FOURIER TRANSFORMATION WITH RESPECT TO PHASE OF THE CONDITIONAL DISTRIBUTION OF ONE OF THE SEMI-MARKOVIAN RANDOM WALK PROCESSES

Tamilla Nasirova[1], Elshan Ibayev[2]

Baku State University, Institute of Cybernetics, Baku, Azerbaijan
[1]*nasirovatamilla@rambler.ru*, [2]*elshan1980@yahoo.com*

Let on the probability space $(\Omega, \Im, P(\cdot))$ is given the sequence $\{\xi_k(\omega), \zeta_k(\omega)\}_{k=1}^{\infty}$ of the independent identically distributed and independent between themselves positive random variables $\xi_k(\omega)$ and $\zeta_k(\omega)$.

We construct the following process

$$Y(t,\omega) = z - t + \sum_{i=1}^{k-1} \zeta_i(\omega), \quad if \quad \sum_{i=1}^{k-1} \xi_i(\omega) \le t < \sum_{i=1}^{k} \xi_i(\omega).$$

This process was investigated in [1, 2, 4, 5, 6], etc. In [3] Some asymptotic results is found for ergodic distribution of the semi-markovian random walk with two screen. We'll study the process of semi-markovian random walk with negative drift, nonnegative jumps, delays and delaying screen in a $(a > 0)$. These processes can be directly applied to the queues, stock control, insurance and financial theories.

Let on the probability space $(\Omega, F, P(\cdot))$ are given the sequences $\{\xi_k(\omega)\}_{k=1}^{\infty}$, $\{\eta_k(\omega)\}_{k=1}^{\infty}$, $\{\zeta_k(\omega)\}_{k=1}^{\infty}$, where $\xi_k(\omega)$, $\eta_k(\omega)$, $\zeta_k(\omega)$, $k = \overline{1,\infty}$, are independent identically distributed and independent between themselves random variables. We suppose that $\xi_k(\omega) > 0$, $\eta_k(\omega) > 0$, $\zeta_k(\omega) \ge 0$, $0 < E\xi_k(\omega) < \infty$, $0 < E\eta_k(\omega) < \infty$, $0 < E\zeta_k(\omega) < \infty$ and $E\zeta_k(\omega) > E\xi_k(\omega)$, $k = \overline{1,\infty}$.

Let's construct the process

$$X_0(t,\omega) = \begin{cases} z - t + \sum_{i=1}^{l-1} \zeta_i(\omega) + \sum_{i=1}^{\nu_1+\ldots+\nu_{l-1}+k_l(t)-1} \eta_i(\omega), & if \quad \sum_{i=1}^{\nu_1+\ldots+\nu_{l-1}+k_l(t)-1} [\xi_i(\omega) + \eta_i(\omega)] \le t < \\ & < \sum_{i=1}^{\nu_1+\ldots+\nu_{l-1}+k_l(t)-1} [\xi_i(\omega) + \eta_i(\omega)] + \\ & + \xi_{\nu_1+\ldots+\nu_{l-1}+k_l(t)}(\omega), \\ z + \sum_{i=1}^{l-1} \zeta_i(\omega) - \sum_{i=1}^{\nu_1+\ldots+\nu_{l-1}+k_l(t)} \xi_i(\omega), & if \quad \sum_{i=1}^{\nu_1+\ldots+\nu_{l-1}+k_l(t)-1} [\xi_i(\omega) + \eta_i(\omega)] + \\ & + \xi_{\nu_1+\ldots+\nu_{l-1}+k_l(t)}(\omega) \le t < \\ & < \sum_{i=1}^{\nu_1+\ldots+\nu_{l-1}+k_l(t)} [\xi_i(\omega) + \eta_i(\omega)], l \ge 1, \end{cases}$$

The process decreases beginning from the moment zero from some state $z\,(z \ge 0)$ under the angle $\alpha = 45^0$ (may be $0 < \alpha \le 90^0$) with the size equal to $\xi_1(\omega)$ $(\xi_1(\omega) > 0)$. The random variable $\xi_1(\omega)$ is the duration of the drift of the process. When the drift ceases the process stops in the state $z-\xi_1(\omega)$ for the duration of the random time $\eta_1(\omega)$. The random time $\eta_1(\omega)$ we shall call the lateness. The successive alternations "negative drift and the lateness" to the first jump of the size $\zeta_1(\omega)(\zeta_1(\omega)\geq 0)$ may be realized a random number. This random variable we denote by $\nu_1=\nu_1(\omega)$. Thus we defined the random variables $\xi_1(\omega)$, $\eta_1(\omega)$, $\zeta_1(\omega)$ and $\nu_1(\omega)$. We can define the random variables $\xi_2(\omega)$, $\eta_2(\omega)$, $\zeta_2(\omega)$, $\nu_2(\omega)$; … similarly. Such constructed process we shall call the process of semi-markovian random walk with negative drift, nonnegative jumps and the latenesses.

l $(l\geq 1)$ is the number of the period (a part of the process between two successive jumps is called period); $k_l(t)$ is the number of the negative drifts to the moment t in the l-th period.

