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SOME NOTES ON THE DISTRIBUTIONS OF SOLUTIONS OF EQUATIONS WITH RANDOM COEFFICIENTS

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The differential equations with random coefficients are relatively less studied because the object considered is difficult enough. The last period achievements in the stochastic integration theory (see for example [1], [2]) gives possibility to consider some questions of solvability of this problem. Also it may be to represent the solution for the wide enough class of boundary problems with random coefficients. This paper is devoted to one of such kind problem.

Let $\{\Omega, \Im, P\}$ be the fixed probability space; $L_2 = L_2([0,1])$ - the space of square integrable functions by Lebesque measure, with usual scalar product and norm; $\tilde{L}_2 = \tilde{L}_2([0,1]\times\Omega)$ - the space of random functions defined on $[0,1]\times\Omega$ with values in R and finite second moment: $\|x\|_{\sim}^2 = E\int_0^1 x^2(t)dt < \infty$. The scalar product in this space has the form $(x, y)_{\sim} = E\int_0^1 x(t)y(t)dt$. Further w_t is a Wiener process; A_t is a random linear (maybe unbounded) operator (see [3]) with definitional domain $D = D(A)$, which is dense in $\tilde{L}_2$ and independent from $t \in [0,1]$ and $\omega \in \Omega$. Furthermore, assume that A_t is the generating operator of strongly continuous evolution family $u(t,s)$. This family represents the bounded random linear operator for any $t, s \in [0,1]$.

Preliminarily we give some formal computations, which show the basic idea of this work. In the class of generalized functions consider stochastic differential equation

$$\frac{d\xi_t}{dt} = A_t\xi_t + b_t w_t', \qquad \xi_0 = 0 \tag{1}$$

We'll show, that solution of the equation (1) one can to represent in form

$$\xi_t = \int_0^t u(t,s)b_s dw_s \tag{2}$$

where the integral in (2) is interpreted as extended stochastic integral.

Really, by formula of differentiation of the stochastic integral we can write

$$d\xi_t = d\int_0^t u(t,s)b_s dw_s = u(t,t)b_t dw_t + \int_0^t \frac{\partial u(t,\tau)}{\partial t} b_\tau dw_\tau dt =$$

$$= b_t dw_t + \int_0^t A_t u(t,\tau)b_\tau dw_\tau dt = b_t dw_t + A_t\int_0^t u(t,\tau)b_\tau dw_\tau dt = b_t dw_t + A_t\xi_t dt,$$

which is equivalent to (1).

The formula obtained we can to use in various interesting situations. For instance consider second boundary problem for ordinary differential equation with random coefficients:

$$y''(t) + \alpha(t)y(t) + \beta(t) = w_t', \quad y'(0) = y'(1) = 0 \tag{3}$$

where $\alpha(t)$ and $\beta(t)$ are the continuous Gaussian random processes and linearly related to w_t.

For the solution of problem (3) consider direct and inverse initial problems

$$y_1''(t) + \alpha(t)y_1(t) + \beta(t) = 0, \quad y_1(0) = 1, \quad y_1'(0) = 0$$

and

$$y_2''(t)+\alpha(t)y_2(t)+\beta(t)=0\,,\quad y_2(1)=1\,,\quad y_2'(1)=0$$

This problems have unique solutions (see [4]) and one can to construct the Green function with the help of this solutions

$$G(t,s)=\begin{cases} y_1(t)y_2(s)v_0^{-1} & \text{if} \quad t\le s \\ y_1(s)y_2(t)v_0^{-1} & \text{if} \quad t>s \end{cases}$$

where the Vronsky determinant $\nu_0 = y_1(1)\ne 0 \ \ (P-a.s)$.

Under conditions listed above the solution of equation (3) exists, is unique and has the following form

$$y(t)=\int_0^1 G(t,s)dw_s\,, \tag{4}$$

where the integral is interpreted as Daletsky-Skorokhod extended stochastic integral.

For strict mathematical basing of questions listed above it is necessary to substantiate understanding of problems (1), (3) and like that, to prove existence of extended stochastic integral in right sides of (2), (4), also argue existence and uniqueness of solutions for problems (1), (3) and truth of equalities (2), (4). This investigation is devoted to this questions.

References

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