SOLUTION OF THE PROBLEM OF THE MIXED-INTEGER PROGRAMMING WITH ONE RESTRICTION

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Is considered the following problem:

$$\sum_{j=1}^{N} c_j x_j \to \max \tag{1}$$

$$\sum_{j=1}^{N} a_j x_j \le b \tag{2}$$

$$0 \le x_j \le d_j,\ j = \overline{1,N}, \tag{3}$$

$$x_j, d_j - \text{integer}\ \ j = \overline{1,n}\ \ (n \le N) \tag{4}$$

Where $c_j > 0, a_j > 0$, $d_j > 0$, ($j = \overline{1,N}$) and $b > 0$ are integer.

Problem (1) – (4) enters into the class of "intractable" problems and the elaboration of newer and more flexible methods has not only theoretical, but as well practical sense.

For the solution of this problem was elaborated algorithm, which includes the following stages:

1) The construction of approximate solution and finding upper and lower bounds of the optimum;
2) The determination of optimal value of the part of unknowns by using method [1];
3) The construction of equivalent problem with essentially narrow range of admissible solutions in comparison with the initial problem;
4) Solution of obtained problem on principle of recurring correlations of dynamic programming.

Let's mark out that elaborated method is the generalization of works [2] and [3] for the case of integer-valued variables $(d_j > 1,\ j = \overline{1,n})$.

Computing experiments have shown efficiency of the elaborated method.

Let f^*, $\underline{f}$ and $\overline{f}$ be optimal value of functional (1) – (4), lower and upper estimations accordingly. Without reducing commonness, it is supposed that unknowns have been regulated in the following sequence:

$$c_{j(i)} \big/ a_{j(i)} \ge c_{j(i+1)} \big/ a_{j(i+1)},\ \ j = \overline{1, N-1}$$

Then the optimal solution $\overline{X} = (\overline{x}_1, \overline{x}_2, ..., \overline{x}_n)$ of the continuous problem (1) – (3) is constructed analytically:

$$\overline{x}_{j(i)} = \begin{cases} d_{j(i)}, i = \overline{1, k-1}, \\ \left(b - \sum_{i=1}^{k-1} a_{j(i)} d_{j(i)}\right) \Big/ a_{j(k)}, i = k \\ 0, i = \overline{k+1, N}. \end{cases}$$

Here number of unknown k is determined from the conditions

$$\sum_{i=1}^{k-1} a_{j(i)} d_{j(i)} \le b < \sum_{i=1}^{k} a_{j(i)} d_{j(i)}.$$

Let's mark out that if $n < k \le N$ then the solution $\overline{X}$ is the solution of problem (1) – (4).

Analogously, obeying to the integer-value constraint it is easily constructed some approximate solutions $\underline{X} = (\underline{x_1}, \underline{x_2}, ..., \underline{x_n})$ of the problem (1) - (4). Thus,

$$\overline{f} = \sum_{j=1}^{N} c_j \overline{x_j}, \quad \underline{f} = \sum_{j=1}^{N} c_j \underline{x_j}.$$

It's obvious that $\underline{f} \le f^* \le \overline{f}$.

Let $s_{j(i)} = c_{j(i)} - \dfrac{c_{j(k)}}{a_{j(k)}} a_{j(i)}, \ i = \overline{1, N}.$

The following theorems have been proven

THEOREM 1. If for certain integer positive value p is satisfied condition

$$p\left|s_{j(i)}\right| > \overline{f} - \underline{f},$$

then for optimal solution $X^* = (x_1^*, x_2^*, ..., x_n^*)$ of the problem (1) – (4)

$$x_{j(i)}^* \ne \overline{x_{j(i)}} - p, \ i = \overline{1, k-1};$$
$$x_{j(i)}^* \ne \overline{x_{j(i)}} + p, \ i = \overline{k+1, N}.$$

Where $p \le d_{j(i)}$ and number $j(i)$ is fixed.

THEOREM 2. Let $h_{ji} = (\overline{f} - \underline{f}) \Big/ \left|s_{j(i)}\right|, \ i = \overline{1, N}$. Then coordinates of the optimal solution of the mixed-integer problem (1)-(4) is situated in the following interval:

$$\max\left\{0; d_{j(i)} - \left\lfloor h_{j(i)} \right\rfloor\right\} \le x_{j(i)} \le d_{j(i)}, \text{ at } i = \overline{1, k-1},$$
$$0 \le x_{j(i)} \le \min\left\{\left\lfloor h_{j(i)} \right\rfloor; d_{j(i)}\right\}, \text{ at } i = \overline{k+1, N}.$$

Where $[z]$ stands for integer part of number z.

It's obvious that if for some i $i=\overline{1,N}$, $h_{j(i)}<1$, then optimal value $x_{j(i)}^{*}=\overline{x_{j(i)}}$.

Hereby, by applying theorem 2 can be defined optimal values of unknowns of problem (1) – (4) and can be restricted interval of variation of variables. Considering this in problem (1) – (4) is obtained equivalent problem with less number of unknowns and with less length of intervals of variation of variables. Therefore, solution of received problem by the use of well–known methods (for example, by method of dynamic programming or by branch and bounds method) demands appreciably less quantity of operations.

Literature

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