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Fuzzy Approach to Analysis and Forecast of Social Mobility

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***Abstract*— In the paper concepts of fuzzy time series and Markov’s fuzzy linguistic chain have been applied for analyze and forecasting social mobility.**

***Keywords*— social mobility, fuzzy time series, Markov’s fuzzy linguistic chain.**

I. INTRODUCTION

According to the definition proposed by the well-known American sociologist P. Sorokin, social mobility represents as “any transition of an individual or social object or value, anything that has been created or modified by human activity, from one social position to another” [1]. Vertical and horizontal mobility are the primary types of social mobility, while the determinant factors of the social mobility in the society are following: historical type of social stratification, condition and development index of the economy, social atmosphere in the country, ideology, traditions, religion, education, family, place of residence and individual characteristics of a person.

Social mobility is measured by means of two basic indicators: 1. Speed of mobility, i.e. number of steps that individuals were able to ascend, or had to descend; 2. Intensity of mobility, i.e. number of individuals that moved along the social ladder in the vertical direction during a certain time span.

Concepts of fuzzy time series and Markov’s fuzzy linguistic chain have been applied in the current paper in order to analyze and forecasting social mobility

II. ANALYSIS OF SOCIAL MOBILITY

In order to analyze social mobility we have developed a table 1 of distribution of population across economical strata in 2005-2010, on the basis of information on value poverty line (PL) in Azerbaijan [6] and grouping scale of population in accordance with income, proposed in [2].

As is shown in the table 1, population is divided into 5 economic strata in accordance with the income grouping scale:

- Absolutely poor – S_1
- Relatively poor – S_2
- Low-income – S_3
- Of moderate means – S_4
- Better off – S_5.

TABLE I. DISTRIBUTION OF POPULATION ACROSS ECONOMIC STRATA

Economic strata	2005		2008		2010	
	PL=42,6	Proportion of population	PL=78,6	Proportion of population	PL=98,7	Proportion of population
Absolutely poor – up to 0,5 PL	up to 21,6	0,001	up to 39,3	0	up to 49,35	0
Relatively poor 0,5 – 1 PL	21,6 – 42,6	0,270	39,3 – 78,6	0,15	49,35– 98,7	0,101
Low–income 1,0 – 2,0 PL	42,6 – 85,2	0,696	78,6 – 157,2	0,773	98,7–197,4	0,789
Of moderate means 2,0 – 3,0 PL	85,2– 127,8	0,027	157,2–165,8	0,050	197,4–296,1	0,089
Better off– more than 3,00 PL	127– greater	0,006	165,8– greater	0,027	296,15– greater	0,021

Analysis of 2005–2010 data on mobility scale indicates that portion of “absolutely poor” population group has been decreased to 0, portion of “relatively poor” population has decreased by 16.9 points, portions of population of “low income” and “of moderate means” have increased by 9.3 and 6.2 points respectively. At the same time if during 2005–2008 portion of “better off” population has increased by 2.1 points, during 2008–2010 its portion has decreased by 0.6 points.

In order to analyze speed of mobility lets consult graph provided on the Fig 1.

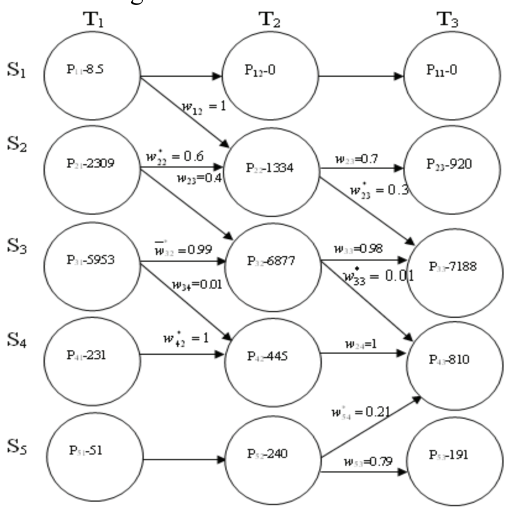

Fig.1. Social mobility speed graph

Baku, Azerbaijan

In the graph $S_i(i=1,...,5)$ denotes economic strata of population, $T_j(j=1,...,3)$ – years of analysis, $P_{ij}(i=1,....,5; j=1,....,3)$ – number of population in the economic strata in the corresponding year, w_{ij} и w_{ij}^* – portion of population moving to a different strata and staying in the same strata correspondingly.

