VOLUME OF INFORMATION IN VARIOUS TYPES OF QUANTUM STATES

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Information is an integral part of the Universe. The fundamental principle of A. Zeilinger [1] quantum mechanics postulates that the basic physical system (in particular, the fundamental particles: quark, electron, photon) contains (has) one bit of information. In physical essence, information is heterogeneity in distribution of matter and energy. Therefore information is indissolubly connected with matter and energy. The universal measure of information physical inhomogenuity, is information entropy by Shannon [2]. Another important characteristic of physical objects is information communication [2], which characterizes interacting of physical objects. The information approach, alongside with physical, allows to receive new, sometimes more general information, in relation to the information received on the basis of only physical laws. Works of the author, for example [3, 4], testify to the efficiency of use of information laws together with physical laws for cognition of the Universe. In the present work estimations of volumes of information in physical systems are given. The physical systems consist of: 1) noninteracting parts (qubits); 2) locally interacting (entangled) parts (qubits); 3) global interacting (entangled) parts (qubits). Parts (qubits) of the system interact in pairs. The entangled states represent physical objects which are of great importance in theoretical and experimental research of many questions of quantum calculations [5].

Let us estimate the volume of information in the system, consisting of n qubits. In the beginning we shall consider systems with equiprobable basic states.

1. Suppose that the system contains n of non interacting qubits. Let the qubit be described by the wave function $\psi = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$, where $|0\rangle, |1\rangle$ - the basic states of the qubit [5]. At measurements the basic states $|0\rangle$, $|1\rangle$ of the qubit will be received with equal probabilities $\frac{1}{2}$.

Uncertainty (information) of the qubit in the state $\psi = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$ is equal to 1 bit: $N_1 = I_1 = -(\frac{1}{2}\log_2\frac{1}{2} + \frac{1}{2}\log_2\frac{1}{2}) = 1.$

Hence, in the system containing n of non interacting qubits with equiprobable basic states the volume of information is proportional to the number of qubits and equasl n bits.

This estimation defines the minimal volume of information in the system of n qubits with equiprobable basic states. It also explains the linear dependence of information volume on mass or the number of particles (elementary systems) in usual substance (fundamental particles - quarks, leptons, photons). The main principle of quantum mechanics of A. Zeilinger [1] follows from it: «we, thus, offer the following principle of quantization of information: the elementary system transfers (contains) 1 bit of information ».

2. Let us consider a case when the system is set by wave functions $\psi = \frac{1}{\sqrt{n}}(\sum_{i=1}^{n}|e_i\rangle)$.

Uncertainty (information) of the system is equal to $N_i = -(\frac{1}{n}\log\frac{1}{n} + ... + \frac{1}{n}\log\frac{1}{n}) = \log_2 n$. If the system contains n objects then the volume of uncertainty (information) of the system is equal to

$N=\sum_{i=1}^{n} N_i = -n(\frac{1}{n}\log\frac{1}{n}+...+\frac{1}{n}\log\frac{1}{n}) = n\log_2 n$. The given dependence characterizes neutron stars and white dwarfs [6]. Neutron stars and white dwarfs are degenerate fermion systems filling the strip of n states $|e_i\rangle$.

3. Global interaction qubits in the system. Suppose that the system contains n of pairs of interacting qubits with equiprobable basic states. The system, consisting of interacting particles (qubits) could be described by the following wave function (suggested by A.D. Panov):

$$\psi_n^+ = \frac{1}{\sqrt{2}}(|0_1\rangle|0_2\rangle...|0_n\rangle + |1_1\rangle|1_2\rangle...|1_n\rangle)$$

Every qubit i has wave function $\psi_i = \frac{1}{\sqrt{2}}(|0_i\rangle + |1_i\rangle)$ ($|0_i\rangle, |1_i\rangle$-basic states of i-th qubit).

The mutual information of each pair of interacting qubits i, j is equal to one bit [7]. We shall show it. Bonded(confusing) state of qubits i, j, the state of qubit j is fully determined, if the state of qubit i is known, and, conversely, the state of qubit i is fully determined, if the state of qubit j is known.

