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ABOUT FINSLER EXTENSIONS OF RELATIVITY THEORY

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It is well-known, that the change of only one axiom in Euclidean geometry results in non-Euclidean geometry. Finsler geometry could be obtained in the same way. But convenient way to do this is not to use the systems of axioms of Euclid or Hilbert, but their modern analogue based on the concept of linear space with postulated scalar product on it.

Usually axioms of scalar product are written in the following way:

Scalar product axioms

1. $(A,B) \in R$
2. $(A,B) = (B,A)$
3. $(A,B+C) = (A,B) + (A,C)$
4. $(A,kB) = k(A,B)$

$$(A,A) = |A|^2 - \textit{fundamental quadratic form}$$

Alltogether they are equivalent to a bilinear symmetric form for components of two vectors. Depending on, whether the quadratic form associated with the particular scalar product will be positively definite or not, the resulting geometry will be either Euclidean or pseudo Euclidean.

If we define multilinear symmetric form of n vectors on the linear space rather than a bilinear form:

Generalization of the scalar product axioms for the polylinear Finsler spaces [1]

1. $(A,B,...,Z) \in R$
2. $(A,B,...,Z) = (B,A,...,Z) = ... = (Z,...,B,A)$
3. $(A,B+C,...,Z) = (A,B,...,Z) + (A,C,...,Z)$
4. $(A,kB,...,Z) = k(A,B,...,Z)$

$$(A,A,...,) = |A|^n - \textit{fundamental } n-\textit{ary form}$$

we can speak about flat finslerian or pseudo-finslerian rather than Euclidean or pseudo Euclidean. We can prove this by substituting the same vector into the particular multilinear form n times.

So if we change only one axiom and take multilinear form instead of only bilinear, we get quite a wide class of finslerian and pseudo-finslerian spaces with nonquadratic metric functions.

It should be emphasized, that the proposed approach to axiomatics of finslerian spaces differs in a substantial way from the standard formalism that is based on the results of Berwald, Taylor, Synge [2] and uses the finslerian metric tensor, which has two indices and depends not only on a point, but also on a direction. In our case it is replaced by an n-ary metric tensor, which depends only on a point. Probably we could speak not about finslerian, but about some other geometry with another name. But we prefer to keep the old terminology.

We can see best the advantages of this new formalism in the approach to finslerian spaces when considering the notion of angle. As it is well known, in the usual finslerian approach the notion of angle meets serious contradictions. In our case the contradictions do not arise, because we can derive the angle between two vectors A and B just from the multilinear form evaluated on the corresponding unit vectors. From this it becomes clear, that in the finslerian geometry two individual vectors should be characterized not by one, but by several angles This is quite natural because the spaces under consideration become nonisotropic.

Trilinear symmetrical form

$$(A,B,C)=\frac{1}{6}(a_1b_2c_3+a_1b_3c_2+\ldots+a_3b_2c_1)$$

$$\textit{length}\quad \lambda=\Psi(A,A,A)$$

$$\textit{angle}\quad \alpha=f_1(A',A',B')\qquad \lambda(A')=1\quad \lambda(B')=1$$

$$\beta=f_2(A',A',B')$$

However the advantages of the new axiomatics are not limited by the notion of angle. For example, just in the case of trilinear symmetric form we can introduce a new notion in addition to the notions of angle and length. We can call it tringle and it describes the mutual properties of the three vectors. In quadratic geometries tringles can't be considered, but, starting from qubic finslerian spaces, they are as natural as angles and lengths.

