Gradient Methods to Solve Integer Vector Optimization Problems

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***Abstract*— The multicriterion problem of integer optimization is examined. Developed and proved exact and approximate algorithms, which are the extensions of gradient methods for vector integer optimization problems, and in case of one-criteria optimization coincide with them.**

Keywords*— *vector optimization; gradient methods; integer optimization

Recent decades are characterized by rapid development of the theory of choice and decision making in the presence of many criteria. Vector (multi criteria) optimization problems are widely used as mathematical models of search variants and decisions in the economy, technology, social and in the other areas.

Let the real space F^{ℓ} partially ordered by a closed convex sharp cone K, $K^* = \left\{ y^* \in F^* : \left\langle y^*, y \right\rangle \geq 0 \; \forall y \in K \right\}$ – conjugate to K cone.

Consider the vector optimization problem:

$$\min\left\{ f(x) \mid x \in X \right\}, \qquad (1)$$

where $X \subseteq Z^n \subset R^n$, $X \neq \varnothing$, $|X| < \infty$, f – vector function, $f : E \to F$, i.e. the problem is to find such integer point $\bar{x} \in X$, that $f\left(X\right) \cap \left(f\left(\bar{x}\right) - K \setminus \{0\}\right) = \varnothing$. Denote by $E_K\left(f, X\right)$ a set of solutions of problem (1), and we will call elements of this set the Pareto optimal (efficient) solutions [1].

Let B – convex closed bounded set from R^n, $0 \notin B$. $K^* = \mathrm{con}(\mathrm{conv}\ B)$. Define a support function of set $B : \sigma_B \overset{def}{=} \max\limits_{z \in B}(y, z)$.

Developed and proved exact and approximate algorithms, which are the extensions of gradient methods [2, 3] for vector integer optimization problems, and in case of one-criteria optimization coincide with them.

Algorithm 1

1. Let $x_s \in X$ – s-th approximation of solution.
2. Solve the problem

$$\sigma_B\left(\nabla f\left(x_s\right)\left(x - x_s\right)\right) \to \min_{x \in X}$$

and find the solution $\bar{x}_s$.

3. Find a point $x_{s+1} \in O_\delta\left(\bar{x}_s\right) \cap X$.

4. If $\nabla f\left(\bar{x}_s\right)\left(x - \bar{x}_s\right) \notin -\mathrm{int}\, K \quad \forall x \in X$, then $\bar{x}_s$ – Pareto optimal solution of the problem. If not, then go to p.1 with $s = s + 1$.

Algorithm 2 is intended for the solving of vector problem of integer optimization (1) with a support cone method. The method consists of approximation initial non-linear problem by a sequence of linear optimization problems. Here are used an approximation of feasible set X with sequence of support cones $Q_0 \supset X, Q_1 \supset X, \ldots, Q_k \supset X, \ldots$, with set of vertices $x^0, x^1, \ldots, x^k, \ldots$, and for all points next conditions must be satisfied:

$$f(x^k) \subset \left\{ \min f(x) \mid x \in Q_k \right\}, \quad x^{k-1} \notin Q_k,\ k = 1, 2, \ldots$$

References

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