Application Saaty Pair Comparisons Method to the Investments Distribution in Parameters of Ecological Sustainability

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***Abstract*— Ecological sustainability - ability of ecological system to keep the properties and parameters of modes in conditions of operating internal and external indigations. Quite often ecological sustainability is considered as a synonym of stability. With increase of anthropogenesis influence to the environment the urgency of development of adequate systems of nature protection actions increases. During optimization of quality of an environment there is a problem of an estimation of ecology-economic efficiency of nature protection actions. Thus the last defensibility to consider from positions of investment projects (IP) and, accordingly, apply to them available approaches and analytical methods. On the basis of fuzzy model for an establishment of an index of ecological sustainability of an environment the technique definition of relative weight coefficients of its basic factors and them priority for the balanced financing is offered.**

Keywords*— *ecological sustainability; Saaty method; pair comparisons; weight coefficients

I. INTRODUCTION

The further development of modern manufacture depends on a condition nature-resource potential of the country and an environment as a whole. By forecasts of scientists ecological state constantly worsens. Rash and uncontrollable from the State economic activities can lead to irreversible consequences. Alongside with legal mechanisms of regulation the scientifically-proved methods by means of which experts - workers of Bodies of the State Executive Authority, could influence to the state of affairs in the field of environmental protection are necessary.

The integrated condition of an environment depends on quality of air, water, the land and biosphere. Definition weights of major factors of the environmental performance are a great importance not only at the analysis of an ecological situation, and also during management of investment processes in this sphere.

In 2005 by scientists Yale and Colombian Universities the Index of Environmental Sustainability for 146 countries of the world has been calculated [1]. By their results on the best ecological situation was Finland - on the 1-st place, and Azerbaijan - on the 99-th place.

In the paper [2] index ESI (Environmental Sustainability Index) is established by method of fuzzy logic inference for Azerbaijan, where have been taken into consideration four basic factors, influencing to the ESI: the Air Quality Index (AQI), the Water Quality Index (WQI), the Land Quality Index (LQI) and the Quality of the Biodiversity Index (EBI). The result is received in the form of fuzzy set.

In an ordinary life we often collide with cases when there are no elementary properties and attributes, which define concepts interesting us. Happens, difficultly to range a degree of property displays of the considered elements. As the membership degrees are considered on the given real set instead of in absolute sense, intensity of the membership can be defined from paired comparisons of considered elements.

Among indirect methods of definition of the membership function the greatest distribution was received with a method of pair comparisons Saaty. Complexity of use of this method consists in necessity of a finding of own vector of a matrix of pair comparisons, which is set by means of specially offered scale. And these complexities increase with growth of dimension of universal set, on which the linguistic term is set.

In the given paper the method using a matrix of pair comparisons of elements of set also is considered.

II. STATEMENT OF THE PROBLEM

On the basis of fuzzy interpretation of ESI index to establish relative weight coefficients of factors of ecological sustainability of regions and, that, to establish their priority scale for the balanced financing.

III. DEFINITION OF WEIGHT CHARACTERISTICS OF FACTORS OF ECOLOGICAL SUSTAINABILITY

Construction of the membership function of on the basis of pair comparisons is based on the assumption of the experts judgments about the importance of the criteria, influencing formation of property, described by fuzzy terms of the environmental sustainability states. Values (expert estimations) are set on nine-ball scale Saaty. It is formed as follows [3].

In our case (an estimation of ecological state ESI) there are 4 basic parameters: quality of air - AQI, waters - WQI, a biodiversity - EBI and the lands - LQI. The matrix is

constructing by relative estimations of ranks $\frac{r_i}{r_j} = a_{ij}\ (i, j = \overline{1.4})$. It possesses following properties:

1. diagonal;
2. It is back symmetric concerning diagonals; $a_{ij} = \frac{1}{a_{ji}}$
3. It is transitive, i.e. $S_{ik} \cdot S_{kj} = S_{ij}$.

TABLE I. THE VALUES SET BY SCALE SAATY

Numerical estimation (aij)	**Qualities estimations (comparison r_i and r_j)**
1	Absence of advantage r_i above r_j
3	Weak advantages r_i above r_j
5	Essential advantages r_i above r_j
7	Clear advantages r_i above r_j
9	Absolute advantages r_i above r_j
2,4,6,8	Intermediate comparative estimations

Presence of these properties leads to that at popularity of elements only one line, it is possible to fill all matrixes.

