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STEIN'S METHOD, NORMAL APPROXIMATION AND MODERATE DEVIATIONS

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Stein's method, which was introduced by Charles Stein in 1972, can be described as follows. In approximating $\mathcal{L}(W)$ by $\mathcal{L}(Z)$ we write

$$Eh(W) - Eh(Z) = E\{Lf_h(W)\}$$

for a class of bounded functions h where L is a linear operator and f_h a bounded solution of the equation

$$Lf = h - Eh(Z).$$

The error $E\{Lf_h(W)\}$ can then be bounded by studying the solution f_h and exploiting the probabilistic properties of W.

For normal approximation, $\mathcal{L}(Z) = N(0,1)$ and

$$Lf(w) = f'(w) - wf(w). \quad \text{(Stein(1972))}$$

For Poisson approximation, $\mathcal{L}(Z) = Po(\lambda)$ and

$$Lf(w) = \lambda f(w+1) - wf(w). \quad \text{(Chen(1975))}$$

In this talk we will show how Stein's method can easily be applied to prove the Berry-Esseen theorem for sums of independent random variables, and then discuss extensions to locally dependent random variables (Chen and Shao (2004)).

Next we will present a general theorem for Cramér-type moderate deviations for dependent random variables (Chen, Fang and Shao (2010)). This theorem is then applied to obtain the following corollaries.

Corollary 1: Suppose $EW = 0, Var(W) = 1$ and W^* is W-zero-biased, that is,

$$EWf(W) = Ef'(W^*)$$

for all bounded f with bounded f'. Suppose W and W^* are defined on the same probability space such that $|W^* - W| \le \delta$. Then

$$\frac{P(W > x)}{1 - \Phi(x)} = 1 + O(1)(1 + x^3)\delta$$

for $0 \le x \le \delta^{-\frac{1}{3}}$, where Φ is the standard normal distribution function.

Corollary 2: We have

$$\frac{P(W > x)}{1 - \Phi(x)} = 1 + O(1)(1 + x^3)\delta$$

for $0 \leq x \leq \delta^{-\frac{1}{3}}$ for the following three examples.

(1) The number of ones in the binary expansion of a random integer from $\{0, 1, \ldots, n-1\}$;

$$\delta = \frac{1}{\sqrt{k}}, \quad 2^{k-1} < n \leq 2^k.$$

(2) The stationary distribution of the anti-voter chain on a complete graph V;

$$\delta = n^{-\frac{1}{2}}, \quad n = |V| = \text{number of vertices in the complete graph } V.$$

(3) The total magnetization in the Curie-Weiss model (without external field);

$$\delta = n^{-\frac{1}{2}}, \quad n \text{ spins}.$$

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