Trilinear symmetrical form

$$(A,B,C)=\frac{1}{6}(a_1b_2c_3+a_1b_3c_2+\ldots+a_3b_2c_1)$$

$$\textit{length}\quad \lambda=\Psi(A,A,A)$$

$$\textit{angle}\quad \alpha=f_1(A',A',B')$$

$$\beta=f_2(A',A',B')$$

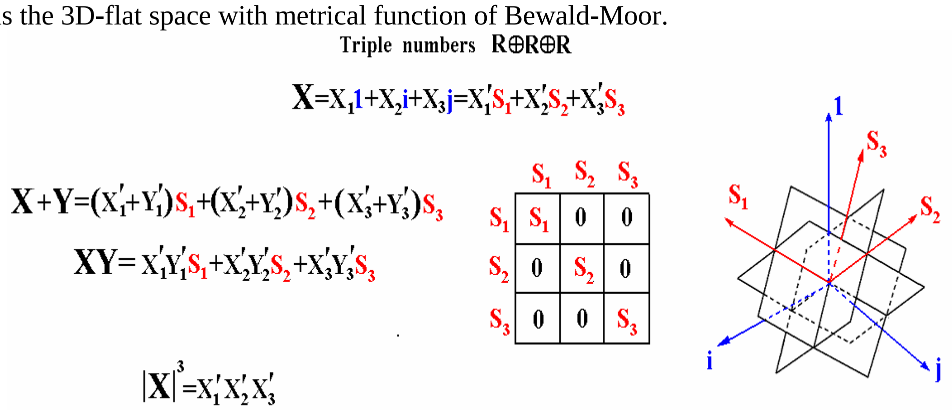
$$\lambda(A')=1\quad \lambda(B')=1$$

$$\textit{tringle}\quad \tau=\varphi(\alpha(A'',B'',C'')$$

$$\alpha(A'',B'')=0\quad \alpha(B'',C'')=0\quad \alpha(C'',A'')=0$$

It should be noted, that the idea of introduction tringles and further extensions was proposed by Peter Rashevsky in the article " Polymetric geometry ", *In the Proceeding of the Simenar on Vector and Tensor Analysis and its Applications to Geometry, Mechanics and Physics, Eds. V.F.Kagan, V, OGIZ, M, L, 1941, (in Russian).* When the notion of multiangles appears in the lexicon of geometers who deal with finslerian spaces, this automatically leads to extension of the fundamental continuous symmetries, because isometric and conform transformations are completed by tringle-invariant transformations etc. The most interesting is that these new symmetries will appear in the spaces of dimensions not higher than 4, and this is important for physics, because the existence of additional dimensions was not yet proved.

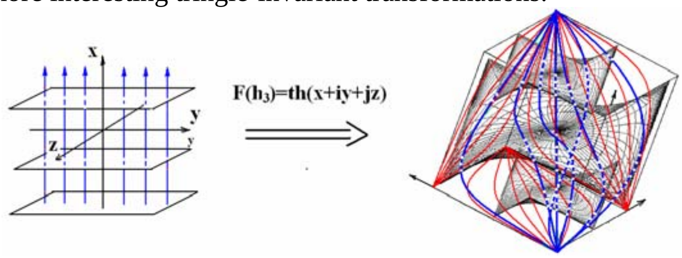
The simplest example of pseudofinslerian space that contains triples as well as lengths and angles is the 3D-flat space with metrical function of Bewald-Moor.



A commutative-associative algebra corresponds to this space, and elements of this algebra could be called triple numbers in the analogy with double numbers.

In this space as well as in the space of double numbers there is an infinite group of conformal transformations, and this represents a difference with the 3D-Euclidean and pseudo Euclidean spaces, where conformal transformations form only 10-parameters group.

On the next slide an example of an elementary conformal transformation is shown. This transformation is connected with an analytical function of hyperbolic tangent and it transforms the infinite linear space into the interior of the unit cube. Obviously these transformations are not trivial, and there are even more interesting tringle-invariant transformations.



Naturally, there exists a four-dimensional pseudofinslerian space with Bewald-Moor metrics.

