# Mathematical and Computer Modelling of Adaptive Systems Identification and Control with Power Objects on the Basis of Wiener-Hammerstein Model

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***Abstract*— In this article problems of mathematical and computer modeling of adaptive systems identification and control in the class of Wiener-Hammerstein models on the basis of personal computers for such nonlinear stochastic dynamic power objects are considered. The problem of identification is reduced to minimization of criterion of quality identification on parameters where the optimal two-stage recurrent algorithm of identification - on the basis of the average method of the least squares is applied. The model of Wiener-Hammerstein constructed in this way is applied in the closed contour of identification and control at creation Ad SIC to track active power desirable value by means of TSRIA, AMLS and adaptive algorithm of control - AdAC. The effective functioning of constructed Ad SIC is checked on the basis of statistics.**

***Keywords*— *modeling; identification; adaptive control: tracking; power objects***

## I. Introduction

In this article problems of mathematical and computer modeling **of adaptive systems of identification and control (Ad SIC) class of Wiener-Hammerstein models on the basis of personal computers (PC)** for such nonlinear stochastic dynamic objects which are Electric furnaces (EF) manufactures of ferroalloys, EF manufactures of super pure metals, the synchronous generator of a heating plant (SGHP) manufactures of the electric power, a part of large-scale power complexes are considered. On the basis of output-input statistics « active power - force of current », taken while normal functioning of the specified objects, Wiener-Hammerstein nonlinear stochastic dynamic model is constructed which is in the class of the block-oriented models [1, 2, 3]. The structure of Wiener-Hammerstein model is represented as a consecutive connection of two linear dynamic blocks between which as a nonlinear static element is square-law function (structural identification). After defining the structure of model in the opened contour of control, the problem of identification is reduced to minimization of criterion of quality of identification on parameters where where as the algorithm of identification, which is similar [4, 5], the optimum **two-stage recurrent algorithm of identification - TSRIA** on the basis of **the average method of the least squares - AMLS** (parametrical identification)] is applied. The Wiener-Hammerstein model constructed in this way is applied in the closed contour of identification and control at creation Ad SIC to tracking active power desirable value by means of TSRIA, AMLS [4, 5], and **adaptive algorithm of control - AdAC** [6]. It is checked on concrete to statistics the effective functioning of constructed Ad SIC.

The power objects are considered as control object in the conditions in which normal functioning on an input is observed the operating process force of a current $X(t) \in R^1$, and on an output - $Y(t) \in R^1$ operated process active power and are assumed to be stationary and permanently connected in dispersive sense aligned ergodique processes which are certain on likelihood space $(\Omega, F, P)$.

During the discrete moments of time $n = 1, 2, \cdots, N, \cdots$ input/output data $\{x_n, y_n\}$ is measured. Sequence

$$\{x_n, y_n\}_{n=1}^{\infty} \tag{1}$$

represents infinite sample of statistically independent supervision casual $X(t)$ and $Y(t)$ processes.

On the basis of realization (1) the model in which by way of consecutive connection following subsystems are included:

1. The linear dynamic block of Wiener model

$$\begin{gathered} y_{1,k} = \sum_{i=1}^{m} g(i) y_{k-i} + \sum_{i=1}^{l} h(i) x_{k+1-i} = \\ = g^T \mathbf{y}(k-1) + h^T \mathbf{x}(k), \quad k = l+1, l+2, \cdots, N, \\ \mathbf{y}(k-1) = [y_{k-1}, \cdots, y_{k-m}]^T \in R^m, \\ \mathbf{x}(k) = [x_k, \cdots, x_{k-l+1}]^T \in R^l, \end{gathered} \tag{2}$$

where $g = [g(1), \cdots, g(m)]^T \in R^m, h = [h(1), \cdots, h(l)]^T \in R^l$ - is a vector of unknown weight factors of the linear block of model of Wiener.

