \beta_{i+3}\gamma_{i+1}\right) + B_i\alpha_{i+2}\gamma_{i+1} + C_i\gamma_{i+1} + G_i\right)\Big) / ZN,$$

$$\gamma_i = -H_i / ZN,$$

$$\theta_i = \left(F_i - A_i\left(\alpha_{i+2}\alpha_{i+3}\theta_{i+1} + \beta_{i+3}\theta_{i+1} + \alpha_{i+3}\theta_{i+2} + E_{i+3}\right) - B_i\left(\alpha_{i+2}\theta_{i+1} + \theta_{i+2}\right) - C_i\theta_{i+1}\right) / ZN,$$

Where

$$ZN = A_i\left(\alpha_{i+1}\alpha_{i+2}\alpha_{i+3} + \alpha_{i+1}\beta_{i+3} + \alpha_{i+3}\beta_{i+2} + \gamma_{i+3}\right) + \\ + B_i\left(\alpha_{i+1}\alpha_{i+2} + \beta_{i+2}\right) + C_i\alpha_{i+1} + D_i, \qquad \left(i = \overline{N-3,4}\right). \quad (3.5)$$

At the first stage of computations (direct run) the kenning coefficients $\alpha_i, \beta_i, \gamma_i, \theta_i$ $\left(i = \overline{N-3,4}\right)$ are determined from the formulae (3.5). At the second stage (reverse run) values of the desired function are determined from the formula (3.4) when *i =1,2,3,N-2,N-1,N* the running coefficients are determined from boundary conditions. Assuming that during the estimated time the influence of initial pulse does not reach boundaries $\overline{\omega}_{h\tau}$, of the area whir, the condition $P_i^j = P_i^0 \; (i = 1,2,3, N-2, N-1, N)$ is taken in as many points as is required according to the scheme. From (3.4) we shall have

$$\alpha_i = 0,\ \beta_i = 0,\ \gamma_i = 0,\ \theta_i = P_i^{\circ},\ (i = 1,2,3,\ N-2,\ N-1,\ N).$$

In this way $\alpha_i, \beta_i, \gamma_i, \theta_i$ are computed from the right to the left for *i=N-3,4* using the formulae (3.5) while the solution of P_i is computed from the left to the right for *i=4, N-3* using the formula (3.4).

Further at abrupt and considerable changes in pressure after each iteration for *P* from (2.3) iteration is computed for density ρ

$$\rho^{(s+1)(j+1)} = \rho_{\circ}\left[1 + \beta_{ж}\left(P^{(s+1)(j+1)} - \rho_{\circ}\right)\right],$$

and from (2.4) iteration for porosity *m* is computed, where is taken:

$$\frac{\partial m}{\partial t} = \frac{m^{s(j+1)} - m^j}{\tau}, \qquad m^{s(j+1)} = m_\circ + \beta_c\left(p^{(s+1)(j+1)} - p_\circ\right),$$

while m^j on the layer j is known. From this it follows:

$$m^{(s+1)s(j+1)} = m_\circ + \beta_c\left(p^{(s+1)(j+1)} - p_\circ\right) - \tau_c \frac{m^{s(j+1)} - m^j}{\tau}.$$

Iteration procedure goes on till the condition $\left|P_i^{s+1} - P_i^s\right| \le \varepsilon^T$, is fulfilled,
where $\varepsilon^T -$ required accuracy of computations.

On the basis of the above algorithm for performing numerical experiments software in Turbo-Pascal language is worked out.

4. Conclusions

A numerical algorithm is obtained and software is worked out for solving the problem of filtrating oil having abnormal properties with consideration for pressure, velocity and porous medium relaxation.

References

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