If to put $\nu_i(\omega)=1, \eta_i(\omega)=0, i\geq 1$ and $k_l(t)=1$, $l\geq 1$, the process $Y(t,\omega)$ can be obtained from the process $X_0(t,\omega)$.

Let's delay process $X_0(t,\omega)$ with screen in $a\,(a>0)$ ::

$$X_a(t,\omega)=X_0(t,\omega)-\sup_{0\leq s\leq t}(0,X_0(s,\omega)-a).$$

The process we shall call the process of semi-markovian random walk with negative drift, nonnegative jumps, delays and delaying screen in $a\,(a>0)$.

One of the realizations of the process $X_a(t,\omega)$ will be in the following form:

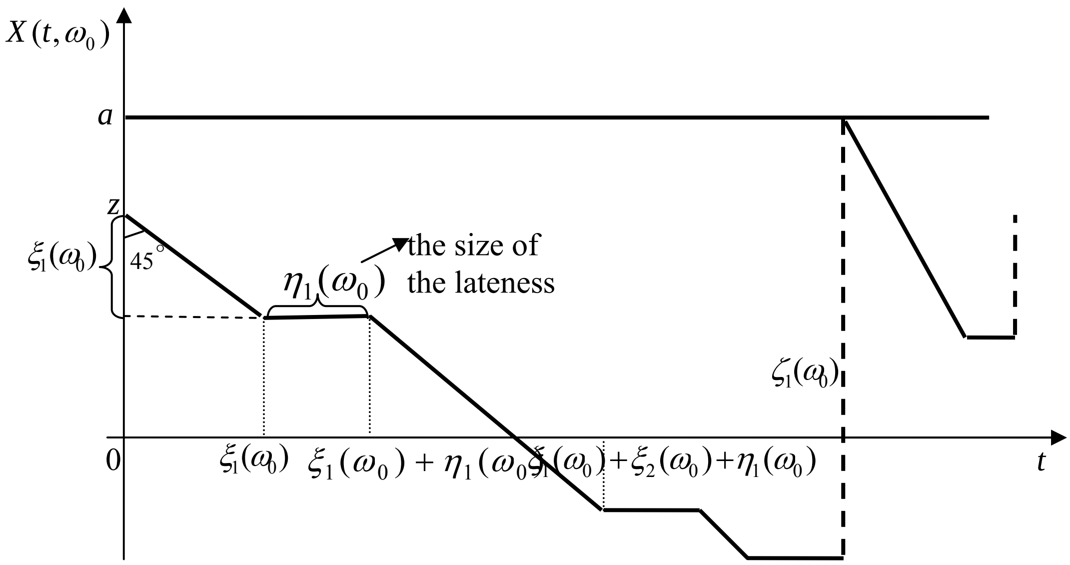

Fig 1

We denote

$$R_a(t,x)=P\{X_a(t,\omega)<x\},\quad t\geq 0,\quad x\in R,$$

$$R_a(t,x|\,z)=P\{X_a(t,\omega)<x\,|\,X_a(0,\omega)=z\},\quad x\in R,$$

$$\widetilde{R}_a(\theta,x|\,z)=\int_{t=0}^{\infty}e^{-\theta t}R_a(t,x|\,z)dt,\qquad \theta>0,\ \widetilde{\widetilde{R}}_a(\theta,\beta|\,z)=\int_{x=-\infty}^{a}e^{i\beta x}d_x\widetilde{R}_a(\theta,x|\,z),$$

$$\varphi(\theta) = Ee^{-\theta\eta_k(\omega)},\ \ \theta > 0,\quad k = \overline{1,\infty},\ \rho = P\{\zeta_k(\omega) > 0\}, k = \overline{1,\infty}.$$

It is obvious that

$$P\{\nu_i(\omega) = k\} = (1-\rho)^{k-1}\rho, k = \overline{1,\infty},\quad i = \overline{1,\infty}.$$