As is shown in the graph, number of steps for which it was possible to ascend from S_1 to S_2, from S_2 to S_3, from S_3 to S_4 – is only one; in the meantime there is a case when portion of "better off" population S_5 descended to a lower strata S_4.

III. Forecasting Value of Poverty Line

In order to forecast value of poverty line, concept of fuzzy time series initially proposed by Q.Song and B.S.Chissorn [3,4], and further developed by a number of authors, has been applied. However the greatest contribution to the development of this concept was done by S.M.Chen [5].

In order to forecast value of poverty line (PL) according to Azerbaijan data we have used time series reflecting this indicator for 2001–2010 (tab. 2):

TABLE II. Value of poverty line

Years	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010
PL in AZN	24	35	35,8	38,8	42,6	58	64	78,6	89,5	98,7

In the first phase mean the average length of time series is determined (tab. 1):

$$AD(x_{11}......., x_n) = \frac{1}{n-1}\sum_{i=1}^{n-1}|x_i - x_{i+1}| = 8.3 \approx 8\,.$$

In the second phase $S = \frac{AD}{2} = 4$ is determined.

In the third phase universal set $U = [\overset{TS}{LS} - S; \overset{TS}{RS} + S] = [20;116]$ is determined.

In the universal set U according to AD and S following bases of the trapezoid are determined:

$A_1 = (20; 24; 32; 40)$;
$A_2 = (32; 40; 48; 56)$;
$A_3 = (48; 56; 64; 72)$;
$A_4 = (64; 72; 80; 88)$;
$A_5 = (80; 88; 96; 100)$;
$A_6 = (96; 104; 112; 116)$.

Applying trapezoidal membership function, degrees of membership have been determined.

On the following stage, applying membership functions mentioned above, degrees of membership of actual numbers of time series corresponding to linguistic variables A_i are calculated.

TABLE III. Degrees of membership of actual numbers

Year	PL	A_1	A_2	A_3	A_4	A_5	A_6	Fuzzy analogue	Forecast PL
2001	24,0	1	0	0	0	0	0	A_1	28
2002	35,0	0.63	0,37	0	0	0	0	A_1	33.9
2003	35,8	0.53	0,47	0	0	0	0	A_1	35.5
2004	38,8	0.15	0,85	0	0	0	0	A_2	41.6
2005	42,6	0	1	0	0	0	0	A_2	44
2006	58,0	0	0	1	0	0	0	A_3	60
2007	64,0	0	0	1	0	0	0	A_3	60
2008	78,6	0	0	0	1	0	0	A_4	76
2009	89,5	0	0	0	0	1	0	A_5	92
2010	98,7	0	0	0	0	0,66	0.34	A_5,A_6	97,44

Fuzzy analogues A_i are determined on the basis of maximum degree of membership values.

S.M.Chen determines fuzzy set as following universal discourse [5]:

$A_1 = 1/u_1+0.5/ u_2+0/u_3+0/u_4+0/u_5+0/u_6$;
$A_2 = 0.5/u_1+1/ u_2+0.5/u_3+0/u_4+0/u_5+0/u_6$;
$A_3 = 0/u_1+0.5/ u_2+1/u_3+0.5/u_4+0/u_5+0/u_6$;
$A_4 = 0/u_1+0/ u_2+0.5/u_3+1/u_4+0.5/u_5+0/u_6$;
$A_5 = 0/u_1+0/ u_2+0/u_3+0.5/u_4+1/u_5+0.5/u_6$;
$A_6 = 0/u_1+0/ u_2+0/u_3+0/u_4+0.5/u_5+1/u_6$;

On the basis of fuzzy analogues of data from table 2, following groups of fuzzy relations have been determined:

Group 1: $A_1 \rightarrow A_1$; Group 2: $A_1 \rightarrow A_2$;
Group 3: $A_2 \rightarrow A_2$; Group 4: $A_2 \rightarrow A_3$;
Group 5: $A_3 \rightarrow A_3$; Group 6: $A_3 \rightarrow A_4$;
Group 7: $A_4 \rightarrow A_5$; Group 8: $A_5 \rightarrow A_6$;

