NUMERICAL ANALYSIS OF THE SOLUTION TO A PROBLEM OF OPTIMAL CONTROL BY TRANSIENT PROCESSES ON DIFFERENT CLASSES OF ADMISSIBLE CONTROLS AND AT DIFFERENT CONSTRAINTS OF TECHNOLOGICAL CHARACTER

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In this work numerical analysis of the solution to a problem of optimal control by transient processes in pipelines, arising at transfer from one steady-state regime to another, at different constraints of technological character put on control actions and on phase variables, is conducted. Admissible controls, which represent the values of water-flow rate on the ends of a pipeline, are considered both on the class of piecewise continuous and on the class of piecewise constant functions. The results of numerical solution to the optimal control problem, when the control is given on the class of continuous functions, are stated. Further we considered the results of the solution to the given problem for the case when the control is given on the class of piecewise constant functions, and the moments of time at which the switching of the rate occurs are also optimized.

Problem statement. Consider the process of oil transportation on linear part of a horizontal pipeline of the length l, diameter d and wall friction coefficient λ. The regime of fluid-flow is considered isothermal, laminar; oil is considered incompressible, having kinematic viscosity ν. There are pump houses at both ends of the pipeline, which provide the given regime of transit.

Unsteady laminar movement of the incompressible fluid with constant density for practical purposes is sufficiently adequately described by the following linear system of differential equations:

$$-\frac{\partial p}{\partial x}=\frac{\partial \omega}{\partial t}+2a\omega, \qquad -\frac{\partial p}{\partial t}=c^2\frac{\partial \omega}{\partial x} \qquad x\in(0,l), \qquad t\geq t_0, \tag{1}$$

where $p=p(x,t)$, $\omega=\omega(x,t)$ are the pressure and velocity of fluid movement correspondingly at the point of the pipeline $x\in(0,l)$ at the moment of time $t>t_0$, c is the velocity of sound in the environment, $\lambda -$is the coefficient of wall friction, $2a=\lambda\omega/2d={}^{32\nu}\!/_{d^2}$.

Suppose that till some definite moment of time t_0 we had a steady-state regime, determined by the conditions

$$\omega(x,t)=\omega_0=const, \quad p(x,t)=p_0(x), \qquad x\in[0,l], \quad t\leq t_0, \tag{2}$$

where the known function $p_0(x)$ at the given fluid-flow rate ω_0 is determined by geometrical dimensions of the pipeline and by properties of the fluid itself (oil).

Conditions (2) are provided due to the maintenance of the regime

$$\omega(0,t)=\omega(l,t)=\omega_0, \qquad t\leq t_0 \tag{3}$$

by pump houses. Suppose that as a result of requests to switch the pipeline to a new steady-state regime the following conditions should take place:

$$\omega(x,t)=\omega_T=const, \;\; p(x,t)=p_T(x), \qquad t\geq T, x\in[0,l], \tag{4}$$

where T is the time at which a new steady-state regime will begin proceeding. Necessary change of transit regimes in pipelines should be provided due to the changes in working regimes of pump houses, namely, due to the change in the velocity of the movement of raw materials o n the ends of the pipeline

$$\omega(0,t)=u_1(t), \qquad \omega(l,t)=u_2(t), \tag{5}$$

provided that the accomplishment of some technological and technical constraints takes place:

$$\underline{u}_1 \le u_1(t) \le \bar{u}_1, \qquad \underline{u}_2 \le u_2(t) \le \bar{u}_2, \quad t \ge t_0. \tag{6}$$

For the avoidance of hydraulic shock it is necessary to observe the following technological constraints on the process passing along the length of the pipeline and along the entire interval of control of the process:

$$p(x,t) \le \bar{p}, \qquad x \in [0,l], \quad t \ge t_0, \tag{7}$$

where $\bar{p}$ is the given maximum admissible value of the pressure depending on the characteristics of the materials of which the pipeline was made. One can transform constraint (7) into a constraint on maximum admissible value of the velocity $\bar{\omega}$:

$$\omega(x,t) \le \bar{\omega}, \qquad x \in [0,l], \quad t \ge t_0. \tag{8}$$

Take the following functional as a figure of merit of the conduction of transient process of raw material transit:

$$J(u,T) = T + \int_T^{T+\Delta T} \int_0^l \left\{ r_1 \left[p(x,t) - p_T(x) \right]^2 + r_2 \left[\omega(x,t) - \omega_T \right]^2 \right\} dx dt \to \min. \tag{9}$$

Here ΔT is the beforehand given length of time interval at which observance over the transit process and over the ascertainment of the presence of δ-steady-state regime is conducted. Thus the considered problem of control by transient process consists in determining the admissible values of the control $u(t) = (u_1(t), u_2(t))$ with respect to (6), and the time T of transient process, at which the solution to (1) satisfies technological constraints (8) and optimizes functional (9).

