ALGORITHM OF ESTIMATION OF EVOLVING MODELS PARAMETERS

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The system of models effectively used by mankind is so complicated that its single-valued decomposition seems impossible. In connection with that we will confine ourselves to the decomposition on the basis of rather robust criterion: we will refer the models which are not invariant relative to the time shift to class "A," and all the rest ones – to class "Б". The next step is construction of the subclasses of quantitative models $А_к \subset А$ and $Б_к \subset Б$.

The first "a" class countably-cyclic model was apparently fallen heir to by Homo sapiens from Homo habilis in the form of the primitive calendar. The further perfection of this class stretched out for thousands years. The prevalence of "Б" class measuring models which gave rise to the measure theory and prepared the axiomatic basis for probability theory [8a] in the 20th century was prepared by the labour of zikurats and pyramids builders and by land surveyors. The thesis that model is the idealized form of reality probably came from the environment of those tireless toilers. Unfortunally we do not have the direct, evident and perfect knowledge of reality which could serve as a reliable basis for estimating model assumptions as a representation of essential sections of reality. The fact mentioned, which showed itself vividly in the conflict of Ptolemaic and Copernican models, brought to the origin of "instrumentalism." Pursuant to the latter, the rational knowledge exists in the form of empirically true statements the content of which cannot be concluded from their agreement with the obvious facts of the world, but it is deduced from the fact that they have successfully passed the test by definite methods [1.2]. Instrumentalism does not consider the model functions of the logical scheme **{(Theory/model, data) → Prediction}** to be significant ones. However, in accordance with the prevailing nowadays point of view the stated model function is the most important as the model correlates at least countable variety of predictions to the final set of data. Neither semeromorphic nor instrumental approach could explain the reasons for auxiliary status of such geometric models as phase portraits $\{p \otimes q\}$ by A. Poincaré, diagrams by Josiah Willard Gibbs, R. Feynman and R. Penrose. It were not the representatives of "exact" knowledge, but philosophers Carnap, Bunge, Frank, Reichenbach and others who clarified a little the painful question. Namely, it was shown that the geometric instrument declaring causal description in fact eliminated time from the reality models. The situation noticeably changed when the needs of sociology, economy and informatics induced the interest to the models of processes developing in time [3, 4]. The reference to the needs of social sciences is excessively optimistic though: the majority of European sociologists referring to the absence of reproducible procedures of measurements and high complexity of social processes treat the quantitative estimations and mathematic models sceptically [6, pp. 77 – 79]. In the judgment of the authors, the scepticism of the sociologists is the reaction to a number of unsuccessful attempts: a) The attempts to construct "social physics" failed by reason of "complexity of sociology objects." Actually, it is not the matter of complexity, but the matter of specificity of interaction between the elements of social medium, the role of material carriers being less than the role of information exchange [3.3a]. The social physics fall-through entailed the failure of b) the attempts to transfer the measurement methods in classical physics, which objects being operationally defined, to the measurements in social sciences operating the objects allowing description in indistinct terms. As a result of the failures mentioned there appeared the prejudice about the acceptability of the measurements based on the already defined system of synthetic relationships exclusively. This ambiguity was pointed out by G. Hegel [7, pp. 250 – 261].

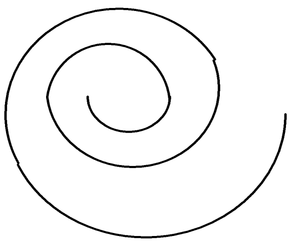
Meanwhile the realisation of the scheme **{(Theory/model, data) → Prediction}** demands measuring, and as it provided to be, the algorithm of the mentioned ones was created by the transactions of the mathematicians in the 19th century: a certain group of transformation corresponds to every noncontradictory classification (as well as any measuring procedure). Actually:

The concept of equality complies with the transitive conditions $(a = b \,\&\, b = c) \to a = c$ and symmetry conditions $a = b \to b = a$, and it is specified by the elements of the group of transformation. Meanwhile the stated conditions turn into the group axioms: the axiom of existence associativity of inverse element and axiom of units existence $g \cdot e = g$ (the latter results from the realization of reflexive property $a = a$). It stands to reason, the idea of group of transformation has common sense in this case. Group means the aggregate of automorphisms (representations) of a certain set of objects of themselves; the two objects turning one into another are considered congruous. The concept of equation plays a special role here which can be explained if the groups of automorphisms are used. The latter will help to subdivide the set of objects into subclasses containing quantitatively identical objects which, in its turn, renders it possible to compare objects and obtain quantitative characteristics.

