ON INFINIT DIMENSIONAL ELASTICITY THEORY

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It's well known that the methods of the theory of random processes and infinite dimensional stochastic analyses are effective when studying problems of the mathematical physics (see [1]). Success, achieved in last two decades in the above mentioned theories gives an opportunity to consider specific questions in mathematical physics having an applied origin and to investigate solids in infinite dimensional spaces.

Here we consider the possibility to pose generalizations of two- and three-dimensional problems of the classical elasticity theory (see [2, 3]) to infinite dimensional linear spaces.

Let $H_+ \subset H_0 \subset H_-$ be a Hilbert-Schmidt triple, μ be a measure on the Borel σ-algebra of the space H_- which satisfies some smoothness properties (has the logarithmic derivative with respect to every fixed direction in the space H_+). $\{e_k\}_{k=1}^{\infty}$ is a fixed orthonormal basis. We consider a convex continuous geometrical solid V with the boundary S which is supposed to be smooth enough. The measure μ generates a surface measure on S, which is denoted by μ_S. The latter measure satisfies some smoothness conditions. Let us consider vector field $z(x)$ on S . A result of its action on a part $T \subset S$ is $F = \int_T z(t)\mu_S(dt),$ and we call it stress. If $n(x)$ denotes the outer normal in the point x the stress is denoted by z^n, too. For a fixed x and any natural k, $k = 1,2,3,...$ let us consider a hyperplane Γ_k of codimension 1 passing through x and orthogonal to e_k. The stress which is generated by z at x on the hyper plane Γ_k is denoted by $z_k = z_k(x)$. The stress operator $A = A(x)$ (and its extension) is defined by the equality

$$Ae_k = z_k,\ k = 1.2.3,... \ . \tag{1}$$

Theorem 1. The operator A is selfadjointed and satisfies the equality

$$z^n = An. \tag{2}$$

Remark 1. In the finite-dimensional case (2) turns out to be the well-known Cauchy condition concerning the stress components and the stress itself.

If we consent that equilibrium in the infinite dimensional space means the vanishing of the sum of all the acting forces then it can be shown that the following assertion is valid.

Theorem 2. If the operator $A(x)$ has a kernel derivative operator, then

$$trA'(x) - l_\mu(A(x), x) + z(x) = 0. \tag{3}$$

Remark 2. The relation (3) is an analogue of the so called equilibrium equalities. The summand $l_\mu(A(x), x)$, the logarithmic derivative, arises due the choice of the measure space. It vanishes in the finite-dimensional case with the Lebesgue measure and (3) reduces to the usual equilibrium condition.

Similar observations could be done when deformation is considered.

References

1. Yu. L. Daletskii, S.V. Fomin, Measures and differential equations in infinite-dimensional spaces. Nauka, Moscow, 1983 (Russian). 384 p.
2. N.I. Muskhelishvili, Some basic problems of the mathematical theory of elasticity. Nauka, Moscow, 1966 (Russian). 708 p.
3. V.V. Novozhilov, Elasticity theory. Sudprom, Moscow, 1958 (Russian). 370 p.