Problem transformation with the use of change of variables. Reduce the system of differential equations (1) to a hyperbolic equation of the second order. Enter unknown function $y(x,t)$ such that

$$\omega(x,t) = c^{-1}\, \partial y(x,t)/\partial t, \quad p(x,t) = (-c)\, \partial y(x,t)/\partial x, \; x \in (0,\ell), t \in (0,T]. \tag{10}$$

It is not difficult to check that (1) can be written in the form:

$$\frac{\partial^2 y}{\partial t^2} - c^2 \frac{\partial^2 y}{\partial x^2} + 2a \frac{\partial y}{\partial t} = 0, \quad x \in (0,\ell), t \in (0,T]. \tag{11}$$

Initial conditions (2) taking into account (10) can be reduced to the form:

$$\frac{\partial y(x,0)}{\partial t} = c\omega(x,0) = c\omega_0 = \varphi_0, \qquad x \in [0,l], \tag{12}$$

$$y(x,0) = -\frac{1}{c}\int_0^x p(s,0)ds = -\frac{1}{c}\int_0^x p_0(s)ds = \pi_0(x), \qquad x \in [0,l]. \tag{13}$$

Final conditions (4) are reduced to the form:

$$\frac{\partial y(x,T)}{\partial t} = c\omega(x,T) = c\omega_T = \varphi_T, \qquad x \in [0,l],$$

$$y(x,T) = -\frac{1}{c}\int_0^x p(s,T)ds = -\frac{1}{c}\int_0^x p_T(s)ds = \pi_T(x), \quad x \in [0,l].$$

Boundary conditions in this case are reduced to the form:

$$y(0,t) = c\int_0^t \omega(0,\tau)d\tau = c\int_0^t u(\tau)d\tau = v(t), \tag{14}$$

$$y(l,t) = c\int_0^t \omega(l,\tau)d\tau = c\int_0^t \omega_T d\tau = c\omega_T t + K = \mu(t), \quad t \in [0, T+\Delta T], \tag{15}$$

where once again entered function $v(t)$ can be considered as a control action with respect to system (11). The connection between the initial control $u(t)$ and a new one $v(t)$ is evident from (14):

$$u(t) = \frac{1}{c}\dot{v}(t), \qquad t \in [0, T + \Delta T]. \tag{16}$$

Functional (9) after substitution (1) takes on the form:

$$J(v,T) = T + \int_{T}^{T+\Delta T}\int_{0}^{l}(r_1\left[(-c)\frac{\partial y(x,t)}{\partial x} - p_T(x)\right]^2 + r_2\left[\frac{1}{c}\frac{\partial y(x,t)}{\partial t} - \omega_T\right]^2)dxdt\,, \tag{17}$$

and constraints (6), (8) are substituted by the inequalities:

$$\underline{u}t \le v(t) \le \overline{u}t, \qquad \frac{\partial y(x,t)}{\partial t} \le c\overline{\omega}, \quad x \in (0,l), \quad t \in [0, T + \Delta T]. \tag{18}$$

One can consider the problem (11)-(18) as a problem of optimal control on quick-action concerning the distributed system at the given values of phase coordinates at indefinite moment of process completion T, which is considered as an optimized parameter, and with a control at boundary conditions. What distinguishes the problem from the investigated ones similar to it is the structure of the target functional (17).

For solving the given problem one can apply two approaches. According to the former the problem with indefinite time of process completion can be substituted with a sequence of problems with fixed final time. In other words one can consider the final time T as an additional parameter and solve the sequence of similar optimization problems for different values of T. That value T from this series at which the figure of merit reaches minimum will be the solution to the problem with non-given final time. According to the latter T is considered as a control component, and for finding its optimal value a gradient procedure is applied. We used the first approach for solving the given problem.

For numerical solving the optimal control problem with fixed time some finite-difference approximation of the whole problem is used. The finite-dimensional problem of mathematical programming, with constraints in the form of inequalities, that arises as a result of approximation of the original continuous problem, can be solved applying first order methods, particularly, conjugate gradient method in combination with outer penalty function method for taking into account the first constraint of (18) on phase variables, and with gradient projection method for taking into account the second constraint of (18) on control actions.

Results of numerical experiments. While carrying out numerical experiments it was assumed that the length of the pipeline was $l = 1$, the velocity of sound in the environment was $c = 0.5$, friction coefficient $a = 0{,}015$ (all the initial data are given in dimensionless quantities), $h_x = 0.1$, $\tau = 0.05; 0.1$.

The dependence of the time of the process establishment from the change of the quantity of the interval of admissible velocities – boundary controls for the case of control on a set of continuous functions, and also the dependence of the time of the process establishment from the number of constancy intervals of the control and from constraints on admissible values of the control for the case of control on a set of piecewise constant functions, are depicted on the given diagrams.

On the diagrams represented in picture 1, the obtained optimal values of the boundary control by transient processes in the pipeline, arising at the transfer from initial regime with velocity $\omega_0 = 1$ to final regime with velocity $\omega_T = 3$, and realized at different constraints on control actions (control is considered on the class of continuous functions), are depicted as an example.

Optimal control realized at constraints (6), where $\underline{u_1} = 0.5$, $\overline{u_2} = 7$, is depicted in figure 1a, $\underline{u_1} = 0.3$, $\overline{u_2} = 5$ - in figure 1b, $\underline{u_1} = 0.3$, $\overline{u_2} = 4$ - in figure c, $\underline{u_1} = 0.7$, $\overline{u_2} = 3.3$ - in figure 1d.