Quadranumbers $R\oplus R\oplus R\oplus R$

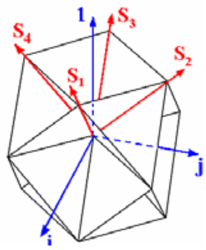
$$\mathbf{X}=X_1\mathbf{1}+X_2\mathbf{i}+X_3\mathbf{j}+X_4\mathbf{k}=X'_1S_1+X'_2S_2+X'_3S_3+X'_4S_4$$

$$\mathbf{X}+\mathbf{Y}=(X'_1+Y'_1)S_1+(X'_2+Y'_2)S_2+(X'_3+Y'_3)S_3+(X'_4+Y'_4)S_4$$

$$\mathbf{XY}=X'_1Y'_1S_1+X'_2Y'_2S_2+X'_3Y'_3S_3+X'_4Y'_4S_4$$

$$|\mathbf{X}|^4=X'_1X'_2X'_3X'_4$$

	1	i	j	k
1	1	i	j	k
i	i	1	k	j
j	j	k	1	i
k	k	j	i	1

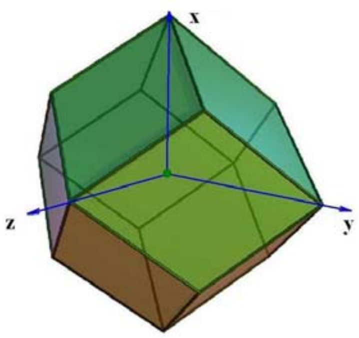


It corresponds also to commutative-associativa algebra, but now with four components. At the first look this algebra is similar to the quaternion algebra, but this is not true. The quaternion algebra isn't commutative and its conform group has only 15 parameters But the algebra of 4-numbers as well as the algebra of complex, double and triple numbers has infinite conform group. But the most important is that there are non-trivial symmetry groups with invariant tringles and their 4 vector generalizations.

Probably the best property of the space of 4-numbers is that it can be considered as effective generalization of Minkowski space.

Just now my colleague proved that the Lorenz group is a subgroup of the complexified conformal group of a space with Bewald-Moor metrics[3]. Also it seems that for compact conformal image of the 4D-space with the Berwald-Moor metrics, connected with analytical function hyperbolic tangent, there is an limit correspondence with the analogous space that is conformally connected with theMinkowski space. In this way perspectives open for a generalization of the theory of relativity from pseudoriemannian space to a psudofinslerian one. And here also opens the possibility to solve the deep philosophical problem of the general relativity theory, where conservation laws have only local nature. In the space-time with the Berwald-Moor metrics one can notice the translational invariance because this space has infinite symmetry groups. The translation invariance can be preserved and this will with the use of the Noether theorem result in global energy-impulse conservation.

One of the basic consequences of change from the Minkowski metric to the pseudofinslerian metric of Bewald-Moor is that at the observer's view the sphere is changed to the rombododecahedron.

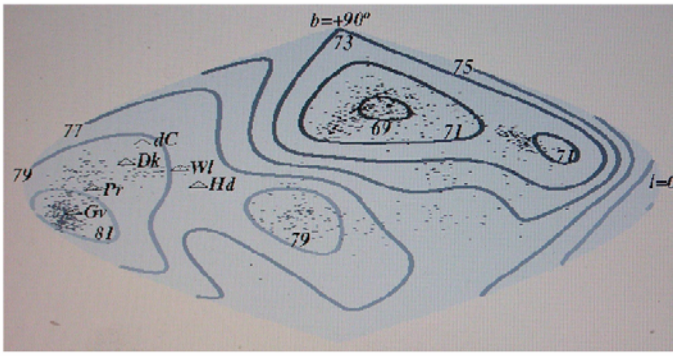


But the specific feature of this rombododecahedron reveals only at distances that are comparable with sizes of the universe under observation. At lesser sizes this polyhedron is smoothed and it can't be distinguished from the ordinary sphere.

The vertices of the rombododecahedron produces on the observer's view 14 special points. These points represent some kind of attractors, whose properties become observable only at cosmological distances.