Let

$$\left.\begin{aligned} P\{\xi_1(\omega) < t\} &= \left[1 - e^{-\mu t}\sum_{i=1}^{m^- -1}\frac{(\mu t)^i}{i!}\right]\varepsilon(t), \mu > 0, m^- = \overline{1,\infty}, \\ P\{\zeta_1(\omega) < t\} &= \left[1 - \rho e^{-\lambda t}\right]\varepsilon(t), \lambda > 0. \end{aligned}\right\} \tag{1}$$

where

$$\varepsilon(t) = \begin{cases} 0, & t < 0, \\ 1, & t > 0. \end{cases}$$

$\tilde{\tilde{R}}_a(\theta,\beta | z)$ is the Fourier transformation with respect to phase of the coditional distribution of process $X_a(t,\omega)$ .Our aim to find $\tilde{\tilde{R}}_a(\theta,\beta | z)$.

Theorem. If $\{\xi_k(\omega),\eta_k(\omega),\zeta_k(\omega)\}, k = \overline{1,\infty}$, is the sequence of independent identically distributed and independent between themselves random variables $\xi_k(\omega),\ \eta_k(\omega),\ \zeta_k(\omega)$, where $\xi_k(\omega) > 0,\ \eta_k(\omega) > 0, \zeta_k(\omega) \ge 0,\quad k = \overline{1,\infty}$ .Then $\tilde{\tilde{R}}_a(\theta,\beta | z)$ satisfies the following integral equation.

$$\begin{aligned} \tilde{\tilde{R}}_a(\theta,\beta | z) &= e^{-\theta z}\int_{x=-\infty}^{z} e^{(i\beta+\theta)x} P\{\xi_1(\omega) > z - x\}dx - \\ &- \frac{1-\varphi(\theta)}{\theta}e^{-\theta z}\int_{x=-\infty}^{z} e^{(i\beta+\theta)x} d_x P\{\xi_1(\omega) < z - x\} - \\ &- (1-\rho)\varphi(\theta)e^{-\theta z}\int_{y=-\infty}^{z} e^{\theta y}\tilde{\tilde{R}}_a(\theta,\beta | y) d_y P\{\xi_1(\omega) < z - y\} + \\ &+ \rho\varphi(\theta)\tilde{\tilde{R}}_a(\theta,\beta | a)\int_{t=0}^{\infty} e^{-\theta t} P\{\zeta_1(\omega) > a - z + t\} dP\{\xi_1(\omega) < t\} + \\ &+ \rho\varphi(\theta)\int_{t=0}^{\infty} e^{-\theta t}\int_{y=z-t}^{a}\tilde{\tilde{R}}_a(\theta,\beta | y) d_y P\{\zeta_1(\omega) < y - z + t\} dP\{\xi_1(\omega) < t\}. \end{aligned}$$

In the case (1) this integral equation has following solution:

$$\begin{aligned} \tilde{\tilde{R}}_a(\theta,\beta | z) &= \frac{i\beta\mu^{m^-}\rho\varphi(\theta)}{A(\beta)\left\{[\mu+\theta+K_1(\theta,\rho)]^{m^-} - \mu^{m^-}\varphi(\theta)\right\}} \times \\ &\times \left\{\frac{(i\beta+\mu+\theta)^{m^-} - \mu^{m^-}}{i\beta+\theta} + \frac{\mu^{m^-}[1-\varphi(\theta)]}{\theta}\right\} e^{[i\beta - K_1(\theta,\rho)]a + K_1(\theta,\rho)z} + \end{aligned}$$

$$+\frac{i\beta-\lambda}{A(\beta)}\left\{\frac{(i\beta+\mu+\theta)^{m^-}}{i\beta+\theta}+\frac{\mu^{m^-}[1-\varphi(\theta)]}{\theta}\right\}e^{i\beta z}$$

where

$$A(\beta)=\sum_{i=2}^{m^-}C^i_{m^-}\left\{(i\beta)^{i+1}-\lambda(i\beta)^i\right\}(\mu+\theta)^{m^- -i}+m^-(\mu+\theta)^{m^- -1}(i\beta)^2-$$

$$-\left\{m^-\lambda(\mu+\theta)^{m^- -1}-(\mu+\theta)^{m^-}+\mu^{m^-}(1-\rho)\varphi(\theta)\right\}i\beta-\lambda\left\{(\mu+\theta)^{m^-}-\mu^{m^-}\varphi(\theta)\right\}.$$

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