2. The nonlinear static element of Wiener model is presented as square function

$$y_{2,k}=y_{1,k}^2=[\sum_{i=1}^{m} g(i)y_{k-i}+\sum_{i=1}^{l} h(i)x_{k+1-i}]^2= \\ =\sum_{i=1}^{m}\sum_{j=1}^{m} g(i)g(j)y_{k-i}y_{k-j}+ \\ +\sum_{i=1}^{m}\sum_{j=1}^{l} g(i)h(j)y_{k-i}x_{k+1-j}+ \\ +\sum_{i=1}^{l}\sum_{j=1}^{l} h(i)h(j)x_{k+1-i}x_{k+1-j}. \tag{3}$$

Finally, Wiener model becomes

$$y_{2,k}=y_{1,k}^2=z_k^T\theta,\quad k=l+1,l+2,\cdots,N,\cdots, \tag{4}$$

$$z_k=[z_{1,k}^T;z_{2,k}^T;z_{3,k}^T]^T\in R^d, d=mm+ml+ll, \tag{5}$$

$$z_{1,k}=\mathbf{y}(k-1)\mathbf{y}^T(k-1)= \\ \begin{bmatrix} y_{k-1}y_{k-1},\cdots,y_{k-1}y_{k-m} \\ \vdots \\ y_{k-m}y_{k-1},\cdots,y_{k-m}y_{k-m}\end{bmatrix}= \\ =[z_1(k),z_2(k),\cdots,z_m(k); \\ z_{m+1}(k),\cdots,z_{2m}(k);\cdots,z_{mm}(k)]^T\in R^{mm}, \tag{6}$$

$$z_{2,k}=\mathbf{y}(k-1)\mathbf{x}^T(k)= \\ =\begin{bmatrix} y_{k-1}x_k,\cdots,y_{k-1}x_{k-l+1} \\ \vdots \\ y_{k-m}x_k,\cdots,y_{k-m}x_{k-l+1}\end{bmatrix}= \\ =[z_{mm+1}(k),\cdots,z_{mm+l}(k); \\ z_{mm+l+1}(k),\cdots,z_{mm+ml}(k)]^T\in R^{ml}, \tag{7}$$

$$z_{3,k}=\mathbf{x}(k)\mathbf{x}^T(k)= \\ =\begin{bmatrix} x_kx_k,\cdots,x_kx_{k-l+1} \\ \vdots \\ x_{k-l+1}x_k,\cdots,x_{k-l+1}x_{k-l+1}\end{bmatrix}= \\ =[z_{mm+ml+1}(k),\cdots,z_{mm+ml+l}(k); \\ \cdots,z_{mm+ml+ll}(k)]^T\in R^{ll}, \tag{8}$$

$$\theta=[(\theta^{(1)})^T;(\theta^{(2)})^T;(\theta^{(3)})^T]^T\in R^d, \tag{9}$$

$$\theta^{(1)}=\begin{bmatrix} g(1)g(1),\cdots,g(1)g(m) \\ \vdots \\ g(m)g(1),\cdots,g(m)g(m)\end{bmatrix}= \\ =[\theta_1,\cdots,\theta_m,\theta_{m+1}\cdots\theta_{2m},\cdots, \\ \theta_{(m-1)m+1},\cdots,\theta_{mm}]^T\in R^{mm}, \tag{10}$$

$$\theta^{(2)}=\begin{bmatrix} g(1)h(1),\cdots,g(1)h(l) \\ \vdots \\ g(m)h(1),\cdots,g(m)h(l)\end{bmatrix}= \\ =[\theta_{mm+1},\cdots,\theta_{mm+l},\theta_{mm+l+1},\cdots, \\ \theta_{mm+2l},\cdots,\theta_{mm+ml}]^T\in R^{ml}, \tag{11}$$

$$\theta^{(3)}=\begin{bmatrix} h(1)h(1),\cdots,h(1)h(l) \\ \vdots \\ h(l)h(1),\cdots,h(l)h(l)\end{bmatrix}= \\ [\theta_{mm+ml+1},\cdots,\theta_{mm+ml+l},\theta_{mm+ll+l+1}, \\ \cdots,\theta_{mm+ml+ll}]^T\in R^{ll}. \tag{12}$$

Coordinates of a vector (9) - (12) represent product of components of vectors $g=[g(1),\cdots,g(m)]^T\in R^m$], $h=[h(1),\cdots,h(l)]^T\in R^l$] and on themselves, and represent values of unknown weight factors of each model input (4) and are required to be defined.