In order to represent fuzzy relations, lets apply fuzzy implication $\min(A_i A_j)$, where $i, j = \overline{1,6}$.

$$\underset{(A_1 \rightarrow A_1)}{R_1 =} \begin{Vmatrix} 1 & 0.5 & 0 & 0 & 0 & 0 \\ 0.5 & 0.5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{Vmatrix} \qquad \underset{(A_1 \rightarrow A_2)}{R_2 =} \begin{Vmatrix} 0.5 & 1 & 0.5 & 0 & 0 & 0 \\ 0.5 & 0.5 & 0.5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{Vmatrix}$$

$$\underset{(A_2 \rightarrow A_2)}{R_3 =} \begin{Vmatrix} 0.5 & 0.5 & 0.5 & 0 & 0 & 0 \\ 0.5 & 1 & 0.5 & 0 & 0 & 0 \\ 0.5 & 0.5 & 0.5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{Vmatrix} \qquad \underset{(A_2 \rightarrow A_3)}{R_4 =} \begin{Vmatrix} 0 & 0.5 & 0.5 & 0.5 & 0 & 0 \\ 0 & 0.5 & 1 & 0.5 & 0 & 0 \\ 0 & 0.5 & 0.5 & 0.5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{Vmatrix}$$

$$\underset{(A_3\to A_3)}{R_5=}\begin{Vmatrix}0&0&0&0&0&0\\0&0.5&0.5&0.5&0&0\\0&0.5&1&0.5&0&0\\0&0.5&0.5&0.5&0&0\\0&0&0&0&0.5&0\\0&0&0&0&0.5&1\end{Vmatrix}\qquad \underset{(A_3\to A_4)}{R_6=}\begin{Vmatrix}0&0&0&0&0&0\\0&0&0.5&0.5&0.5&0\\0&0&0.5&1&0.5&0\\0&0&0.5&0.5&0.5&0\\0&0&0&0&0&0\\0&0&0&0&0&0\end{Vmatrix}$$

$$\underset{(A_4\to A_5)}{R_7=}\begin{Vmatrix}0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0.5&0.5&0.5\\0&0&0&0.5&1&0.5\\0&0&0&0.5&0.5&0.5\\0&0&0&0&0&0\end{Vmatrix}\qquad \underset{(A_5\to A_6)}{R_8=}\begin{Vmatrix}0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0.5&0.5\\0&0&0&0&0.5&1\\0&0&0&0&0.5&0.5\end{Vmatrix}$$

Applying the union operator, as a result we obtain the resulting matrix $R=\cup R_k$:

$$R=\begin{Vmatrix}1&1&0.5&0&0&0\\0.5&1&1&0.5&0.5&0\\0.5&0.5&1&1&0.5&0.5\\0&0.5&0.5&0.5&1&0.5\\0&0&0&0&0.5&1\\0&0&0&0&0.5&0.5\end{Vmatrix}$$

As a further step, lets apply following model for fuzzy forecasting:

$$A_i=A_{i-1}\circ R\,,$$

where under the "$\circ$" operation lets choose operation maxmin i.e. $A_i=\cup(A_i\cap R)$.

For our prognosis:

$$A_{2011}=A_{2010}\circ R=(0000\,0.66\,0.34)\circ R=(0000\,0.34\,0.66)$$

Defuzzification operation for A_{2011} is determined by the following equation $y(t)=\dfrac{\sum_{i=1}^{n}\alpha_i\overline{A}_i}{\sum_{i=1}^{n}\alpha_i}$ and where $\overline{A}_t$ mean value of variable in *t*-year.

$$Y_{2011}=\frac{0.34\cdot 92+0.66\cdot 108}{0.34+0.66}=102.56$$

Calculated forecasted value (101.1 AZN) of the poverty line and the grouping scale of population allows us to evaluate parameters of the economic strata for 2011.

- Absolutely poor: 51.28
- Relatively poor: 51.28÷102.56
- Low–income: 102.56÷205.12
- Of moderate means: 205.12÷307.68
- Better off – more than 307.68.

