The Second International Conference "Problems of Cybernetics and Informatics"
September 10-12, 2008, Baku, Azerbaijan. Section #4 "Applied Stochastic Analysis"
www.pci2008.science.az/4/16.pdf

FRACTAL ANALYSIS OF NETWORK PROCESSES

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The analysis of signals based on fractal concept develops intensively for the last time [1-4]. The quantitative analysis of fractal concept is based on mathematical technique of integrodifferentiation of fractional order. In the present paper the results of application of a fractal analysis to the processes in high-speed computer networks are given. For analysis of dynamics of information transmission, the operator method and especially Laplace transform are widely used. Formally, a dynamic system can be represented as a system converting the casual input process $\xi\ (t)$ into casual output process $f\ (t)$. This transfer of the input signal $\xi\ (t)$ into output signal $f\ (t)$ $f\ (t)$ = W [$\xi\ (t)$] one can formally represent by conversion $f\ (t)$ = W [ξ (t)], where W[.] is some, generally speaking, nonlinear operator. For simplicity we will be confined to considering of linear systems. To find the connection between input $\xi\ (t)$ signal and output $f\ (t)$ one we can proceed out of linear differential equation

$$\partial_{0\tau}^{\alpha} f(\tau) + Af(\tau) = 0, f(\tau = 0) = A, 0 < \alpha \le 1 \, , \tag{1}$$

which, after Laplace transform, in operator form will be $F\ (s) = W\ (s)\ \xi\ (s)$, where s is the complex parameter, $F\ (s)$ and $\xi\ (s)$ are the Laplace images of output and input signals correspondingly, $W\ (s)$ is the transfer function. The transfer function defines reaction of a system on impulse signal, which simulates reaction of a system when influencing the delta-impulse at the initial moment of time. The structure of the transfer function is defined by properties of a system.

Let the signal $f\ (t)$ enter a device with impulse transitive function of the degree form $h\ (t) = kt^{\beta-1}$, where β is a fractal parameter. Such transfer function at zero initial conditions is determined by integral of fractional order

$$u(\tau) = \frac{1}{\Gamma(\beta)} \int_0^{\tau} \frac{f(\xi)}{(\tau - \xi)^{1-\beta}} \, d\xi \, . \tag{2}$$

If, for example, as input signal functions representing solutions of traditional differential equations are used, the multiple computation of the integral (2) becomes impossible. Draw attention that integral (2) represents actually integral of fractional order and processes generated by the considered transfer function correspond to behavior of a system with partial memory. Therefore, selecting a solution of fractional order as a function $f\ (\tau)$, one should expect possibility of constructive analysis of network processes.

As a specific example of such approach, one can regard a network with packet switching, in which

$$f(\tau) = A\tau^{\alpha-1} E_{\alpha,\alpha}(-b\tau^{\alpha}) \, , \tag{3}$$

where $\tau = t / t_0, t_0 -$ the characteristic time of a process. Parameter β describes condition of a router. Substituting (3) into (2) we will obtain

$$u(\tau) = A \frac{1}{\Gamma(\beta)} \int_0^{\tau} \frac{\xi^{\alpha-1} E_{\alpha,\alpha}(-b\xi^{\alpha})}{(\tau - \xi)^{1-\beta}} \, d\xi = A\tau^{\alpha+\beta-1} E_{\alpha,\alpha+\beta}(-b\tau^{\alpha}) \, . \tag{4}$$

Let us represent the conversion (4) by the following way: $f(\tau)\overset{\beta}{\Rightarrow}u(\tau)$. Processes in the computer network with formation of multiple virtual connections one can represent in the form

$$f(\tau)\overset{\beta_0}{\Rightarrow}u(\tau)\overset{\beta_1}{\Rightarrow}...\overset{\beta_n}{\Rightarrow}u_n(\tau)\overset{\beta_{n+1}}{\Rightarrow}... \tag{5}$$