3. As a nonlinear static element of Hammerstein model is represented by Wiener's model under the formula (4).

4. The linear block Hammerstein model looks like

$$y_{3,n}=\sum_{i=1}^{\kappa} a_iy_{2,n-i}=\sum_{i=1}^{\kappa} a_i\theta^Tz_{n-i}=Z^T(n)w, \\ n=l+\kappa+1,l+\kappa+2+,\cdots,N,\cdots, \\ w=[w_1,\cdots,w_\kappa]^T\in R^{\kappa d}=[(a_1\theta)^T,\cdots,(a_\kappa\theta)^T]^T\in R^{\kappa d}, \\ Z(n)=[z_{n-1}^T,\cdots,z_{n-\kappa}^T]^T\in R^{\kappa d}, a=[a_1,\cdots,a_\kappa]\in R^\kappa. \tag{13}$$

Finally, Wiener-Hammerstein model will have the following form

$$\hat{y}_n=Z^T(n)w+\eta(n),\ n=N_1+1,N_1+2,\cdots,N,\cdots, \tag{14}$$

where $N_1$ that number or a step of iteration on which process of identification in the open contour of control has come to the end. In system (14) $\eta_k$ - a handicap on an output a zero average, final dispersion and not correlated with input, and output.

## II. MODELING IN THE CLOSED CONTUR

**Wiener-Hammerstein advancing model** is considered **on** $p$ **step** [6] as a model for the closed contour it in the form

$$\tilde{y}_{n+p}=Z^T(n+p)w+\beta u_n+\rho_{n+p}, \\ n=N_1+l+\kappa+1,\cdots,\rho_{n+p}=\sum_{k=1}^{p-1} c_k\eta_{n+p-k}, c_0=1, \tag{15}$$

where $Z(n)$ - the entrance vector of model (15), its components are defined with the formula (5); $u_n$ - an operating input at which the force of a current on each phase of electric objects acts; $\beta$ - factor of control; $\rho_{n+p}$ - a handicap sliding in average, are defined similarly from [6].

Lets assume that the sequence $y_n^*$ of the limited random variables of active power of power objects desirable value and $y_{n+p}^*$ $F_n$ can be measured. We will define **the criterion of control for tracking** in the form

$$J(u_n) = M\{\left|\tilde{y}_{n+p} - y^*_{n+p}\right|^2 \mid F_n\} + \lambda u_n^2,\ \lambda \ge 0;$$
$$n = N_1 + l + 1, N_1 + l + 2, \cdots. \tag{16}$$

The Problem of adaptive control in this case consists in defining such optimal control, $u_n^*$ which satisfies the condition

$$u_n^* = \arg\min_{u_n \in R^1} J(u_n) \tag{17}$$

From a condition of a minimum (17) the formula of optimum control becomes

$$u_n^* = (\beta^2 + \lambda)^{-1}\beta[y^*_{n+p} - Z^T(n+p)w],$$
$$n = N_1 + l + 1, N_1 + l + 2, \cdots \tag{18}$$

Also is **on** $p$ **step advancing optimal control** [6].

For definition $w$ it is necessary to solve a problem of parametrical identification in the closed contour of control. With this purpose TSRIA is used where the first stage is realized on the basis of AMLS which can be presented as **on** $p$ **a step advancing algorithm** AMLS for the closed contour.