Let's admit in the first experiment for parameters experts following values (in the increasing order) ranks 2,3,6,7 accordingly are offered.

The matrix in this case will look as follows:

$$A = \begin{vmatrix} 1 & 2/3 & 2/6 & 2/7 \\ 3/2 & 1 & 3/6 & 3/7 \\ 6/2 & 6/3 & 1 & 6/7 \\ 7/2 & 7/3 & 7/6 & 1 \end{vmatrix} \qquad (1)$$

For a finding of own vector w and own value λ of a matrix (1).

$$\begin{aligned} &Aw = \lambda w, \\ &(A - \lambda E)w = 0, \end{aligned} \qquad (2)$$

it is necessary to calculate first of all a determinant of a matrix $(A - \lambda E)$, that is

$$|A - \lambda E| = \lambda^4 - 4\lambda^3 - 6.167\lambda^2 + 8.141\lambda - 1.328 = 0.$$

Applying iterative procedure for a finding of maximal own number $\lambda_{\max}$ and own vector, connected with it (W), we shall calculate values λ, using program Excel a graphic representation of a polynomial of IV degree. From the estimation λ maximal value it has appeared $\lambda'_{\max} = 4.95$. The stroke above λ specifies to the number of experiment. This estimation $\lambda'_{\max}$ satisfies to condition $\lambda' \geq n(n = 4)$, but it is not completely satisfies to a principle of a coordination, i. e. value for λ has turned out not so close to 4. The estimation $\lambda' = 4$ would be ideal.

Then we shall assume that in the second experiment ranks for criteria get out on - to other. Balls are distributed as follows:

1 ball - for quality of the land (LQI), 2 balls - for a parameter of quality of a biodiversity (EBI), 6 balls - to parameters of quality of air (AQI) and 7 balls – for parameter of quality of water (WQI).

The matrix of pair comparisons (matrix B) in the second experiment will look so:

$$B = \begin{pmatrix} 1 & 1/2 & 1/6 & 1/7 \\ 2 & 1 & 2/6 & 2/7 \\ 6 & 3 & 1 & 6/7 \\ 7 & 7/2 & 7/6 & 1 \end{pmatrix} \qquad (3)$$

Let's calculate own value for a matrix B.

Maintaining the integrity of the specifications

$$|B - \lambda E| = \begin{vmatrix} 1-\lambda & 1/2 & 1/6 & 1/7 \\ 2 & 1-\lambda & 2/6 & 2/7 \\ 6 & 3 & 1-\lambda & 6/7 \\ 7 & 7/2 & 7/6 & 1-\lambda \end{vmatrix} = \lambda^4 - 4\lambda^3 - 8 = 0. \qquad (4)$$

For this purpose we shall solve the equation (4) rather to λ''. We find for own value of a matrix B following estimations:

$$\begin{aligned} &\lambda''_1 = -1.776 \\ &\qquad\qquad \Rightarrow \qquad \lambda''_{\max} = 4.1148. \\ &\lambda''_2 = 4.1148 \end{aligned} \qquad (5)$$

This value λ'' specifies that a coordination of expert's judg ments higher as:

$$\lambda''_{\max} - 4 = 4.1148 - 4 = 0.1148 \qquad (6)$$

Let's find own vector $w = (w_1, w_2, w_3, w_4,)$ corresponding to $\lambda = 4.1148$.

Let's solve system of the linear equations

$$\begin{cases} (1 - 4.1148)w_1 + 1/2\, w_2 + 1/6\, w_3 + 1/7\, w_4 = 0, \\ 2w_1 + (1 - 4.1148)w_2 + 1/3\, w_3 + 2/7\, w_4 = 0, \\ 6w_1 + 3w_2 + (1 - 4.1148)w_3 + 6/7\, w_4 = 0, \\ 7w_1 + 7/2\, w_2 + 7/6\, w_3 + (1 - 4.1148)w_4 = 0. \end{cases} \qquad (7)$$

For this purpose we shall exclude from system of the linear equations the first equation, having added normalizing condition [4]. Then the system will accept a following kind:

$$\begin{cases} 2w_1 - 3.1148w_2 + 0.333w_3 + 0.2856w_4 = 0, \\ 6w_1 + 3w_2 - 3.1148w_3 + 0.8571w_4 = 0, \\ 7w_1 + 3.5w_2 + 1.167w_3 - 3.1148w_4 = 0, \\ \sum_{i=1}^{4} w_i = 1. \end{cases} \quad (8)$$

Solving system of the linear equations by Gauss method, we receive for own vector following values:

$$w_1(LQI) = 0.069,$$
$$w_2(EBI) = 0.124,$$
$$w_3(AQI) = 0.372,$$
$$w_4(WQI) = 0.434.$$

Own vector W, corresponding to $\lambda = 4.1148$ is equal:

$$W = (0.069; 0.124; 0.372; 0.434)$$

Values W are weight coefficients of ecological factors of quality of the land (*LQI*), qualities of a biodiversity (*EBI*), qualities of air (*AQI*) and qualities of water (*WQI*) accordingly.