Diagrams, on which obtained optimal values of the boundary control at transfer from the regime with the velocity $\omega_0 = 1$ to the regime with the velocity $\omega_T = 2$, and realized at different constraints on control actions (the control is considered on the class of piecewise constant functions) are depicted, are represented in figure 2. The optimal control realized at constraints (6), where $\underline{u}_1 = 0.5$, $\overline{u}_2 = 7$, is depicted in figure 2a, and, $\underline{u}_1 = 0.3$, $\overline{u}_2 = 5$ - in figure 2b.

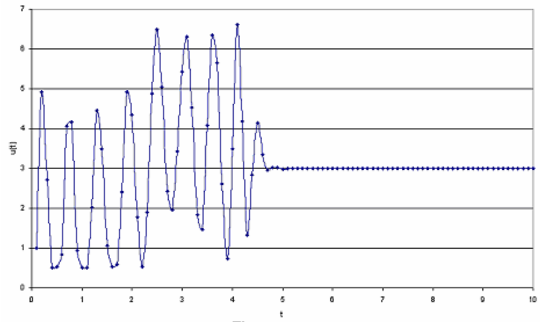


Fig.1.А

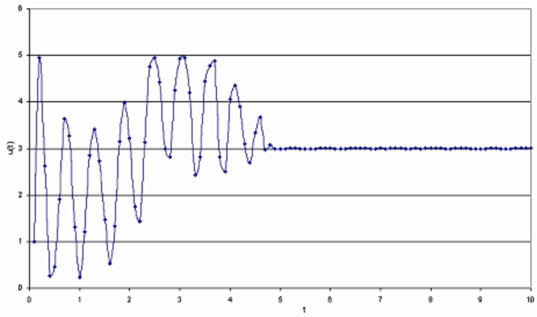
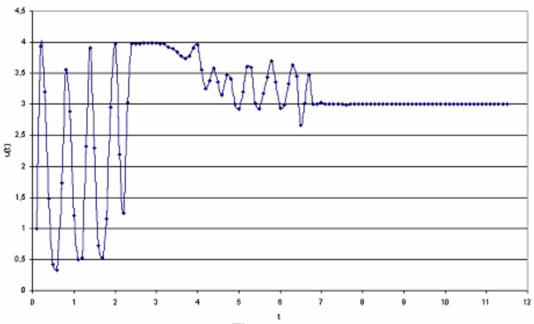


Fig.1.Б



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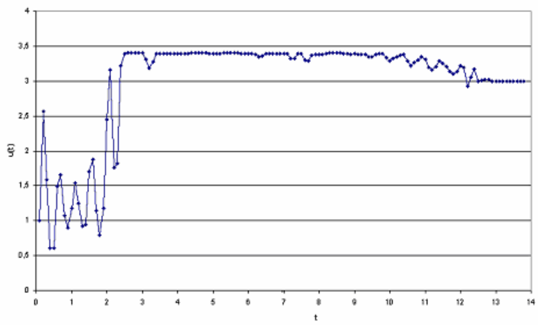

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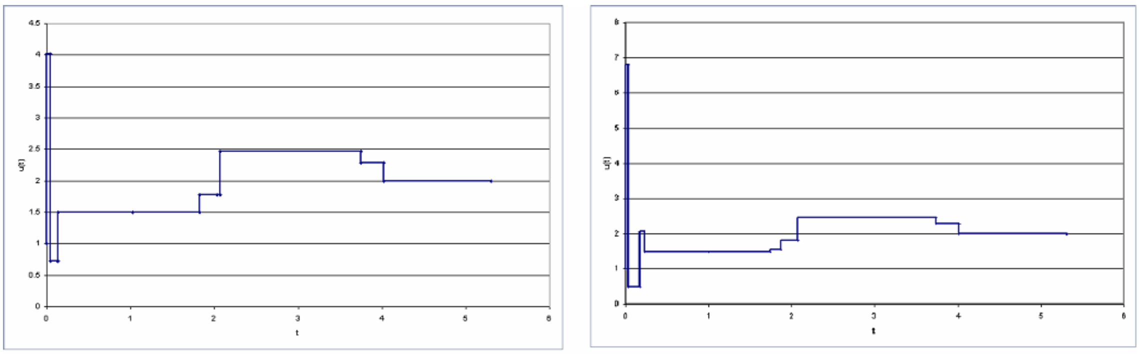

Fig. 2.

Literature

1. Charni I.A. Nonsteady movement of real liquid in the pipes. M., Nedra, 199 p., 1975.
2. F.P.Vasil'yev. Methods of solving extreme problems. Moscow, Nauka, 1981.
3. E.Polak. Numerical methods of optimization. Moskow, Mir, 1974. (in Russian)
4. A.E.Bryson, Yu-Chi Ho. Applied optimal control. Optimization, estimation and control. Waltham, Massachusetts, Toronto, London, 1969.
5. J.L. Lions. Optimal Control of Systems Governed by Partial Differential Equations, Springer, Berlin, 1971.