$$r_n^{-1}w_{n+p} = r_{n-1}^{-1}w_{n+p-1} +$$
$$+[\tilde{y}_{n+p} - (\beta^2+\lambda)^{-1}\beta^2(y^*_{n+p} - Z^T(n)w_n)]Z(n),$$
$$r_n = r_{n-1} + a_n r_{n-1} Z(n) Z^T(n) r_{n-1},$$
$$r_{N_1} > 0,\quad a_n = [1 + Z^T(n) r_{n-1} Z(n)]^{-1}, \tag{19}$$

where the operating input of model (15) is estimated **by adaptive algorithm of control - AdAC** in the form

$$\tilde{u}_n = (\beta^2+\lambda)^{-1}\beta[y^*_{n+p} - Z^T(n)w_n]. \tag{20}$$

At the second stage from a vector of an estimation $w_{n+p}$ using the formula (19) are formed matrixes with $(\kappa \times mm)$, $(\kappa \times ml)$ and $(\kappa \times ll)$ dimensions and singular decomposition method is applied to define the separateness of vectors $a = [a_1, \cdots, a_\kappa] \in R^\kappa$ $\theta^{(1)}; \theta^{(2)}; \theta^{(3)}$,

$g = [g(1), \cdots, g(m)]^T \in R^m]$, $h = [h(1), \cdots, h(l)]^T \in R^l$ [7].

For this purpose we form matrixes

$$\Theta^{(n+p)}_{a\theta^{(1)}} = \begin{pmatrix} a_{1,n+p}\theta_1^{(n+p)}, \cdots, a_{1,n+p}\theta_{mm}^{(n+p)} \\ \vdots \\ a_{\kappa,n+p}\theta_1^{(n+p)}, \cdots, a_{\kappa,n+p}\theta_{mm}^{(n+p)} \end{pmatrix}, \tag{21}$$

$$\Theta^{(n+p)}_{a\theta^{(2)}} = \begin{pmatrix} a_{1,n+p}\theta_{mm+1}^{(n+p)}, \cdots, a_{1,n+p}\theta_{mm+ml}^{(n+p)} \\ \vdots \\ a_{\kappa,n+p}\theta_{mm+1}^{(n+p)}, \cdots, a_{\kappa,n+p}\theta_{mm+ml}^{(n+p)} \end{pmatrix}, \tag{22}$$

$$\Theta^{(n+p)}_{a\theta^{(3)}} = \begin{pmatrix} a_{1,n+p}\theta_{mm+ml+1}^{(n+p)}, \cdots, a_{1,n+p}\theta_{mm+ml+ml}^{(n+p)} \\ \vdots \\ a_{\kappa,n+p}\theta_{mm+ml+1}^{(n+p)}, \cdots, a_{\kappa,n+p}\theta_{mm+ml+ll}^{(n+p)} \end{pmatrix}. \tag{23}$$

Considering singular decomposition of a matrix $\Theta^{(n+p)}_{a\theta^{(1)}}$ $\Theta^{(n+p)}_{a\theta^{(2)}}$, $\Theta^{(n+p)}_{a\theta^{(3)}}$ the formula (21) – (23) [7], we receive following estimations:

$$\begin{aligned}
&\hat{a}_{n+p} = s_\mu \mu_{1,n+p};\quad \mu_{1,n+p} = [\mu_{1,1,n+p}, \cdots, \mu_{1,\kappa,n+p}]^T \in R^\kappa, \\
&\hat{a}_{n+p} = [\hat{a}_{1,n+p}, \cdots, \hat{a}_{\kappa,n+p}]^T \in R^\kappa, \\
&\hat{a}_{1,n+p} = s_\mu \mu_{1,1,n+p}; \cdots, \hat{a}_{\kappa,n+p} = s_\mu \mu_{1,\kappa,n+p}; \\
&\hat{\theta}^{(1),(n+p)} = s_\mu \sigma_{1,n+p} \nu_{1,n+p}; \\
&\nu_{1,n+p} = [\nu_{1,1,n+p}, \cdots, \nu_{1,mm,n+p}]^T \in R^{mm}, \\
&\hat{\theta}^{(2),(n+p)} = s_\mu \sigma_{2,n+p} \nu_{2,n+p}; \\
&\nu_{2,n+p} = [\nu_{2,mm+1,n+p}, \cdots, \nu_{2,mm+ml,n+p}]^T \in R^{ml}, \\
&\hat{\theta}^{(3),(n+p)} = s_\mu \sigma_{3,n+p} \nu_{3,n+p}; \\
&\nu_{3,n+p} = [\nu_{3,mm+ml+1,n+p}, \cdots, \nu_{3,mm+ml+ll,n+p}]^T \in R^{ll}.
\end{aligned} \tag{24}$$

It is proved, similar to [8] that estimations (24) in moyenne quadratique and on probability converge to the theoretical values.