Values of membership function of fuzzy set « Ecological condition » we shall result in a tabulated kind (tab.2) [5]:

TABLE II. VALUES OF MEMBERSHIP FUNCTION OF FUZZY SET «ECOLOGICAL STATE»

W	***LQI***	***EBI***	***AQI***	***WQI***
Subnormal fuzzy set	0.069	0.124	0.372	0.434
Normal fuzzy set	0.15	0.18	0.867	1

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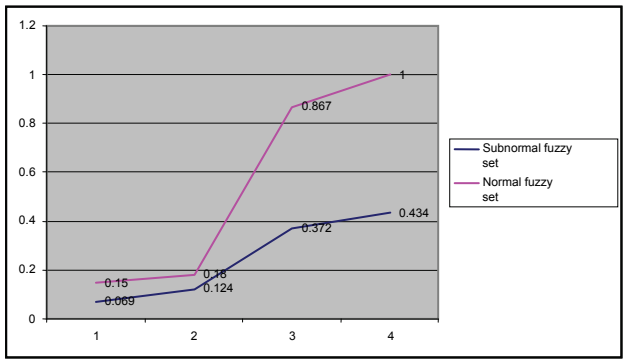

Figure 1. The schedule of the membership function of fuzzy set «Ecological state»

Thus, using the received results, we shall lead some comparisons with statistical data on ecology of Azerbaijan for 2000-2006-th years (tab.3).

Making division of capital investments under each factor on the general investments, we shall receive final distribution of the weights of ecological factors by years (tab.4).

TABLE III. EXPENSES FOR ECOLOGY IN THE AZERBAIJAN REPUBLIC (ONE THOUSAND AZN)

	2000	**2001**	**2002**	**2003**	**2004**	**2005**	**2006**
LQI	293.2	328.7	34.1	136.2	100.0	98.6	439.4
AQI	825.9	177.1	1426.1	879.1	204.8	1503.9	1601.9
WQI	603.6	765.3	944.7	1949.8	1706.5	1309.8	6838.1
EBI	301.4	371.0	321.3	434.8	398.4	1026.0	1544.4
TOTAL	2024.1	1624.1	2726.2	3399.9	2409.7	3938.3	10423.8

TABLE IV. WEIGHT DISTRIBUTION OF THE FACTORS OF ECOLOGICAL SUSTAINABILITY

	2000	**2001**	**2002**	**2003**	**2004**	**2005**	**2006**
LQI	0,1449	0,2002	0,0125	0,0401	0,0415	0,0250	0,0422
AQI	0,4080	0,1079	0,5231	0,2586	0,0850	0,3819	0,1537
WQI	0,2982	0,4660	0,3465	0,5735	0,7082	0,3326	0,6560
EBI	0,1489	0,2259	0,1179	0,1279	0,1653	0,2605	0,1482
TOTAL	1	1	1	1	1	1	1

IV. CONCLUSION

Apparently from the resulted calculations distribution of the weight coefficients of ecological sustainability factors by results of statistical data essentially differ from ours, as distribution of investments during 2000-2006-th was carried out by a residual principle. We offer the proved approach for an establishment of the balanced investment in improvement of ecology of the republic, leaning {basing} to adequate fuzzy models of ecological sustainability of an environment.

Together with it is necessary to note, that the offered technique of distribution of investments under basic papers of ecological safety is not optimal, since the base fuzzy model for calculation of an index of ecological stability of an environment is not completely adequate. For increase of a degree of reliability of distribution of the weight coefficients factors of ecological sustainability the given model is necessary for testing for an optimality of its output parameters.

Thus, it is necessary to note, that though the offered approach does not apply for optimality, but purely any it also cannot be named, since used fuzzy implicative rules [2] are characterized by the sufficiency and consistency.

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