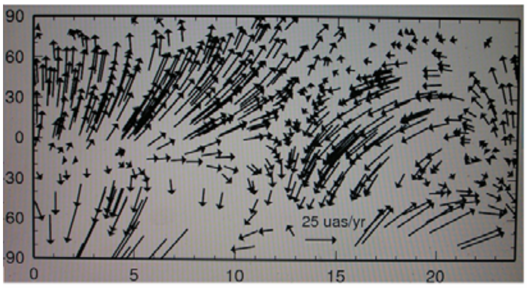
Our hypothesis about connection of the real world with pseudofinslerian geometry allows an experimental test. But it is possible only on large space-time intervals. One of the predictions of the finslerian geometry is the quadruple and octople anisotropy in the Hubble parameter. The observations of Canadian astrophysics show that there is at least a quadruple effect in distribution of the Hubble parameter.

M. L. McClure, C. C. Dyer, Anisotropy in the Hubble constant as observed in the HST Extragalactic Distance Scale Key Project results.
arXiv:astro-ph/0703556v1 21 Mar 2007

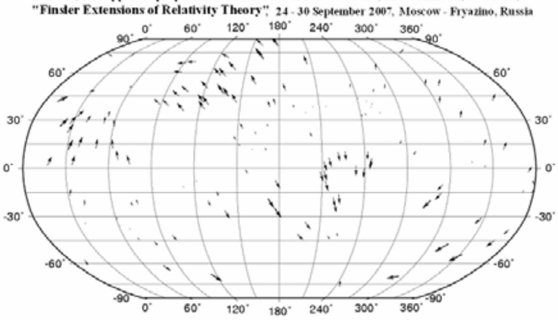


Another unusual prediction of the finslerian model is the quadruple and octople anisotropy in quasar parameters distribution. The results of McMillan from NASA and Oleg Titov from Australian observatory allows to fix a quadruple anisotropy in circular motion of quasars. Probably there exists also an octople component.

MacMillan D.S. Qasar Apparent Proper Motion Observed by Geodetic VLBI Netwerks. NVI, Inc., NASA Goddard Space Flight Center, Grenbelt. arXiv: astro-ph/0309826

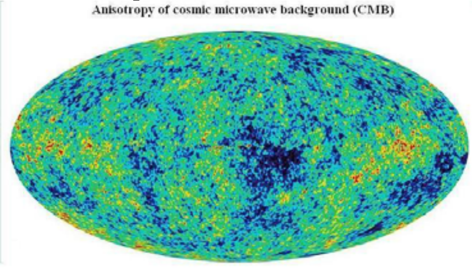


Titov O., The apparent proper motions of reference radio sources (GEOSCIENCE, Australia)
"Finsler Extensions of Relativity Theory", 24 - 30 September 2007, Moscow - Fryazino, Russia

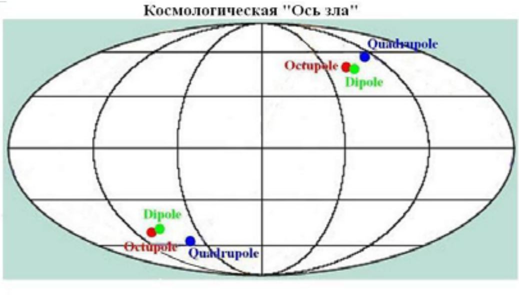


Probably the most beautiful prediction of the new geometrical model of the space-time is more complex Doppler-effect when observer moves in the relict radiation. While in pseudorimannian geometry Doppler-effect results in dipole kinematic part of anisotropy of relict background temperature, in finslerian case the quadruple and octople kinematic components should appear. The observations of NASA satellite WMAP evidence is the existence of the predicted correlation of axes of all lower multipoles.

Anisotropy of cosmic microwave background (CMB)



Космологическая "Ось зла"



In addition to this effect, that gets the name of "Evil Axe", one can predict also annual variations of amplitude and phase of all three multipoles, that are due to the motion of the Earth around the Sun. If the being prepared European "PLANK" satellite will observe such variations, the finslerian nature of our space-time will be proven.

References

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2. H. Rund: The Differential Geometry of Finsler spaces, Springer-Verlag, Berlin 1959.
3. G.I. Garas'ko, D. G. Pavlov, Hypercomplex Numbers in Geometry and Physics, 1, 9, 2008.