MATHEMATICAL MODEL OF MOVING PARTICLES WITH INTERACTION

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The paper is the continuation of investigations carried out in [1-4]. A mathematical model of motion of two particles on an infinite straight line without overtaking was considered in [1]. In this model the right particle makes a random walk with parameters ($\overline{r},\overline{l}$) and the left particle with parameters (r,l). It is proved that in the stationary regime the left particle also makes a random walk with parameters ($\overline{r},\overline{l}$).

In [2] this model is generalized for a more general case. Here the particles move with parameters (r_k,l_k) $(k=1,2,3,\ldots)$, where k a distance to next particle is. It is shown that there exist such (r,l) that each separately considered particle in the stationary regime makes a random walk with these parameters. Similar results for the motion of two particles with equidistant points of a circle are obtained in [3].

In [3] for a symmetric model a stationary distribution of distance between particles is investigated and it form is found. It is proved that each separately considered particle makes a random walk with parameters ($r,l=1-r$). Mathematical model of motion of there particles is considered in [4]. In stationary regime, invariant character of random walk of separately considered particle is proved for non-symmetric model. Distribution of distance between particles is found. For non-symmetric model where the number of particles is greater than 3 the analytical investigations are connected with some difficulties, therefore modeling is one of effective investigation methods.

Description of the model. Let's consider a model of motion of S ($S<N$) particles on N closen and equidistant points of a circle. Motion may occur on any closed trajectory, but for convenience we take motion on equidistant points of a circle with distance between points $d=2\pi R/N$. It is assumed that at each point of a circle there can be no more than one particle. Particles can change a position at discrete time $t\in T=\{0,h,2h,...\}$ in counterclockwise direction. Motion happens without overtaking. Assume that $h=1$. Introduce the notation: $\xi_{i,t}$ is a coordinate of a particle i at time t. Position of each particle, i.e. its coordinates is fixed at initial position $t=0$. Distance between neighboring particles is denoted by $\rho_{i,t}=\xi_{i+1,t}-\xi_{i,t}$. At discrete time t the distance $\rho_{i,t}$ from the particle i to the particle $i+1$ is determined by the formula:

$$\rho_{i,t}=\begin{cases}\xi_{i,t}-\xi_{i+1,t}, & if \quad \xi_{i,t}>\xi_{i+1,t}\\ N-(\xi_{i+1,t}-\xi_{i,t}), & if \quad \xi_{i+1,t}>\xi_{i,t}\end{cases}$$

Denote $\varepsilon_{i,t}=\xi_{i,t+h}-\xi_{i,t}$ i.e. $\varepsilon_{i,t}=1$ if the particle make a jump at time t otherwise $\varepsilon_{i,t}=0$. Introduce $\varepsilon_i(t)$, that accepts "0" and "1". Random walk of particles happens by the following law:

$$
\begin{aligned}
&P\left\{\varepsilon_{i,t}=1 \middle| \rho_{i,t}=k\right\}=r_k;\ k=\overline{1,N-S+1};\\
&P\left\{\varepsilon_{i,t}=0 \middle| \rho_{i,t}=k\right\}=l_k;\ k=\overline{1,N-S+1};\\
&P\left\{\varepsilon_{i,t}=1 \middle| \rho_{i,t}=1,\varepsilon_{i+1,t}=1\right\}=r_1;\\
&P\left\{\varepsilon_{i,t}=0 \middle| \rho_{i,t}=1,\varepsilon_{i+1,t}=1\right\}=l_1;\\
&P\left\{\varepsilon_{i,t}=1 \middle| \rho_{i,t}=1,\varepsilon_{i+1,t}=0\right\}=0;\\
&\left\{\varepsilon_{i,t}=0 \middle| \rho_{i,t}=1,\varepsilon_{i+1,t}=0\right\}=1;
\end{aligned}
\tag{1}
$$