Applying the conversion (4) to the sequence (5), where as a function *f (τ)* appears function $A\tau^{\alpha-1}E_{\alpha,\alpha}(-b\tau^{\alpha})$, and using (4) one can see that the following result takes place:

$$u_n(\tau)=A\tau^{\alpha-1+\sum_{I-1}^{N}\beta_I}E_{\alpha,\alpha+\sum_{i-0}^{n}\beta_i}(-b\tau^{\alpha}) \tag{6}$$

where $E_{\alpha,\beta}(-z^{\alpha})=\sum_{n=0}^{\infty}(-1)^n\frac{z^{\alpha n}}{\Gamma(\alpha n+\beta)}$ is the Mittag – Leffler function. The correlation (6) determines analytically conversion of a traffic in the computer networks. Incidentally the conceptual difference of the suggested approach from traditional approaches is the fact that parameters *α, β* can be the functions of time and other parameters of the considered problem. Besides, it is important that with parameter *α* the structure of input signal is simulated. We will consider certain examples of such possibility further. The parameter *β* describes the state of the router. The correlation (4.6) is interesting at standpoint of topological properties of the network. Spesifically, if to choose two different transfer routes, for example, the route $\sum_{i=1}^{n}\beta_i^{(1)}$ and the route $\sum_{i=1}^{m}\beta_i^{(2)}$, they can turn out as "equivalent", if $\sum_{i=1}^{n}\beta_i^{(1)}=\sum_{i=1}^{m}\beta_i^{(2)}$. Naturally, this equality should be considered in the dynamic state, because the functions $\beta_i^{(1)}$ and $\beta_i^{(2)}$ are complex functions of time and parameters of the router. Thus, the considered quantitative model allows representing the dynamic state of high-speed computer networks in analytical form. The using of the Mittag – Leffler function gives eventually unlimited possibilities for analysis of a signal. Specifically, considering the case when $1<\alpha(t)\le 2$ and supposing that $\alpha(t)=[(1-e-d)\varphi(t)+(e-d+3)]/2,(|\varphi(t)|<1)$ one can define conversion of a signal for different $\beta,\varphi(t)$. So, in the Fig. 1 the conversion of a signal in accordance with (4.1) for the function $\varphi(t)=\cos(k(\cos(f(t))$ and for $\beta=cos(7\cos(f(t))$ are given.

Selecting different α,β one can reproduce a signal of any complicate structure. It is very important from the point of view of producing the parameter of router. Really, if the traffic structure before and after a router is known exactly, determination of value of the parameter *α* by input signal give possibility to restore the significance of *β*, which describes the state of the router by output signal. Besides, the suggested method of describing of the router state can delineate the dynamic state of a router too. Simultaneously it is determined the time dependence of the parameter *β*.

The concrete application of these results to the real state of the network INTERNET router is very interesting. Specifically the parameters *α* and *β* can be defined by the measured signals on input and output of the router. It requires separate researches demanding additional technical solutions. In such formulation of the problem the parameters *α* and *β* are the most important characteristics of the state of a network. Incidentally, experimental definition of their time dependence can be basis of an integral analysis of the dynamic state of a network.

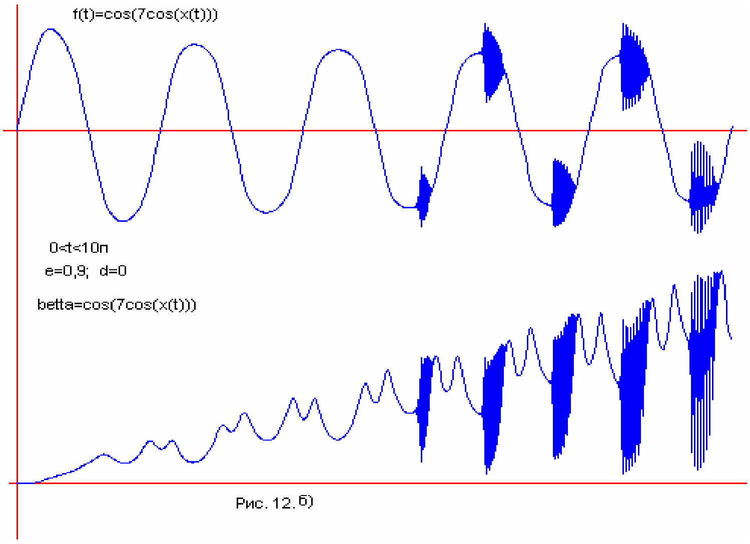

Fig. 1. Transfer of signal determined by the correlation (4.3).

Thus, the suggested approach to analysis of processes in high-speed computer networks gives possibility of construction of adequate quantitative models with account of effects of a system memory.

Literature

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