Research of Synergetic Models of Non-Linear Economy

Valdemar Vitlinskyy[1], Yuriy Kolyada[2], Vitalii Tukalo[3]
V. Getman Kyiv National Economic University, Kyiv, Ukraine
[1]*wite101@meta.ua*, [2]*kolyadau@i.ua*, [3]*vinil87@i.ua*

***Abstract*— Indicated mathematical model structures describe dynamics of economical proceedings. Unity of approaches in receiving these models, as well as their quality and quantitativity analysis promote elaboration of adaptive strategy of computer nonlinear economical dynamics modeling, results of which help to adopt relevant administrative economical decisions.**

***Keywords*— synergetic models; dynamic structure; Lorentz model**

Number of synergetic mathematical models [1] are covered by the dynamic structure

$$\dot{x} = Ax + f(t,x), \tag{1}$$

where $x = x(t)$ and $x = \{x_1, x_2, x_3\}$ is a variables vector; $\dot{x} = dx/dt$ is derivative in time *t*; value $A\{a_{ij}\} =, i, j = 1,2,3$ is a constant matrix;

$$f(t,x) = \{f_1(t,x), f_2(t,x), f_3(t,x)\}$$

is a nonlinearities vector function. As an example, for

$$A = \begin{pmatrix} -\sigma & \sigma & 0 \\ r & -1 & 0 \\ 0 & 0 & -b \end{pmatrix}$$

and

$$f(.) = \begin{pmatrix} 0 \\ -xz \\ xy \end{pmatrix},$$

where σ =10; *b*=8/3 scalar parameter *r* takes different numerical values. The famous Lorenz model is derived from equation (1). It is also used in economic theory [2].

But the economy inherent drift of its components (elements), which suggests to consider the dynamic structure of the following form

$$\dot{x} = A(t)x + f(t,x) \tag{2}$$

and

$$\dot{x} = A(t,x)x + f(t,x). \tag{3}$$

Some or all elements of the matrix A may be functions of time or time and variables, but not just constants. Thus, almost all synergistic effects of behavior of solutions of mathematical models are described in this way, such as ceasing the process of formation of strange attractor in the classical Lorentz model [3].

For integrals of linear differential systems

$$\dot{x} = A(t)x \tag{2a}$$

obtained the inequality

$$\|x(t)\| \le n\|x(t_0)\| \exp\left\{\int_{t_0}^{t} \varphi(t)dt\right\},$$

where

$$\varphi(t) = \sum_{i=1}^{n}\sum_{j=1}^{n} |a_{ij}(t)| + \max \operatorname{Re} a_{ij}(t),$$

assuming the existence of a continuous matrix *A*(*t*) for

$$\forall t \in \Delta, \quad \Delta = [t_0, +\infty).$$

This majorant rating applies to the dynamic structure of the following form.

If there is a continuous matrix *A*(*t*) for the dynamic model

$$\dot{x} = A(t,x)x, \tag{3a}$$

such that its elements are performed throughout the range of inequality:

$$|a_{ij}(t,x)| \le n|a_{ij}(t)|, \; i \ne j$$

$$\operatorname{Re}|a_{ij}(t,x)| \le \operatorname{Re} a_{ij}(t), \; i,j = 1,2,\dots,n.$$

For nonlinear dynamic model

$$\dot{x} = F(t,x), \tag{4}$$

the right part of which is continuous together with its partial derivatives $\partial F_i(t,x)/\partial x_i$ and $F(t,0) = 0$ implements, there is continuous on Δ matrix $A(t)$ such that there are inequalities:

$$\left|\frac{\partial F_i(t,x)}{\partial x_j}\right| \le \left|a_{ij}(t)\right|,\ i \ne j$$

$$\operatorname{Re}\frac{\partial F_i(t,x)}{\partial x_j} \le \operatorname{Re} a_{ij}(t)\ .$$

The question of equivalence of mathematical models (2) and (2a), (3) and (3a) in terms of equivalent behavior of the solutions have been considered in [5].

Image of a nonlinear model (4) of the dynamic process as

$$\dot{x} = Ax + \Phi(t,x)$$

facilitated the development of effective procedures for numerical integration of stiff equations in mathematical modeling is the rule rather than exception. These ways of learning synergetic model classes (1) - (3) of the whole economy, promote adaptive mathematical modeling of nonlinear economic dynamics.

REFERENCES

1. Magnitsky N.T. The theory of dynamical chaos. - M., 2011, 320 p. (in Russian)
2. Zhang, V.B. Synergetic economics: Time and changes in nonlinear economic theory / V.B. Zhang. – M. Mir, 1999, 320 p. (in Russian)
3. Coordinate – wise considering self – restriction of variables in the Lorenz models / Vitalii Tukalo, Valdemar Vitlinskyy, Yuriy Kolyada// The Third International Conference "Problems of Cybernetics and Informatics", September 6-8, 2010, Baku, Azerbaijan. Section № 3 "Modelling and Identification", pp. 111-113.
4. Georghiu N. sur les integrals des systemes // Annalele stint ale llniver. Al. I Cuza, Sect. 1a, t. XI A, pp. 273-278, 1965.
5. Bylov B.F. The principle of linear starting for systems of differential equations / B.F. Bylov, D.M. Grobman // Successes in mathematical science, 1962, T. XVII, Vol. 3 (105), pp. 159-161.
6. Pavlov B.V. A method of numerical integration of ordinary differential equations / B.V. Pavlov, A. J. Povzner. // ZhVMMF, 1975, V.13, № 3, pp. 1056-1059.

[1] Kolyada Y.V. Adaptive modeling paradigm of economic dynamics: monograph / Y.V. Kolyada, K.: KNEU, 2011, 297 p.