Let:

$$r^{(k)}+l^{(k)}=1, k=\overline{1,N-S+1};$$

$$r^{(k)}=P\left\{\varepsilon_{i,t}=1 \middle| \rho_{i-1,t}=k\right\};$$

$$l^{(k)}=P\left\{\varepsilon_{i,t}=0 \middle| \rho_{i-1,t}=k\right\}=1-r^{(k)} \quad k=\overline{1,N-S+1};$$

$$\pi_k=\lim_{t\to\infty}P\left\{\rho_{1,\mathrm{t}}=k\right\}, \quad k=\overline{1,N-S+1};$$

Non-symmetric model. Let the parameters of motion of a particle $r,l(r+l=1)$, and the motion parameters of another particle depend on the distance to the previous particle. If the distance from the particle i to the particle $i+1$ composes k then probability of the jump of the particle $i+1$ equals r_k, but with probability l_k the particle stands in its place. While moving the first particle may retard the motion of next particles and if the first particle overtakes the last one, it pushes it slightly. In other words the first particle plays the part of a leader. The particles move according to the following principles:

$$
\begin{aligned}
&P\left\{\varepsilon_{i,t}=1 \middle| \rho_{i,t}=k\right\}=r;\ k=\overline{2,N-1};\\
&P\left\{\varepsilon_{i+1,t}=1 \middle| \rho_{i+1,t}=k\right\}=r_k,\quad \left(k=\overline{2,N-1}\right);\\
&P\left\{\varepsilon_{i,t}=1 \middle| \rho_{i,t}=1,\ \varepsilon_{i+1,t}=1\right\}=r;\\
&P\left\{\varepsilon_{i,t}=1 \middle| \rho_{i,t}=1,\ \varepsilon_{i+1,t}=0\right\}=0;\\
&P\left\{\varepsilon_{i+1,t}=1 \middle| \rho_{i+1,t}=1,\ \varepsilon_{i,t}=1\right\}=r_1;\\
&P\left\{\varepsilon_{i+1,t}=1 \middle| \rho_{i+1,t}=1,\ \varepsilon_{i,t}=0\right\}=0;
\end{aligned}
$$

For this model in stationary condition from relation (1) with using total probability formula we get the following recurrent formulae:

$$\pi_1 r^{(1)} l_1=\pi_2 l^{(2)} r;$$

$$\pi_k\left(r_k l^{(k)}+l_k r^{(k)}\right)=\pi_{k-1} r^{(k-1)} l_{k-1}+\pi_{k+1} l^{k+1} r_{k+1} \quad k=\overline{2,N-S+1};$$

$$\pi_{N-S+1} r_{N-S+1} \left(1 - r_1^{S-1}\right) = \pi_{N-S} r^{(N-S)} l_{N-S} .$$

Denote A_k:

$$A_1 = 1, \quad A_k = \prod_{j=1}^{k} \left(\frac{r^{(j-1)} l_{j-1}}{r_j l} \right)^{k-1} \qquad A = \sum_{j=1}^{N-S+1} A_j .$$

Then $\pi_k = \dfrac{A_k}{A}$ is a solution of recurrent equations.

If number of particles equals S, the values of π_k remain unknown, since $r^{(k)}$ should be calculated for them. In order to use the relations $\pi_{k_1}, \pi_{k_2}, \ldots, \pi_{k_{s-1}}$ with parameters r_k, l_k where $k = \overline{1, N-S+1}$, we must compose recurrent relations. Knowing π_k one can find the values of $r^{(k)}$ for concrete models, wherein the calculation of these parameters up to now made certain difficulty. The use of computers for realization of the given algorithm allows to avoid calculating character difficulties that creates premises for successful application of the suggested algorithm in practical problems.

In the paper we given results of modeling in a computer of a system of moving (random) particles on a ring. Analysis of the results of modeling allows to exclude a fact on binomial walk of separately considered particle, when the number of particles is more than three.

Literature

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