Speaking of subdividing the similar objects into subclasses by the group of automorphisms we mean subdividing into adjacency classes by the elements of group of automorphisms {g}. In other words, if {g} is given in a certain set {f}, it is subdivided into subsets {fg} containing {f} elements identified by some $g \in \{g\}$, i.e. all $gf_g = f_g$. Thus, {f} is subdivided into the elements classes which are comparable, i.e. quantitatively similar as per the definition above. Such division is necessary at any measurements as the similar objects are compared.

As follows from the above, from the point of view of mathematics constructing the system of synthetic relationships with the aim of classification (measuring) is equivalent to the direct constructing of the group of acceptable transformations. In addition, the purposeful application of the group structure allows to avoid elementary but hardly revealed errors. To show that the class of evolving measurable models is not empty we will construct the elementary measuring model in which the equations of evolution parameters (both technological and social processes) are formed up similarly by choosing the scale-invariant representation. Let $\psi -$ be the observed extensive parameter of the evolving system, then we can put down $\frac{1}{\psi}\frac{d\psi}{dt} = \mu(t)$ at rather general conditions, where $\mu -$ is the difference of the specific rates of origin and dropout of the system elements. It can easily be seen that $\psi(t) = \psi_0 \exp\int_0^t \mu(t)dt$.

Let's introduce the non-dimentional parameters $\eta = \frac{\psi}{\psi_0}$ and $\theta = \int_0^t \mu(t)dt$, then the equation of the observed evolution parameter can be put down in the form of $\eta = e^{\theta}$. Supposing interpreting monotony η and θ as polar coordinates $r \leftrightarrow \eta$ and $\varphi \leftrightarrow \theta$, we obtain the following: the subset of one-dimensional equations of evolution parameters is isomorphic to the class of spirals. Meanwhile, the evolving systems similar accurate within the scalar factor $\mu(t)$ are located on the same branch of spiral. That is the switching over to the scale-invariant representation gives one of the basis of classification. In the simplest case $\mu = const$, i.e. $\theta = \mu \cdot t$ and $\psi = \psi_0 e^{\mu t}$ which corresponds to the level of equiangular spiral.



It's noteworthy that the models of estimating the world reserves of oil and natural gas are usually based on the modelling equations. Specifically, the performance model [5] is based on

Verfluchst's equation. From our point of view the introduction of the modelling equations is the additional hypothesis not allowing direct testing.

The class of spirals had been studied by Lee and Klein as a W-class eigen subset of the plane curves allowing sliding on them as far back as in the 1870s. In Lee's group theory terms W-curves are invariants of the projective transformations. Elementary representatives of W-class curves are: straight line, circumference, exponent and equiangular spiral. The two remarks are suitable here: a) our $r \leftrightarrow \eta$ and $\varphi \leftrightarrow \theta$ only differ from "synergetic coordinates for evolution description" in the name [4; 4a]; b) the model of economic growth by Simon Kuznets can evidently be transformed into the form of spiral in synergetic coordinates.

