METHODS FOR THE SOLUTION OF THE OPTIMAL STABILIZATION OF THE STATIONARY SYSTEM BY STATIC OUTPUT FEEDBACK

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In the present work an iterative method is offered for solution of the stabilization of the linear system with uncompleted information. This problem is considered in works [1-5], where in each step the Lyapunov's equation is solved. But here instead of this procedure we offer an iterative method.

Linear quadratic optimal output regulator problem with feedback low in stationary was considered [1-5] . In [2] convex programming apparatus is used and [4,5] adjoin gradient are applied . In this work Lyapunov equation is solved in each step witch can have negative influence to the accuracy of the solution. In the present work an iterative approach is offered instead of the solution of the Lyapunov equation.

Let the object's motion be described by the stationary system of finite – difference equations

$$x(i+1) = \Psi x(i) + \Gamma u(i), \qquad i = 0,1,2,..., \tag{1}$$

where functional is requirement to minimize

$$J = \sum_{i=0}^{\infty} \left(x'(i) Q x(i) + u'(i) R u(i) \right), \tag{2}$$

assuming the law of the feedback circuit

$$u(i) = Kx(i) \tag{3}$$

of the closed loop system (1),(3).

Here $x(i) - n -$dimensional vector of phase coordinates of the object, $u(i)$ is m – dimensional vector of controlling influence, Ψ, Γ, $Q = Q' \geq 0$, $R = R' > 0$, a constant matrices corresponding dimensions.

The solution of the problem (1)-(3) is reduced to solution of the following nonlinear system of the algebraic equations

$$K = -(R + \Gamma' S \Gamma)^{-1} \Gamma' S \Psi \quad , \tag{4}$$

where $S = S' > 0$ is solution of algebraic Riccati equation

$$S = \Psi' S \Psi - \Psi' S \Gamma (R + \Gamma' S \Gamma)^{-1} \Gamma' S \Psi + Q . \tag{5}$$

For the finding of the decision (5) there are different methods: a method of own vectors [6], method of Shura [7], a method of the signum functions [8]. One of effective methods is the iterative circuit in which it is proved convergence of the decision [9]

$$S_{i+1} = \Psi' S_i \Psi - \Psi' S_i \Gamma (R + \Gamma' S_i \Gamma)^{-1} \Gamma' S_i \Psi + Q \ , \tag{6}$$

$$K_i = -(R + \Gamma' S_i \Gamma)^{-1} \Gamma' S_i \Psi \ , \tag{7}$$

where at any entry conditions $S_0 > 0$ the iterative circuit converges.

Such iterative circuit facilitates a finding of decisions. Therefore it is meaningful to distribute this circuit for the decision of a problem of discrete optimum control on an output feedback.
It is observed the vector

$$y(i) = Cx(i)$$

$y(i) - r -$ a vector of the output (measurements) $x_0 -$ is a random vector with zero mathematical expectation and covariance matrix $X_0 =< x_0 x_0' >$.Here the symbol < > -means operator of the averaging. C a constant matrices.
The problem consists in determining of the controlling law with static output feedback

$$u(i) = Fy(i) = FCx(i)\,, \tag{8}$$

providing the asymptotical stability of the system (1,8) satisfying the condition of asymptotic stability system (1),(3). In the work [1], solution problem (1), (8), (2) is reduced to solution of the following nonlinear system of the algebraic equations

$$L = \left(\Psi + \Gamma FC\right)' L\left(\Psi + \Gamma FC\right) + Q + C'FRFC, \tag{9}$$

$$U = \left(\Psi + \Gamma FC\right) U \left(\Psi + \Gamma FC\right)' + X_0 \ , \tag{10}$$

$$F = -\left(R + \Gamma' L \Gamma\right)^{-1} \Gamma' L \Psi U C' \left(CUC'\right)^{-1} \tag{11}$$

It is known, that finding F it is necessary to solve the equations (9)- (11) For the decision of the equations (9)-(11) it is possible to offer the iterative algorithm where initial approximate solution F_0 should be chosen so that eigenvalues of the closed system $(\Psi + \Gamma\, F_0\, C)$ laid inside of individual circle. In this algorithm on each iteration Lyapunov's (9) (10) algebraic equations are solved .

$$F_i = -\left(R + \Gamma' L_i \Gamma\right)^{-1} \Gamma' L_i \Psi U_i C' \left(CU_i C'\right)^{-1} \quad , \tag{12}$$

$$L_i = \left(\Psi + \Gamma F_i C\right)' L_i \left(\Psi + \Gamma F_i C\right) + Q + C' F_i' R F_i C, \tag{13}$$

$$U_{i+1} = \left(\Psi + \Gamma F_i C\right) U_i \left(\Psi + \Gamma F_i C\right)' + X_0 \,, \tag{14}$$

Thus, for the decision of a problem (1), (8) (2) the following computing algorithm is offered.

Algorithm 1

Step1. We choose initial approach $L_0 > 0;\ U_0 > 0$ accordingly F_0 so that eigenvalues of a matrix $(\Psi + \Gamma\, F_0 C)$ laid inside an unit circle

Step 2. We calculate F_0 on (12)

Step 3. We calculate L_i, U_i on (13),(14)

Step 4. The condition $\left\| F_{i+1} - F_i \right\| < \varepsilon$ is checked If the condition is satisfied, procedure of calculation stops, differently we pass to a step 2.

Here $\|\cdot\|$ is norm of a matrix, ε is the set positive number.

Example. Matrixes $\Psi,\ \Gamma, C, Q, R$ appearing in (1), (2), (8) look like

$$\Psi = \begin{bmatrix} 2 & 1 & 0 \\ 0 & -0.1 & 1 \\ 0 & 0 & 3 \end{bmatrix}; \Gamma = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}; Q = \begin{bmatrix} 10 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 10 \end{bmatrix}; R = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}; C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

We choose initial approach $L_0 = I; U_0 = I$ where I - an unit matrix. With these data, solving a problem (1), (8), (2) it is received:

$$F = \begin{bmatrix} -1.74277688047887 & -0.37934272471665 \\ 0.0006658209882 & -2.8350876761572 \end{bmatrix},$$ value of functional is

$$J = 78.28046546698863.$$

Eigenvalue of matrix

$$\lambda(\Psi + \Gamma F C) = (0.27164254548986; 0.1431239705; -0.092663).$$

In work [2]

$$F = \begin{bmatrix} -1.9 & -0.137 \\ 0.00082 & -2.9 \end{bmatrix},$$ value of functional is $J = 79.344866$.

Comparison of these two results shows, that the offered algorithm improves result of work [2].

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