For the state $\psi = \frac{1}{\sqrt{2}}(|0_i\rangle|0_j\rangle + |1_i\rangle|1_j\rangle)$ probabilities of realization of basic states are equal to $P(|0_i\rangle) = P(|1_i\rangle = \frac{1}{2}$, $P(|0_j\rangle)) = P(|1_j\rangle) = \frac{1}{2}$; probabilities of realization of states pairs are equal to $P(|0_i\rangle|0_j\rangle) = P(|1_i\rangle|1_j\rangle) = \frac{1}{2}$. Joint probabilities are defined by the matrix

$$\mathrm{P_{jon}} = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix}.$$

Uncertainty (information entropy) of qubits i, j is equal to 1 bit

$$N_i = N_j = -(\frac{1}{2}\log\frac{1}{2} + \frac{1}{2}\log\frac{1}{2}) = 1\text{(bit)}.$$

Uncertainty (information entropy) of joint probability distribution of qubits i states and j qubits is equal to $N_{ij} = -(\frac{1}{2}\log\frac{1}{2} + \frac{1}{2}\log\frac{1}{2}) = 1$(bit).

The mutual information of qubits i and j is equal to

$$I_{ij} = N_i + N_j - N_{ij} = 1 + 1 - 1 = 1 \text{ (bit)}.$$

The volume of mutual information in the system from n in pairs interacting qubits with equiprobable basic states, described by wave function $\psi_n = \frac{1}{\sqrt{2}}(|0_1\rangle|0_2\rangle...|0_n\rangle + |1_1\rangle|1_2\rangle...|1_n\rangle)$, is equal to $I_{n\,св} = \frac{n\cdot(n-1)}{2}$ bits.

The volume of information in the system from n in pairs interacting qubits, consists of n bits of information contained in qubits and $\frac{n\cdot(n-1)}{2}$ bits of mutual information between qubits. The full volume of information in the system containing in pairs interacting qubits with equiprobable basic states, is equal to $I_n = n + \frac{n\cdot(n-1)}{2} = \frac{n\cdot(n+1)}{2}$ bits.

The given estimation defines the maximal volume of information in the system from n qubits with equiprobable basic states. In the system containing in pairs interacting qubits with equiprobable basic states, the volume of information is proportional to the square of qubits number and is equal to

$I_n = \frac{n\cdot(n+1)}{2}$ bits.

When $n >> 1$ the volume of information in system consisting of n in pairs interacting qubits is equal to one second of a square of interacting qubits number $I_n \approx \frac{n^2}{2}$. It explains quadratic dependence of the volume of information on mass in black holes.

The system consisting of n in pairs interacting qubits with equiprobable basic states, contains more information on n bits, than the system consisting of $n-1$ qubits

$$I_n - I_{n-1} = \frac{n\cdot(n+1)}{2} - \frac{(n-1)\cdot n}{2} = n$$

4. Local interaction of qubits in the system. Let us consider a case when in the system from n qubits $\frac{n}{k}$ groups are allocated by k qubits and each of k qubits interacts only with their group of qubits (we consider, that n is divided by k). Then the volume of information in the group consisting of k in pairs interacting qubits with equiprobable basic states, is equal to $\frac{k\cdot(k+1)}{2}$ bits. Hence, considered system from n qubits contains

$I_{n/k} = \frac{n}{k}\frac{k\cdot(k+1)}{2} = \frac{n\cdot(k+1)}{2}$ bits.

This explains linear dependence of information volume on mass in compound particles of usual substance (for example, in elementary particles - mesons, baryons, and also atoms).

At $k=1$ the system contains the minimal volume of information: $I_{n/1} = n\frac{1\cdot(1+1)}{2} = n$.

At $k=n$ the system contains the maximal volume of information: $I_{n/n} = \frac{n}{n}\frac{n\cdot(n+1)}{2} = \frac{n\cdot(n+1)}{2}$.