Classification and writing an equation is only the first step of the quantitative analysis. The next step is is verification, measuring in some representative scale being the necessary condition. The condition for scale realization is the availability of group Φ of acceptable transformations of the straight line into itself. In other words, the theory of measuring is the theory of invariants relative to the groups of acceptable transformations. We would remind you that g function is called invariant if $g(x_1, x_2, ..., x_n) = g(\varphi(x_1), \varphi(x_2), ..., \varphi(x_n))$ is fulfilled for all x_i $\varphi \in \Phi$, where $i = 1,2,...,n$. Under the defenition the data is measured in group Φ scale if the sets $(x_1, x_2, ..., x_n)$ and $(\varphi(x_1), \varphi(x_2), ..., \varphi(x_n))$ bear the same information. It is possible to define a limited number of uninterrupted groups of transformation on the straight line. Particularly, the Mobius group and the projective group coinside on it. The latter circumstance allows to reduce the task of building the evolving processes scales to the task of finding the mixed invariants of the linear fractional group acting on the straight line $g = \dfrac{ax+b}{cx+d}$, $g^{-1} = \dfrac{dx-b}{a-cx}$. We build the solution by superposition of contingencies 1 and 2 to the type of invariants $z = I(t, a, b, c, ...)$, i.e. the invariants include:

1) The variable t and parameters a, b, c…
2) The function y(t) not subject to variations and derivatives y/, y//, …

In our case it is sufficient to add the three parameters a, b, c, and the following anharmonic relation occurs to be invariant:

$$z = I(t, a, b, c, ...)$$

$$z = \left.\frac{t-a}{t-b}\frac{c-a}{c-b}\right];$$

$$y, \frac{dy}{dz}, \frac{d^2y}{d^2z}, ...$$

$$y^{/} = \frac{dz}{dt}\frac{dy}{dz}, y^{//} = \frac{d^2z}{dt^2}\frac{dy}{dz} + \left(\frac{dz}{dt}\right)^2 \frac{d^2y}{dz^2}$$

$$y^{///} = \frac{d^3z}{dt^3}\frac{dy}{dz} + 3\frac{dy}{dt}\frac{d^2y}{dt^2}\frac{d^2y}{dz^2} + \left(\frac{dz}{dt}\right)^3 \frac{d^3y}{dz^3}$$

$$z = \lg(t-a) - \lg(t-b) + \lg\frac{t-b}{t-a}$$

$$\frac{dz}{dt} = \frac{1}{t-a} - \frac{1}{t-b} = \frac{a-b}{(t-a)(t-b)}$$

The invariant operator of the group will take on the form $\dfrac{(t-a)(t-b)}{a-b}\dfrac{d}{dt}$, then

$y_1 = \frac{(t-a)(t-b)}{a-b} y^{/}$ is the invariant of the the group containing the first derivative.

$y_2 = \frac{(t-a)^2 (t-b)^2}{(a-b)^2} y^{//} + \frac{(t-a)(t-b)(2t-a-b)}{(a-b)^2} y^{/}$ is the invariant with the second derivative.

Now the formation of the evolution scale will not cause any difficulties:

Let's designate $\mu = \mu_{исходная}$, then it is convenient to introduce the parameter of evolution description $\chi = \frac{\mu_{исходная}}{\mu_{текущее}}$. Such description has a drawback at $\mu_{текущее} = 0$ (there's no development) this proportion is difficult to interpret, so we will introduce the noncontradictory description parameter $\tilde{\chi} = 1 - \frac{1}{\chi}$. It can be shown that the natural requirement of evolution resourses limitation leads to the possibility of normalization of $\tilde{\chi} \in [0,1]$, but the scale itself remains the scale of intervals. In this scale the central moments (dispersion, etc.) have the objective sense, but the initial ones (average and suchlike) as well as the computing origin have the relative sense. Thus, speaking of variation (relative error) of measuring is just meaningless. It is easy to see that dimensionless equations and the analogs of proportion $\tilde{\chi}$ can be constructed for the wide class of monotone extensive parameters of the evolving systems.

In conclusion, we would like to advance an argument in favour of interval and group approach:

While measuring any quantities, even physical ones, we obtain the intervals containing the values of these quantities. Meanwhile, the quantity varies from measuring to measuring. The two interpretations are possible: a) There is only one exact value, all the rest are the measuring errors. b) Every value from the range of possible ones is considered like the true one, and the measuring errors are also possible, but they have the same order for the whole interval which is the basis for their exclusion. Interpretation b) is closer to Kolmogorov's point of view [8]. If the condition of invariance relative to transformation of the fixed group is taken as axiom, interpretation b) becomes the only one.

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