In order to forecast of the share of population in each strata we will apply Markov's fuzzy linguistic chain.

III. Forecasting Social Mobility by Fuzzy Markov Linguistic Chain

As the process of social mobility inherently resembles theory of Markov's chains, we have used engines of this theory [7], [8], [9] in order to forecast social mobility.

For this purpose, first of all, we will fuzzificate specific share of population in different economic strata as linguistic variables: low L(0–0.3), average M(0.29–0.66), high (H)0.65–1. Fuzzy interpretation of these linguistic variables correlates to following membership functions:

$$\mu_1={}^{1}\!/_{0}+{}^{0.6}\!/_{0.1}+{}^{0.4}\!/_{0.2}+{}^{0}\!/_{0.3}$$

$$\mu_M={}^{0}\!/_{0.29}+{}^{0.5}\!/_{0.4}+{}^{1}\!/_{0.5}+{}^{0}\!/_{0.66}$$

$$\mu_H={}^{0}\!/_{0.65}+{}^{0.25}\!/_{0.7}+{}^{1}\!/_{0.85}+{}^{0}\!/_{1}\,.$$

Vector of the current (2010) condition of economic stratas: $S_{2010}=(0,0.101,0.789,0.089,0.021)$, corresponds to the linguistic vector (L, L, H, L, L).

Discrete matrix mobility of conditions of economic strata to another strata according go the analysis (fig. 1) can be represented as follows:

$$T_{2011}=\begin{array}{c|ccccc|}&S_1&S_2&S_3&S_4&S_5\\\hline S_1&0&1&0&0&0\\S_2&0&0.8&0.2&0&0\\S_3&0&0&0.8&0.2&0\\S_4&0&0&0&0.8&0.2\\S_5&0&0&0&0.2&0.8\end{array}$$

Discrete matrix of mobility can be represented by means of linguistic variables as follows:

$$T_{2011}=\begin{Vmatrix}L&H&L&L&L\\L&H&L&L&L\\L&L&H&L&L\\L&L&L&H&L\\L&L&L&L&H\end{Vmatrix}$$

For forecasting of conditions of economic strata in 2011 Markov's fuzzy chain has been applied: $S_{2011}=S_{2010}\circ T_{2011}$, where element S_i, is calculated via the following equation:

$$S_i=\underset{n}{\cup}(S_i\wedge t_j)$$

Fuzzy logical vector is forecasted by means for equation S_i:

$$S_{2011}=(L,L,H,L,L)\,,$$

where elements of this vector are determined as follows:

$$S_1=\cup\left[(L\wedge L),(L\wedge L),(H\wedge L),(L\wedge L),(L\wedge L),\right]=L$$
$$S_2=\cup\left[(L\wedge H),(L\wedge H),(H\wedge L),(L\wedge L),(L\wedge L),\right]=L$$
$$S_3=\cup\left[(L\wedge L),(L\wedge L),(H\wedge H),(L\wedge L),(L\wedge L),\right]=H$$
$$S_4=\cup\left[(L\wedge L),(L\wedge L),(H\wedge L),(L\wedge H),(L\wedge L),\right]=L$$
$$S_5=\cup\left[(L\wedge L),(L\wedge L),(L\wedge L),(L\wedge L),(L\wedge H),\right]=L$$

As is obvious from the results obtained, vector of conditions of economic strata repeats conditions of year 2010. This is mainly, connected to the fact that the mobility matrix covers situation for 2005–2010.

Let's assume that individuals, who make decisions in field of social policy, have decided to improve its conditions. In this case mobility matrix will take following shape:

$$T_{2011}^{opt.} = \begin{Vmatrix} L & H & L & L & L \\ L & M & M & L & L \\ L & L & M & M & L \\ L & L & L & M & H \\ L & L & L & L & H \end{Vmatrix}$$

And the state vector: $S_{2011}^{opt} = (L, L, M, M, M)$

IV. CONCLUSION

Results of the research of social mobility by means of fuzzy logic instruments enable decision makers in field of socioeconomic systems management to analyze situation in this sphere more thoroughly and make corrections of the direction in light of the occurring changes. Further research proposes elaboration of the system of fuzzy models considering state of economic and finance system.

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