ANALYSIS OF MULTIRATE SYSTEMS WITH SHARED RESERVATION OF CHANNELS AND QUEUES OF WIDEBAND CALLS

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The models of multirate systems that handle narrow-band and wide-band calls are investigated. Narrow-band calls are serviced by single channel whereas wide-band calls are require m channels simultaneously, m>1. In absence the necessary number of free channels wide-band calls are join queue but narrow-band calls are handle in accordance the scheme with pure lost. Narrow-band call is accepted if number of free channels more than given threshold. Algorithmic approach to calculate the quality of service of such models is developed and results of numerical experiments are shown.

Key words: *multirate system, narrow-band call, wide-band call, and design algorithm*

1. Introduction. Queuing system in which heterogeneous calls require a random number of channels simultaneously is called a multirate one (Multi Rate Queue, MRQ). These systems are adequate models of modern multimedia communication networks. Quality of service (QoS) in such systems is mainly defined by accepted call admission control (CAC). Therefore various CAC are suggested in different works. More detail references might be finding in [1]-[5]. In mentioned works as well as in other works models of MRQ with pure losses (i.e. without of queues) are investigated.

In known works frequently for save the wide-band calls (*w*-calls) either guard channels scheme (shared reservation) or scheme which is based on restriction of acceptance of narrow-band calls (*n*-calls) are used. Recently in [6] another guard channels scheme (isolated reservation) was proposed. In last CAC an individual (private) zone of channels is used for *w*-calls only and both types of calls uses common zone of channels. At that threshold for number of calls in common zone of channels might be defined. Note that in [6] models of MRQ with pure losses are considered also.

Another preventive rule for save the *w*-calls is organizing a buffer (finite or infinite) for it's waiting in queue. In available literature these kinds of MRQ models is not sufficiently investigated. In this work a simple computational procedures to calculate the QoS parameters of MRQ with queues of *w*-calls and shared reservation scheme of guard channels are proposed. Here for develop appropriate procedures an approach of the work [6] is used.

2. Models of MRQ and computational procedures

Consider the MRQ with $N>1$ channels for handle flow of *n*- and *w*-calls. Narrow calls are handled by single channel whereas for handles of single inelastic *w*-call simultaneously m, $m>1$, free channels are required. All channels that are used for handle single *w*-call are start and end of the servicing process simultaneously. Narrow-band calls are handled in accordance the scheme with pure lost, i.e. non-accepted *n*-calls are blocked. However non-accepted *w*-calls are waits in queue with either finite or infinite size.

CAC for *n*-calls is defined as follows. If at the arriving epoch of *n*-call number of free channels is more than a given threshold then this call is accepted; otherwise it will be lost. For effective use the channels capacity the value of defined threshold should be multiple of m, i.e. arrived n-call is accepted if at this epoch number of free channels is more than quantity mA, $1 \le A \le \overline{A}$, where

$$\overline{A} = \begin{cases} [N/m]-1 \text{ if } \mathrm{mod}(N,m)=0, \\ [N/m] \text{ otherwise.} \end{cases}$$

Here [x] is whole part of x. Handling of w-calls is performing as follows. If at the arriving epoch of w-call number of free channels is at least m then this call is accepted immediately; otherwise it will join queue. After departure w-call number of free channels becomes enough for handling one call of this kind since in these cases one of w-call is choice from queue (if any). If after departure n-call number of free channels becomes enough for handling one w-call then one of w-call is choice from queue (if any); otherwise released channel stand idle and w-calls continued wait in queue until releasing enough number of free channels. For the sake of brief assume that in the queue of w-calls the FCFS discipline is used. Here both models with finite and infinite queues are considered.

For obtain tractable results here the Markov models of MRQ are investigated. Let rate of n-calls (w-calls) is λ_n (λ_w) and rate of their handling is μ_n (μ_w).

First consider the model with finite queue. State of the system in arbitrary moment of the time in stationary mode is described by two-dimensional vector $\boldsymbol{k}=(k_n, k_w)$, where k_n (k_w) define the number of n-calls (w-calls) in system. It is clear that

$$0 \le k_n \le N - mA;\ 0 \le k_w \le \left[\frac{N}{m}\right] + R. \tag{1}$$

Regard the admitted CAC we conclude that if system is in state $\boldsymbol{k}=(k_n, k_w)$ then number of w-call in channels $\left(k_w{}^s\right)$ and number of w-calls in queue $\left(k_w{}^q\right)$ are calculate as follows:

$$k_w{}^s = \begin{cases} \left[\dfrac{N-k_n}{m}\right], & \text{if } k_w \ge \left[\dfrac{N-k_n}{m}\right] \\ k_w & \text{otherwise;} \end{cases} \tag{2}$$

$$k_w{}^q = \begin{cases} k_w - \left[\dfrac{N-k_n}{m}\right], & \text{если } k_w \ge \left[\dfrac{N-k_n}{m}\right] \\ 0 & \text{в противном случае.} \end{cases} \tag{3}$$

From (1)-(3) we find that in each possible $\boldsymbol{k}=(k_n, k_w)$ from state space S the following condition should be hold: $0 \le k_n + mk_w{}^s \le N$.