On the third stage for an estimation of separate vectors $g = [g(1), \cdots, g(m)]^T \in R^m]$, $h = [h(1), \cdots, h(l)]^T \in R^l$ from an estimation of vectors $\hat{\theta}^{(1),(n+p)}$, $\hat{\theta}^{(2),(n+p)}$, $\hat{\theta}^{(3),(n+p)}$ from the formula (24) the following matrixes is formed

$$\Theta^{(n+p)}_{gg^T} = \begin{pmatrix} \hat{\theta}_1^{(n+p)}, \cdots, \hat{\theta}_m^{(n+p)} \\ \vdots \\ \hat{\theta}_{(m-1)m+1}^{(n+p)}, \cdots, \hat{\theta}_{mm}^{(n+p)} \end{pmatrix},$$
$$\Theta^{(n+p)}_{gh^T} = \begin{pmatrix} \hat{\theta}_{mm+1}^{(n+p)}, \cdots, \hat{\theta}_{mm+l}^{(n+p)} \\ \vdots \\ \hat{\theta}_{mm+(m-1)l+1}^{(n+p)}, \cdots, \hat{\theta}_{mm+ml}^{(n+p)} \end{pmatrix}, \tag{25}$$
$$\Theta^{(n+p)}_{hh^T} = \begin{pmatrix} \hat{\theta}_{mm+ml+1}^{(n+p)}, \cdots, \hat{\theta}_{mm+ml+l}^{(n+p)} \\ \vdots \\ \hat{\theta}_{mm+mm+(l-1)l+1}^{(n+p)}, \cdots, \hat{\theta}_{mm+ml+ll}^{(n+p)} \end{pmatrix}.$$

By applying singular decomposition of a matrix $\hat{\Theta}^{(n+p)}_{gh^T}$ from the formula (25), we shall receive following estimations:

$$\begin{aligned}
&\hat{g}_{n+p} = s_{\mu}\mu_{3,n+p};\\
&\mu_{3,n+p} = [\mu_{3,1.n+p}, \cdots, \mu_{3,m,n+p}]^T \in R^m,\\
&\hat{g}_{n+p} = [\hat{g}_{1,n+p}, \cdots, \hat{g}_{m,n+p}]^T = \in R^m,\\
&g_{1,n+p} = s_{\mu}\mu_{3,1,n+p}; \cdots, g_{m,n+p} = s_{\mu}\mu_{3,m,n+p};\\
&\hat{h}_{n+p} = [\hat{h}_{1,n+p}, \cdots, \hat{h}_{l,n+p}]^T = \in R^l,\\
&h_{1,n+p} = \sigma_{4,n+p}\nu_{4,n+p};\\
&\nu_{4,n+p} = [\nu_{4,1,n+p}, \cdots, \nu_{1,mm,n+p}]^T \in R^{mm}.
\end{aligned} \quad (26)$$

Similar to [8], it is proved, that estimations (26) in moyenne quadratique and on probability converge to the theoretical values.

Thus, Wiener-Hammerstein model is constructed for the closed contour of control for tracking the desirable value of random variables of active power and it is the following:

$$\begin{aligned}
&\tilde{\tilde{y}}_{n+p} = Z^T(n+p)w_{n+p} + \beta u_n + \rho_{n+p},\\
&n = N_1 + l + \kappa + 1, \cdots.
\end{aligned} \quad (27)$$

Results of computer modeling Ad SIC for active power of electric objects using the model (27) show, that while choosing correctly the coefficients of modification, adaptation and control results in the high accuracy of adaptive control behind tracking of desirable value of active power of electric objects [8].

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