5. In general, volume of information in the system consisting of n qubits with equiprobable basic states, is at least n bits and no more than $\frac{n\cdot(n+1)}{2}$ bits:

$$n \le I_n \le \frac{n\cdot(n+1)}{2}.$$

6. If we consider the system of n qubits with arbitrary probabilities realization of basic states $|0\rangle$, $|1\rangle$, a qubit is described by the wave function $\psi = a|0\rangle + b|1\rangle$. While measuring a qubit basic states of $|0\rangle$, $|1\rangle$ with probabilities $|a|^2$, $|b|^2$. would be received. The qubit's uncertainty (information) in the state ψ is equal to $N_1 = I_1 = -(|a|^2 \log_2|a|^2 + |b|^2 \log_2|b|^2)$.

In general, the volume of information in the system consisting of n qubits, is more or equals to zero of bits and no more than $\frac{n\cdot(n+1)}{2}$ bits: $0 \le I_n \le \frac{n\cdot(n+1)}{2}$.

7. Variations of system configuration of n qubits.
   The system of n qubits can have one of configurations:

7.1. A set consisting of noninteracting qubits

$$\Phi_1 = \{\psi_i\},\ \psi_i = \frac{1}{\sqrt{2}}\left(|0_i\rangle \pm |1_i\rangle\right).$$

7.2. A subset consisting of noninteracting qubits, and a subset of pairwise interacting qubits

$$\Phi_1 = \{\psi_i\} \cup \Phi_2 = \{\psi_{j_1 j_2}\},\ \psi_{j_1 j_2} = \frac{1}{\sqrt{2}}\left(|0_{j_1}\rangle|0_{j_2}\rangle \pm |1_{j_1}\rangle|1_{j_2}\rangle\right).$$

7.3. A set of qubits consisting of a subset of noninteracting qubits; and a subset of pairwise interacting qubits; and subsets of three qubits, interacting in pairs

$$\Phi_1 = \{\psi_i\} \cup \Phi_2 = \{\psi_{j_1 j_2}\} \cup \Phi_3 = \{\psi_{l_1 l_2 l_3}\}.\ \psi_{l_1 l_2 l_3} = \frac{1}{\sqrt{2}}\left(|0_{l_1}\rangle|0_{l_2}\rangle|0_{l_3}\rangle \pm |1_{l_1}\rangle|1_{l_2}\rangle|1_{l_3}\rangle\right).$$

7.4. A subset consisting of noninteracting qubits; and subset of pairwise interacting qubits; and subset of three interacting qubits, and a subsets of $i-1$ qubits, interacting in pairs; and subsets of i qubits, interacting in pairs

$$\Phi_1 = \{\psi_i\} \cup \Phi_2 = \{\psi_{j_1 j_2}\} \cup \Phi_3 = \{\psi_{k_1 k_2 k_3}\} \cup \ldots \cup \Phi_{i-1} = \{\psi_{s_1 s_2 \ldots s_{i-1}}\} \cup \Phi_i = \{\psi_{t_1 t_2 \ldots t_i}\}.$$

$$\psi_{s_1 s_2 \ldots s_{i-1}} = \frac{1}{\sqrt{2}}\left(|0_{s_1}\rangle|0_{s_2}\rangle\ldots|0_{s_{i-1}}\rangle \pm |1_{s_1}\rangle|1_{s_2}\rangle\ldots|1_{s_{i-1}}\rangle\right),\ \psi_{t_1 t_2 \ldots t_i} = \frac{1}{\sqrt{2}}\left(|0_{t_1}\rangle|0_{t_2}\rangle\ldots|0_{t_i}\rangle \pm |1_{t_1}\rangle|1_{t_2}\rangle\ldots|1_{t_i}\rangle\right)$$

7.5. .....

7.6. The system consists of n interacting qubits $K = \{\Phi_n \equiv \psi_{123\ldots i-1 i \ldots n}\}$.

8. The system which consists of k_1 noninteracting qubits, k_2 couples of interacting qubits, k_3 triples of interacting qubits, ... contains the following volume of information

$$I = k_1 \frac{1(1+1)}{2} + k_2 \frac{2(2+1)}{2} + k_3 \frac{3(3+1)}{2} + \ldots + k_i \frac{i(i+1)}{2} = \sum_{j=1}^{i} k_j \frac{j(j+1)}{2} \text{ bit.}$$

In the work [8] estimations of volume of information in the cosmological objects (usual substance, black holes, neutron stars, stars of the Sun type, ..., the Universe as a whole) are given. They are received with use of the results of the present work.

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