Elements of infinitesimal matrix of appropriate Markov chain $q(\boldsymbol{k},\boldsymbol{k}'), \boldsymbol{k},\boldsymbol{k}' \in S$ are calculated by

$$q(\boldsymbol{k},\boldsymbol{k}') = \begin{cases} \lambda_n & \text{если } f(\boldsymbol{k}) > mA, \boldsymbol{k}' = \boldsymbol{k} + \boldsymbol{e_1}\ , \\ \lambda_w & \text{если } \boldsymbol{k}' = \boldsymbol{k} + \boldsymbol{e_2}\ , \\ k_n\mu_n & \text{если } \boldsymbol{k}' = \boldsymbol{k} - \boldsymbol{e_1}\ , \\ k_w{}^s\mu_w & \text{если } \boldsymbol{k}' = \boldsymbol{k} - \boldsymbol{e_2}\ , \\ 0 & \text{в остальных случаях}, \end{cases} \tag{4}$$

where $\boldsymbol{e_1} = (1,0), \boldsymbol{e_2} = (0,1)$; $f(\boldsymbol{k}) := N - k_n - mk_w{}^s$ define the number of free channels in state $\boldsymbol{k} \in S$.

Let stationary probability of state $\boldsymbol{k}$ is denoted by $p(\boldsymbol{k})$. Main QoS parameters of the given models are probability of loss (blocking) of each type of calls. Let *PBn (PBw)* denotes probability of blocking of n-calls (w-calls). These quantities are calculated as follows:

$$PB_n = \sum_{\boldsymbol{k}\in S} p(\boldsymbol{k}) I\left(N \le k_n + m\left(k_w{}^s + A\right)\right), \tag{5}$$

$$PB_w = \sum_{\boldsymbol{k} \in S} p(\boldsymbol{k}) \delta\left(k_w{}^q, R\right) \quad . \tag{6}$$

Here $\delta(i, j)$ denotes Kroneccer' symbols: $\delta(i, j) = \begin{cases} 1 & \text{если}\, i = j \\ 0 & \text{если}\, i \neq j. \end{cases}$

Direct calculation of stationary distribution from balance equations is difficult problems for large-scale models. Therefore here new approximate calculation formulae are suggested.

The following splitting of state space S is considered:

$$S = \bigcup_{i=0}^{N-mA} S_i \ , S_i \bigcap S_j = \varnothing, i \neq j, \tag{7}$$

where $S_i := \{k \in S : k_n = i\},\ i = 0,1,..., N - mA.$

Stationary probability of state (i, j) within class of states S_i is denoted by $\rho_i(j)$. From (4) we conclude that these quantities are calculated as follows:

$$\rho_i(j) = \begin{cases} \dfrac{\nu_w{}^j}{j!} \rho_i(0), & \text{если } j = 1,...,[b_i] \\ \left(\dfrac{\nu_w}{[b_i]}\right)^j \cdot \dfrac{([b_i])^{b_i}}{[b_i]!} \rho_i(0), & \text{если } j = [b_i]+1,...,[b_i]+R \end{cases} \tag{8}$$

where

$$b_i := [(N-i)/m], \nu_w := \frac{\lambda_w}{\mu_w}\ ;\ \rho_i(0) = \left(\sum_{j=0}^{[b_i]} \frac{\nu_w{}^j}{j!} + \frac{([b_i])^{b_i}}{[b_i]!} \sum_{j=[b_i]+1}^{[b_i]+R} \left(\frac{\nu_w}{[b_i]}\right)^j \right)^{-1} \quad . \tag{9}$$

Intensities between classes S_i and Sj which are denoted by $\theta(i, j)$ are calculated by

$$\theta(i, i+1) = \begin{cases} \lambda_n \sum\limits_{j=0}^{[b_i - A]-1} \rho_i(j), & \text{if } \mathrm{mod}(N - i, m) \neq 0, \\ \lambda_n \sum\limits_{j=0}^{[b_i - A]} \rho_i(j), & \text{if } \mathrm{mod}(N - i, m) = 0; \end{cases} \tag{10}$$

$$\theta(i, i-1) = i\mu_n \ ; \tag{11}$$

$$\theta(i, j) = 0, \text{ if } |i - j| > 1. \tag{12}$$

From (10)-(12) stationary probabilities of merged states (i.e. class of states S_i) which are denoted by $\pi(i)$ are calculated as follows:

$$\pi(i) = \frac{\nu_n{}^i}{i!} \prod_{j=1}^{i} \theta(j-1, j) \pi(0) \ , i = 1,..., N - mA, \tag{13}$$

where

$$\nu_n := \frac{\lambda_n}{\mu_n}\ ;\ \pi(0) = \left(1 + \sum_{i=1}^{N-mA} \frac{\nu_n{}^i}{i!} \prod_{j=1}^{i} \theta(j-1, j) \right)^{-1} \quad . \tag{14}$$

Then stationary distribution of initial model is calculated approximately by

$$p(i, j) \approx \rho_i(j) \pi(i), (i, j) \in S. \tag{15}$$

Finally to calculate desired QoS parameters the following approximate formulas are proposed:

$$PB_n \approx \sum_{i=0}^{N-mA}\left(\sum_{j=[b_i-A]}^{[b_i]+R}\rho_i(j)\pi(i)I(\mathrm{mod}(N-i,m)\neq 0)+\sum_{j=[b_i-A]+1}^{[b_i]+R}\rho_i(j)\pi(i)I(\mathrm{mod}(N-i,m)=0)\right) \quad (16)$$

$$PB_w = \sum_{i=0}^{N-mA}\rho_i([b_i]+R)\pi(i) \ . \quad (17)$$

By using the proposed approach the models of MRQ with infinite queues of w-call might be investigate also. For ergodicity of these kinds of models the condition $\nu_w < A$ is obtained.

In the paper the results of numerical experiments are shown. The proposed approach allows solve the problems of improving of QoS parameters also.

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